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ADJUDICATION of the MEDALS of the ROYAL SOCIETY for the year 1895, by the PRESIDENT and COUNCIL.

The COPLEY MEDAL to KARL WEIERSTRASS, For.Mem.R.S., for his Investigations in Pure Mathematics.

A ROYAL MEDAL to JAMES ALFRED EWING, F.R.S., for his Investigations on Magnetic Induction in Iron and other Metals.

A ROYAL MEDAL to JOHN MURRAY, for his services to Biological Science and Oceanography in connection with the "Challenger" Reports, and for his Original Contributions to the same.

The DAVY MEDAL to WILLIAM RAMSAY, F.R.S., for his share in the Discovery of Argon, and for his Discoveries regarding Gaseous Constituents of Terrestrial Minerals.

The Bakerian Lecture, "On the Laws of Connexion between the Conditions of a Chemical Change and its Amount.—III. Further Researches on the Reaction of Hydrogen Dioxide and Hydrogen Iodide," was delivered by A. VERNON HARCOURT, F.R.S., and WILLIAM ESSON, F.R.S.

The Croonian Lecture, "On the Nature of Muscular Contraction," was delivered by Professor T. W. ENGELMANN.

XVII. *The Complete System of the Periods of a Hollow Vortex Ring.*

By H. C. Pocklington, *B.A., Fellow of St. John's College, Cambridge.*

Communicated by Joseph Larmor, *F.R.S.*

Received April 23,—Read May 16, 1895.

1. According to the vortex theory of matter, atoms consist of vortex rings in an infinite perfect liquid, the æther. These rings may be either hollow or filled with rotating liquid. The cross section of the hollow or rotating core is in the simplest case small and the ring is circular. Such vortices have been investigated. It has been shown that they can exist, and that they are stable for certain types of deformation. In this paper the stability of the hollow vortex ring is investigated further, with a view to proving that it is stable for all small deformations of its surface. An attempt is also made to make the vortex theory of matter agree with the kinetic theory of gases as regards the relation between the velocity and the energy of an atom. On the latter theory the energy of an atom varies as the square of its velocity, while on the former theory the energy decreases as the velocity increases. As the two theories differ on a fundamental point, while the consequences of the kinetic theory agree over a wide range with experiment, those of the vortex theory are likely to be in discrepancy therewith. However, no account has been taken of the electric change which an atom must hold if electrolysis is to be explained. This electrification will evidently alter the relation between the energy and the velocity. The nature of the change thus produced is here discussed for the case of a hollow vortex, the surface of which behaves as a conductor of electricity, a representation which is dynamically realised by the theory of a rotationally-elastic fluid æther developed in Mr. Larmor's paper,* "A Dynamical Theory of the Electric and Luminiferous Medium." The small oscillations also are worked out with a view to the discussion of the stability of an electrified vortex.

2. The velocity of translation of the vortex in its steady motion is constant and perpendicular to its plane. By impressing on the whole liquid a velocity equal and opposite to this, the hollow is reduced to rest. Since the cross section of the hollow is small, any small length of it may be regarded as cylindrical. A cylindrical vortex must, by reason of symmetry, have its cross section a circle, so that the cross section

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* J. Larmor, 'Phil. Trans.,' 1894, A, p. 719.

of the hollow of the annular vortex is approximately circular, and the hollow itself approximately a tore.

3. The motion of the fluid is referred to the toroidal coordinates u, v, w, where u, v are given in terms of the cylindrical coordinates ϖ, z, by the equation

$$u + \iota v = \log \frac{\varpi + \iota z + a}{\varpi + \iota z - a} \quad \ldots \ldots \ldots \quad (1),$$

and w is the angular cylindrical coordinate.

The constant a and the position of the axes are so chosen that the hollow is approximately given by $u =$ const.

On solving (1) for $z + \iota\varpi$, and equating real and imaginary parts,

$$\varpi = a \frac{\sinh u}{\cosh u - \cos v} = a \frac{\mathrm{S}}{\mathrm{C} - c} \quad \ldots \ldots \ldots \quad (2),$$

$$z = -a \frac{\sin v}{\cosh u - \cos v} = -a \frac{s}{\mathrm{C} - c} \quad \ldots \ldots \ldots \quad (3),$$

where C, c, S, s are written for $\cosh u$, $\cos v$, $\sinh u$, $\sin v$ respectively.

The square root of the Jacobian of u, v with respect to ϖ, z is ξ where

$$\xi^2 = \frac{1}{\left(\frac{d\varpi}{du}\right)^2 + \left(\frac{d\varpi}{dv}\right)^2} = \frac{(\cosh u - \cos v)^2}{a^2},$$

or,

$$\xi = \frac{\mathrm{C} - c}{a}.$$

If k be put for e^{-u}, k is small near the surface of the hollow, and the values of ϖ, z, ξ are

$$\left.\begin{aligned} \varpi &= a\,(1 + 2k \cos v) \\ z &= 2ak \sin v \\ \xi &= \frac{1 - 2k \cos v}{2ka} \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (4),$$

in which expression the squares and higher powers of k have been neglected.

4. The motion of the liquid, being symmetrical about the axis of the ring, may be expressed in terms of STOKES' current function ψ. Now ψ consists of two terms. One is $-\frac{1}{2}\mathrm{V}\varpi^2$, corresponding to a uniform motion of the liquid with velocity V. The other vanishes at an infinite distance from the ring and, therefore, can be expressed

in the form* $\{2(\mathrm{C} - c)\}^{-\frac{1}{2}}\Sigma \mathrm{A}_n \mathrm{R}_n \cos nv$, where $\mathrm{R}_n = \mathrm{S}\dfrac{d\mathrm{P}_n}{du}$ and P_n is that toroidal function of order n, which is finite when $u = 0$.

Thus,

$$\psi = -\tfrac{1}{2}\mathrm{V}\varpi^2 + \frac{1}{\sqrt{2(\mathrm{C} - c)}}\Sigma \mathrm{A}_n \mathrm{R}_n \cos nv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

The first term under the sign Σ corresponds to a circulation about the critical circle $k = 0$. The succeeding terms correspond partly to the disturbance of this circulation by the free surface, and partly to the disturbance of the uniform flow $\psi = -\frac{1}{2}\mathrm{V}\varpi^2$ by the same surface. Now since the annulus is nearly coincident with a flow surface of the circulation, and since V, as is easily seen from elementary considerations, is small, these terms are small. Their products by small quantities may therefore be neglected.

Hence, expanding by (4),

$$\psi = -\tfrac{1}{2}\mathrm{V}a^2(1 + 4k\cos v) + (1 + k\cos v)k^{\frac{1}{2}}\{\mathrm{A}_0\mathrm{R}_0 + \mathrm{A}_1\mathrm{R}_1\cos v + \text{etc}\}. \quad (6).$$

Since $k = b$ is a flow surface, $\psi = \text{const.}$ when $k = b$, or

$$\text{const.} = \mathrm{A}_0\mathrm{R}'_0 b^{\frac{1}{2}} - \tfrac{1}{2}\mathrm{V}a^2 + (\mathrm{A}_1\mathrm{R}'_1 b^{\frac{1}{2}} + \mathrm{A}_0\mathrm{R}'_0 b^{\frac{3}{2}} - 2\mathrm{V}a^2 b)\cos v + \mathrm{A}_2\mathrm{R}'_2 b^{\frac{1}{2}}\cos 2v + \text{etc.},$$

where R'_s means the value of R_s when k is put equal to b.

Hence, equating to zero the coefficients of $\cos v$, $\cos 2v$, etc.,

$$\mathrm{A}_2 = \mathrm{A}_3 = 0, \qquad \mathrm{A}_1 = \frac{2\mathrm{V}a^2 b^{\frac{1}{2}} - \mathrm{A}_0\mathrm{R}'_0 b}{\mathrm{R}'_1}.$$

Also†

$$\mathrm{R}_0 = -(\tfrac{1}{2}\mathrm{L} - 1)k^{-\frac{1}{2}}, \qquad \mathrm{R}_1 = \tfrac{1}{2}k^{-\frac{3}{2}},$$

where $\mathrm{L} = \log 4/k$, whence, substituting,

$$\mathrm{A}_1 = 4\mathrm{V}a^2b^2 + (\mathrm{L}' - 2)b^2.$$

Substituting the value of A_1 in (6),

$$\psi = -\tfrac{1}{2}\mathrm{V}a^2 - \mathrm{A}_0\frac{\mathrm{L} - 2}{2} + \left\{\frac{2\mathrm{V}a^2b^2}{k} - 2\mathrm{V}a^2k + \mathrm{A}_0\frac{\mathrm{L}' - 2}{2k}b^2 - \mathrm{A}_0\frac{\mathrm{L} - 2}{2}k\right\}\cos v \quad (7).$$

The fluid circulation is the line integral of the velocity taken along any curve enclosing the hollow. Taking the curve to be $k = b$, and putting for the velocity its value $\frac{\xi}{\varpi}\frac{d\psi}{du}$, we have

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* W. M. Hicks, 'Phil. Trans.,' 1884.

† W. M. Hicks, 'Phil. Trans.,' 1884, p. 172.

$$\text{circulation} = \mu = \int_0^{2\pi} \frac{dv}{\xi}\,\frac{\xi}{\varpi}\,\frac{d\psi}{du} = -\int_0^{2\pi} dv\,\frac{\mathrm{C} - \cos v}{\mathrm{S}a}\,b\,\frac{d\psi}{dk}$$

$$= -\int_0^{2\pi} dv\,\frac{\mathrm{C} - \cos v}{\mathrm{S}a}\,b\left[\frac{\mathrm{A}_0}{2b} - \left\{4\mathrm{V}a^2 + \mathrm{A}_0(\mathrm{L}' - 2) - \frac{\mathrm{A}_0}{2}\right\}\cos v\right]$$

$$= -\pi\mathrm{A}_0/a,$$

neglecting quantities of the second order.

Hence

$$\mathrm{A}_0 = -\mu a/\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

5. The motion of the liquid is now completely expressed in terms of μ and V by (7) and (8). We proceed to find the energy of the fluid motion and, with the object of writing down the condition that is satisfied at the free surface, the pressure of the liquid at the surface.

If ψ' is the current function that gives the motion of the liquid when the reversed velocity V is not impressed on the liquid, so that $\psi' = \frac{1}{2}\mathrm{V}\varpi^2 + \psi$, the energy of the actual motion is

$$\tfrac{1}{2}\rho\iint 2\pi\varpi\,d\varpi\,dz\,\frac{1}{\varpi^2}\left\{\left(\frac{d\psi'}{d\varpi}\right)^2 + \left(\frac{d\psi'}{dz}\right)^2\right\}$$

$$= \pi\rho\left[\int ds\,\frac{\psi'}{\varpi}\,\frac{d\psi'}{dn} - \iint d\varpi\,dz\,\psi'\left\{\frac{d}{d\varpi}\left(\frac{1}{\varpi}\,\frac{d\psi'}{d\varpi}\right) + \frac{1}{\varpi}\,\frac{d^2\psi'}{dz^2}\right\}\right],$$

by integration by parts, where, in the double integrals, ϖ and z are to take all values corresponding to space occupied by the liquid, and in the single integral, ds is an element of length of the cross-section of the boundary, dn the element of normal to the boundary, taken as positive when drawn from the liquid, and the integral is taken round the cross-section of the boundary. The double integral in the latter expression vanishes by the equation satisfied by ψ in an irrotational motion, and thus

$$\text{energy} = \pi\rho\int ds\,\frac{\psi'}{\varpi}\,\frac{d\psi'}{dn} = \pi\rho\int_0^{2\pi}\frac{dv}{\xi}\,\frac{\psi'}{\varpi}\,\xi\,\frac{d\psi'}{du},$$

$$= -\pi\rho\int_0^{2\pi} dv\,b\,\frac{1 - 2b\cos v}{a}\left\{\frac{\mathrm{A}_0}{2b} - \left(2\mathrm{V}a^2 + \mathrm{A}_0(\mathrm{L}' - 2) - \frac{\mathrm{A}_0}{2}\right)\cos v\right\}$$

$$\left\{-\mathrm{A}_0\,\frac{\mathrm{L}' - 2}{2} + 2\mathrm{V}a^2b\cos v\right\}$$

$$= \frac{\pi^2\rho\mathrm{A}_0^2(\mathrm{L}' - 2)}{2a} = \tfrac{1}{2}\rho a\mu^2(\mathrm{L}' - 2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9),$$

by (8).

If the pressure at any point of the liquid is Π, and at infinity is Π_∞,

$$\Pi = \Pi_\infty - \tfrac{1}{2}\rho q^2,$$

where q is the resultant velocity.

The components of the velocity perpendicular to the surfaces $v =$ const. and $u =$ const. are $\frac{\xi}{\varpi}\frac{d\psi}{du}$, $\frac{\xi}{\varpi}\frac{d\psi}{dv}$, respectively. But $\frac{d\psi}{dv} = 0$ on the flow surface $k = b$. Hence

$$q^2 = \frac{\xi^2}{\varpi^2}\left(\frac{d\psi}{du}\right)^2 = \frac{(\mathrm{C} - c)^4}{a^4\mathrm{S}^2}\,k^2\left(\frac{d\psi}{dk}\right)^2,$$

$$= \frac{1 - 8k\cos v}{4a^4}\left[\frac{\mathrm{A}_0}{2k} - \left\{\frac{2\mathrm{V}a^2b^2}{k^2} + \mathrm{A}_0\frac{(\mathrm{L}' - 2)\,b^2}{2k^2} + \mathrm{A}_0\frac{\mathrm{L} - 2}{2} - \frac{\mathrm{A}_0}{2} + 2\mathrm{V}a^2\right\}\cos v\right]^2,$$

or, putting $k = b$,

$$q^2 = \frac{\mathrm{A}_0^{\,2}}{16a^4b^2}\left\{1 - 4b\left(\mathrm{L}' - \tfrac{1}{2} - \frac{4\mathrm{V}a^2}{\mathrm{A}_0}\right)\cos v\right\},$$

$$= \frac{\mu^2}{16\pi^2a^2b^2}\left\{1 - 4b\left(\mathrm{L}' - \tfrac{1}{2} - \frac{4\pi\mathrm{V}a}{\mu}\right)\cos v\right\},$$

and thus

$$\Pi = \Pi_\infty - \frac{\mu^2\rho}{32\pi^2a^2b^2}\left\{1 - 4b\left(\mathrm{L}' - \tfrac{1}{2} - \frac{4\pi\mathrm{V}a}{\mu}\right)\cos v\right\} \quad . \quad . \quad . \quad (10).$$

The pressure is zero at the free surface, so that

$$\frac{\mu^2\rho}{32\pi^2a^2b^2} = \Pi\,, \qquad \frac{4\pi\mathrm{V}a}{\mu} = \mathrm{L}' - \tfrac{1}{2},$$

or

$$\mathrm{V} = \frac{\mu}{4\pi a}\left\{\log\frac{16\pi a\sqrt{(2\Pi_\infty)}}{\mu\sqrt{\rho}} - \tfrac{1}{2}\right\},$$

and thus V diminishes with increase of a.

6. To find the small oscillations we assume that the equation to the surface of the hollow when it vibrates is at time t

$$k = b\,\{1 + \beta e^{\iota(nv + mw + pt)}\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11),$$

where $\iota = \sqrt{(-1)}$ and m and n are positive integers, β being small. We then determine the corresponding disturbed motion of the liquid, and finally write down the condition that the pressure at the surface of the liquid is zero. If the resulting equation can be satisfied by any value of p, the assumption as to the form of the disturbed surface is justified, and $2\pi/p$ is the period of the oscillation. The displacement of the surface given by (11) is complex. Noting, however, that the signs of ι and of w may be changed, other possible equations to the surface are seen to be

$$k = b\{1 + \beta e^{\iota(-nv+mw-pt)}\},$$
$$k = b\{1 + \beta e^{\iota(nv-mw+pt)}\},$$
$$k = b\{1 + \beta e^{\iota(-nv-mw-pt)}\},$$

in which equations p has the same value as it has in (11), and is here supposed to be real. The displacements, being small, may be added and subtracted, so that

$$\left.\begin{aligned} k &= b\{1 + \beta \sin(nv + pt) \sin mw\}, \\ k &= b\{1 + \beta \sin(nv + pt) \cos mw\}, \\ k &= b\{1 + \beta \cos(nv + pt) \sin mw\}, \\ k &= b\{1 + \beta \cos(nv + pt) \cos mw\}, \end{aligned}\right\} \quad \ldots \ldots \quad (12)$$

are possible equations for the surface of the hollow at time t. These equations no longer contain imaginaries, and thus they correspond to real oscillations. If m is zero, the disturbance of the surface is symmetrical about the axis of the vortex, and consists of trains of waves, the crests of which are circles parallel to the critical circle, progressing round the hollow. These vibrations will be called the fluted vibrations of the vortex. If n is zero, the disturbance consists of alternate swellings and contractions of the hollow disposed at equal intervals along its length. These vibrations will be called the beaded vibrations of the vortex. As given by (12) they are standing waves. On adding or subtracting the disturbances given by the first and last or the second and third of these equations, beaded vibrations are obtained which progress along the length of the hollow. If both m and n are zero, the disturbance consists of a uniform periodic enlargement of the vortex. This vibration will be called the pulsation of the vortex. If n is unity, the disturbance consists of a displacement of the axis of the hollow, without alteration of its cross section. These vibrations will be called the sinuous vibrations of the vortex.

7. The motion of the liquid is that which subsists when there is no vibration, together with small terms introduced by the vibration. The former has already been expressed in terms of a stream function ψ given by (7). Let the corresponding velocity potential be ϕ_1. The terms due to the vibration must have an acyclic velocity potential. Let this potential be ϕ_2. Since ϕ_2 vanishes at infinity it can be expressed in a series of the form* $D\sqrt{2(C-c)}\ {}_mP_n\ e^{\iota(nv+mw+pt)}$, where ${}_mP_n$ is the associated toroidal function of order n and rank m, so that

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* W. M. Hicks, 'Phil. Trans.,' 1881.

$$ {}_m\mathrm{P}_n = \int_0^\pi \frac{\cos m\theta \, d\theta}{(\mathrm{C} - \mathrm{S}\cos\theta)^{(2n+1)/2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13). $$

We shall assume, our assumption to be justified by the result, that ϕ_2 reduces to the single term $\mathrm{D}\sqrt{2(\mathrm{C}-c)}\,{}_m\mathrm{P}_n\, e^{\iota(nv+mw+pt)}$. Since D is small, we may neglect all but the lowest powers of k in this expression, and thus

$$ \phi_2 = \mathrm{D}\frac{{}_m\mathrm{P}_n}{\sqrt{k}} e^{\iota(nv+mw+pt)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14). $$

The value of D must now be found from the condition that the velocity of the free surface normal to itself is equal to the velocity of the liquid normal to the surface. The velocity of the surface normal to itself is

$$ \begin{aligned} -\frac{1}{\xi}\frac{du}{dt} &= \frac{1}{\xi k}\frac{dk}{dt} \\ &= 2pab\iota\beta e^{\iota(nv+mw+pt)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15) \end{aligned} $$

from (11).

The components of the velocity due to ψ are ;

$$ \left. \begin{array}{l} \text{perpendicular to } u = \text{const.,} \\ \qquad\qquad \dfrac{\xi}{\varpi}\dfrac{d\psi}{dv} = 0 \text{ to our order ;} \\ \text{perpendicular to } v = \text{const.,} \\ \dfrac{\xi}{\varpi}\dfrac{d\psi}{du} \\ \quad = -\dfrac{1-4k\cos v}{2a^2k}\, k\left\{\dfrac{\mathrm{A}_0}{2k} + \left(\dfrac{\mathrm{A}_0}{2} - \mathrm{A}_0\dfrac{\mathrm{L}-2}{2} - \mathrm{A}_0\dfrac{\mathrm{L}'-2}{2}\dfrac{b^2}{k^2} - 2\mathrm{V}a^2 - \dfrac{2\mathrm{V}a^2b^2}{k^2}\right)\cos v\right\}, \\ \text{or at the surface given by (11),} \\ \quad = -\dfrac{1-4b\cos v}{2a^2}\left\{\dfrac{\mathrm{A}_0}{2b} - \dfrac{\mathrm{A}_0}{2b}\beta e^{\iota(nv+mw+pt)} + \left(\dfrac{\mathrm{A}_0}{2} - \mathrm{A}_0(\mathrm{L}-2) - 4\mathrm{V}a^2\right)\cos v\right\} \\ \qquad\qquad = \dfrac{\mu}{4\pi ab}\left\{1 - \beta e^{\iota(nv+mw+pt)} - 2b\left(\mathrm{L}' - \tfrac{1}{2} - \dfrac{4\pi\mathrm{V}a}{\mu}\right)\cos v\right\} ; \\ \text{perpendicular to } w = \text{const.,} \\ \qquad\qquad\qquad\qquad 0 \end{array} \right\} \quad (16). $$

The velocities here, as elsewhere, are considered to be positive if measured in the direction of increase of k, v, w.

The components of velocity given by ϕ_2 are ;

$$\left.\begin{aligned}
&\text{perpendicular to } u = \text{const.,}\\
&\qquad -\xi\frac{d\phi_2}{du} = \frac{\mathrm{D}}{2a}\frac{d}{db}\left(\frac{{}_m\mathrm{P}_n'}{\sqrt{b}}\right)e^{\iota(nv+mw+pt)}\,;\\
&\text{perpendicular to } v = \text{const.,}\\
&\qquad \xi\frac{d\phi_2}{dv} = \frac{\iota n\mathrm{D}}{2ab}\,\frac{{}_m\mathrm{P}_n'}{\sqrt{b}}\,e^{\iota(nv+mw+pt)}\,;\\
&\text{perpendicular to } w = \text{const.,}\\
&\qquad \frac{1}{\varpi}\frac{d\phi_2}{dw} = \frac{\iota m\mathrm{D}}{a}\,\frac{{}_m\mathrm{P}_n'}{\sqrt{b}}\,e^{\iota(nv+mw+pt)}.
\end{aligned}\right\} \quad . \quad . \quad . \quad (17).$$

The direction cosines of the normal to the surface are

$$1, \quad -\iota n\beta e^{\iota(nv+mw+pt)}, \quad -2\iota m\beta e^{\iota(nv+mw+pt)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18).$$

Hence the velocity of the liquid normal to the surface is, from (16), (17), (18),

$$-\frac{\mu}{4\pi ab}\,\iota n\beta e^{\iota(nv+mw+pt)} + \frac{\mathrm{D}}{2a}\frac{d}{db}\left(\frac{{}_m\mathrm{P}_n'}{\sqrt{b}}\right)e^{\iota(nv+mw+pt)},$$

neglecting terms small compared with those retained.

This is equal to the velocity of the surface normal to itself, so that from (15),

$$-\frac{\mu}{4\pi ab}\,\iota n\beta e^{\iota(nv+mw+pt)} + \frac{\mathrm{D}}{2a}\frac{d}{db}\left(\frac{{}_m\mathrm{P}_n'}{\sqrt{b}}\right)e^{\iota(nv+mw+pt)} = 2pab\iota\beta e^{\iota(nv+mw+pt)},$$

which gives

$$\mathrm{D} = 2\left(\frac{\mu n}{4\pi} + 2pa^2b^2\right)\frac{\iota\beta}{b\,\dfrac{d}{db}\left(\dfrac{{}_m\mathrm{P}_n'}{\sqrt{b}}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19).$$

Substituting from this result and (16) and (17), the components of the velocity of the liquid at the surface of the hollow become;

perpendicular to u = const.,

$$\frac{1}{ab}\left(\frac{\mu n}{4\pi} + 2pa^2b^2\right)\iota\beta e^{\iota(nv+mw+pt)}\,;$$

perpendicular to v = const.,

$$\frac{\mu}{4\pi ab}\left\{1 - \beta e^{\iota(nv+mw+pt)} - 2b\left(\mathrm{L}' - \tfrac{1}{2} - \frac{4\pi\mathrm{V}a}{\mu}\right)\cos v\right\} - \frac{n\lambda}{ab}\left(\frac{\mu n}{4\pi} + 2pa^2b^2\right)\beta e^{\iota(nv+mw+pt)}\,;$$

perpendicular to $w =$ const.,

$$-\frac{m\lambda}{a}\left(\frac{\mu n}{4\pi} + 2pa^2b^2\right)\beta\,;$$

where

$$\lambda = \frac{{}_m\mathrm{P}_n'/\sqrt{b}}{b\dfrac{d}{db}\left(\dfrac{{}_m\mathrm{P}_n'}{\sqrt{b}}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20).$$

The equation giving the pressure is

$$\Pi = \Pi_\infty - \tfrac{1}{2}\rho q^2 - \rho\,\frac{d}{dt}(\phi_1 + \phi_2).$$

Since ϕ_1 is independent of t, on substituting from (14) and (19), and equating the pressure to zero,

$$0 = \Pi = \Pi_\infty - \frac{\mu^2\rho}{32\pi^2a^2b^2}\left\{1 - 4b\left(\mathrm{L}' - \tfrac{1}{2} - \frac{4\pi\mathrm{V}a}{\mu}\right)\right.$$
$$\left. - 2\left(1 + n^2\lambda + \frac{8\pi a^2b^2n\lambda p}{\mu}\right)\beta e^{\iota(nv+mw+pt)}\right\} + \rho\lambda p\left(\frac{\mu n}{2\pi} + 4pa^2b^2\right)\beta e^{\iota(nv+mw+pt)} \; (21).$$

In this equation the terms not involving the exponential cancel by the equations defining the steady motion. The equation can therefore be satisfied by some value of p, which fact justifies the original assumption as to the nature of the disturbance of the surface. On dividing by $\beta e^{\iota(nv+mw+pt)}$ we find as equation for p,

$$\frac{\mu^2\rho}{16\pi^2a^2b^2}\left(1 + n^2\lambda + \frac{8\pi a^2b^2n\lambda p}{\mu}\right) + p\lambda\rho\left(\frac{\mu n}{2\pi} + 4pa^2b^2\right) = 0,$$

or

$$p^2 + 2pn\frac{\mu}{8\pi a^2b^2} + \left(n^2 + \frac{1}{\lambda}\right)\left(\frac{\mu}{8\pi a^2b^2}\right)^2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (22).$$

8. It now remains to find the value of λ, defined by (20).

Now

$${}_m\mathrm{P}_n' = \int_0^\pi \frac{\cos m\theta\, d\theta}{(\mathrm{C} - \mathrm{S}\cos\theta)^{(2n+1)/2}},$$

or, expressing in terms of k and remembering that k is small,

$${}_m\mathrm{P}_n' = (2k)^{(2n+1)/2}\int_0^\pi \frac{\cos m\theta\, d\theta}{\{1 - \cos\theta + k^2(1 + \cos\theta)\}^{(2n+1)/2}}$$
$$= (4k)^{(2n+1)/2}\int_0^\epsilon \frac{d\theta}{(\theta^2 + 4k^2)^{(2n+1)/2}} + (2k)^{(2n+1)/2}\int_\epsilon^\pi \frac{\cos m\theta\, d\theta}{(1 - \cos\theta)^{(2n+1)/2}} \quad . \quad . \quad (23),$$

where ϵ is small absolutely, but yet large compared with k. If n is not zero, the latter integral can be neglected in comparison with the former, and the limits of the former can be taken to be 0 and ∞, so that

$$ {}_m\mathrm{P}_n = (4k)^{(2n+1)/2}\int_0^\infty \frac{d\theta}{(\theta^2+4k^2)^{(2n+1)/2}}\,. $$

Now

$$ \int_0^\infty \frac{d\theta}{(\theta^2+\alpha)^{(2n+1)/2}} = \frac{(-1)^{n-1}\left(\dfrac{d}{d\alpha}\right)^{n-1}}{\frac{3}{2}\cdot\frac{5}{2}\cdots\dfrac{2n-1}{2}}\int_0^\infty \frac{d\theta}{(\theta^2+\alpha)^{\frac{3}{2}}} $$

$$ \propto \left(\frac{d}{d\alpha}\right)^{n-1}\left[\frac{1}{\alpha}\,\frac{\theta}{\sqrt{\theta^2+\alpha}}\right]_0^\infty $$

$$ \propto \frac{1}{\alpha^n}\,. $$

Hence, in the case where n is not zero, ${}_m\mathrm{P}_n$ varies as $k^{-(2n-1)/2}$ when k is small, so that

$$ \lambda = -1/n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24). $$

If n is zero, from (23)

$$ {}_m\mathrm{P}_0 = (4k)^{\frac{1}{2}}\int_0^\epsilon \frac{d\theta}{\sqrt{\theta^2+4k^2}} + k^{\frac{1}{2}}\int_\epsilon^\pi \frac{\cos m\theta\, d\theta}{\sin(\theta/2)} $$

$$ = (4k)^{\frac{1}{2}}\log\frac{\epsilon}{k} + 2k^{\frac{1}{2}}\log\frac{4}{\epsilon} - k^{\frac{1}{2}}\int_0^\pi \frac{1-\cos m\theta}{\sin(\theta/2)}\,d\theta. $$

Calling the last integral γ_m we have

$$ \gamma_{m+1} - \gamma_m = 2\int_0^\pi d\theta \sin\frac{2m+1}{2}\,\theta = \frac{4}{2m+1}\,. $$

Also $\gamma_0 = 0$, therefore

$$ \gamma_m = 4\left(1+\tfrac{1}{3}+\tfrac{1}{5}+\ldots+\frac{1}{2m-1}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (25), $$

and

$$ {}_m\mathrm{P}_0 = 2k^{\frac{1}{2}}\log\frac{4}{k} - k^{\frac{1}{2}}\gamma_m, $$

giving

$$ \lambda = -(\mathrm{L}' - \tfrac{1}{2}\gamma_m) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26). $$

9. In the preceding paragraph, $\cos m\theta$ was put equal to unity in the integrals taken from 0 to ϵ. If m is so large that $m\epsilon$ is finite, this step is no longer valid. In this case, however, the second integral of (23) vanishes in comparison with the first, and the upper limit of the first may be taken to be infinity, so that

$$ {}_m\mathrm{P}_n = (4k)^{(2n+1)/2}\int_0^\infty \frac{\cos n\theta\, d\theta}{(\theta^2+4k^2)^{(2n+1)/2}} = \frac{2}{k^{(2n-1)/2}}\int_0^\infty \frac{\cos 2m\chi\, d\chi}{(1+\chi^2)^{(2n+1)/2}}, $$

and

$$ \frac{{}_m\mathrm{P}_n}{\sqrt{k}} = \frac{2}{k^n}\int_0^\infty \frac{\cos 2m\chi\, d\chi}{(1+\chi^2)^{(2n+1)/2}}, $$

a Bessel's function of the second kind of order n with imaginary argument. Denoting this function by the symbol K_n,

$$ \frac{{}_m\mathrm{P}_n}{\sqrt{k}} \propto \mathrm{K}_n(2\,mk), \qquad \lambda = \frac{\mathrm{K}_n(2mb)}{2mb\mathrm{K}_n'(2mb)} \quad . \; . \; . \; . \; . \; . \; . \quad (27). $$

10. Substituting in (22) the value of λ given by (24), (26), (27), we have as equations for p,
if m is finite and n not zero,

$$ p^2 + 2pn\frac{\mu}{8\pi a^2b^2} + n(n-1)\left(\frac{\mu}{8\pi a^2b^2}\right)^2 = 0 \quad . \; . \; . \; . \; . \; . \quad (28); $$

if m is finite and n zero,

$$ p^2 = \frac{1}{\mathrm{L}' - \frac{1}{2}\gamma_m}\left(\frac{\mu}{8\pi a^2b^2}\right)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (29); $$

if m is great, whatever n may be,

$$ p^2 + 2pn\frac{\mu}{8\pi a^2b^2} + \left\{n^2 + \frac{2mb\mathrm{K}_n'(2mb)}{\mathrm{K}_n(2mb)}\right\}\left(\frac{\mu}{8\pi a^2b^2}\right)^2 = 0 \quad . \; . \; . \quad (30). $$

The first of these equations agrees, in the case of $m = 0$, *i.e.*, in the case of fluted vibrations, with that obtained by Mr. Basset,[*] and the second, in the case of $m = 0$, *i.e.*, in the case of pulsations, with that obtained by Mr. Hicks.[†] The first of these equations evidently gives in all cases real values of p. The second also does so, for L' is large and γ_m finite. The third also gives real values of p, for, by a property of the function K_n, easily proved from its definition as a definite integral, K_n is positive and its differential coefficient negative for real positive values of its argument. Hence all the values of p are real. One, however, given by the first of these equations when $n = 1$, is zero. This shows that the corresponding value of p is not, as are the other values of p, of the second order of large quantities, but of some lower order. These equations, then, show that the hollow vortex is stable for all displacements of its surface, with the possible exception of such as involve sinuous vibrations of the slow type. These latter vibrations have been investigated by Professor Thomson,[‡] who proves that the vortex is stable for sinuous displacements, and finds the periods of the sinuous vibrations.

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[*] A. B. Basset, 'Hydrodynamics,' vol. 2, p. 92.

[†] W. M. Hicks, 'Phil. Trans.,' 1884.

[‡] J. J. Thomson, 'Phil. Trans.,' 1882, and "Motion of Vortex Rings," Part I.

11. The electrified vortex. If the vortex carries a charge of electricity on its surface, the surface condition is altered. The effect of the electricity is to cause a tension to act on the surface which at any point is $F^2/8\pi$ per unit area, where F is the electric force. This tension must be balanced by the pressure of the liquid. Hence the surface condition is

$$\Pi = F^2/8\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31).$$

The electric state of the system is defined by the electric potential ϕ. Since ϕ vanishes at infinity, and is an even function of v, its expansion in a series of toroidal harmonics must be*

$$\phi = \sqrt{2(C - c)}\, \Sigma B_n P_n \cos nv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32).$$

The first term in the series is the potential, due to a uniform line distribution on the critical circle $k = 0$. If there were exact coincidence between the surface of the hollow and some equipotential of this distribution, the terms after the first would vanish. As an approximate coincidence subsists, these terms will be small, and their products by small quantities may be neglected.

Expanding (32),

$$\phi = B_0P_0k^{-\frac{1}{2}} + (B_1k^{-\frac{1}{2}}P_1 - B_0k^{\frac{1}{2}}P_0) \cos v + B_2P_2k^{-\frac{1}{2}} \cos 2v + \&c. \quad . \quad (33).$$

Putting $k = b$, ϕ is equal to the constant potential ϕ_0 of the vortex,

$$\phi_0 = B_0P_0'b^{-\frac{1}{2}} + (B_1b^{-\frac{1}{2}}P_1' - B_0b^{\frac{1}{2}}P_0') \cos v + B_2P_2b^{-\frac{1}{2}} \cos 2v + \&c.,$$

so that

$$\phi_0 = B_0P_0'b^{-\frac{1}{2}}, \quad B_1 = B_0 \frac{bP_0'}{P_1'}$$
$$B_2 = B_3 = \ldots = 0.$$

Also,† as

$$P_0 = 2Lk^{\frac{1}{2}}, \qquad P_1 = 2k^{-\frac{1}{2}},$$

to our order, we have

$$B_0 = \frac{\phi_0}{2L'}, \qquad B_1 = \frac{b^2}{2}\phi_0.$$

Substituting these values in (33),

$$\phi = \phi_0 \frac{L}{L'} + \phi_0 k \left(\frac{b^2}{k^2} - \frac{L}{L'}\right) \cos v.$$

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* W. M. Hicks, 'Phil. Trans.,' 1881, p. 618.

† W. M. Hicks, 'Phil. Trans.,' 1884, p. 171.

The force normal to $u =$ const. is thus

$$- \xi k \frac{d\phi}{dk} = \frac{1 - 2k \cos v}{2ak} k \left\{ \phi_0 \frac{1}{k\mathrm{L}'} + \phi_0 \left(\frac{b^2}{k^2} - \frac{1}{\mathrm{L}'} + \frac{\mathrm{L}}{\mathrm{L}'} \right) \cos v \right\}$$

$$= \frac{\phi_0}{2ak\mathrm{L}'} \left\{ 1 + k \left(\frac{b^2}{k^2} \mathrm{L}' + \mathrm{L} - 3 \right) \cos v \right\} \cdot$$

As the force normal to $v =$ const. vanishes on the equipotential $k = b$, the resultant force F is the value when $k = b$ of the above,

$$\mathrm{F} = \frac{\phi_0}{2ab\mathrm{L}'} \{ 1 + b(2\mathrm{L}' - 3) \cos v \} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34).$$

The charge on the vortex is

$$\mathrm{E} = \int_0^{2\pi} \frac{dv}{\xi} 2\pi\varpi \frac{\mathrm{F}}{4\pi},$$

$$= \int_0^{2\pi} \frac{dv}{2\mathrm{L}'} \phi_0 a (1 + 4k \cos v) \{ 1 + b(2\mathrm{L}' - 3) \cos v \},$$

$$= \frac{\pi a \phi_0}{\mathrm{L}'},$$

or,

$$\phi_0 = \frac{\mathrm{L}'}{\pi a} \mathrm{E},$$

so that, substituting in (34),

$$\mathrm{F} = \frac{\mathrm{E}}{2\pi a^2 b} \{ 1 + b(2\mathrm{L}' - 3) \cos v \} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35).$$

The electric energy is

$$\tfrac{1}{2} \phi_0 \mathrm{E} = \frac{\mathrm{L}'}{2\pi a} \mathrm{E}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36).$$

12. We are now in a position to write down the equation giving the steady motion of an electrified vortex. Equating the value of Π, given by (10), to that of $\mathrm{F}^2/8\pi$, given by (35), we have

$$\frac{\mathrm{E}^2}{32\pi^3 a^4 b^2} \{ 1 + 2b(2\mathrm{L}' - 3) \cos v \} = \Pi_\infty - \frac{\mu^2 \rho}{32\pi^2 a^2 b^2} \left\{ 1 - 4b \left(\mathrm{L}' - \tfrac{1}{2} - \frac{4\pi \mathrm{V} a}{\mu} \right) \cos v \right\},$$

which gives, on equating separately the terms involving v and those independent of v,

$$\mathrm{E}^2 + \pi \rho a^2 \mu^2 = 32\pi^3 a^4 b^2 \Pi_\infty \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37),$$

$$\mathrm{V} = \frac{\mu}{4\pi a} \left\{ \mathrm{L}' - \tfrac{1}{2} - \frac{\mathrm{E}^2}{\pi \rho a^2 \mu^2} (\mathrm{L}' - \tfrac{3}{2}) \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38).$$

The energy of the vortex is the sum of three terms. The first is the energy of the electricity, found in (36). The second is the kinetic energy of the liquid, found in (9). The third is the potential energy due to the presence of the hollow. Since the hollow could, if the liquid were brought to rest and the electricity removed, be filled by liquid flowing into it at pressure Π_∞, the last term is the product of Π_∞ and the volume of the hollow. Now, from (4), if k is small we have

$$(\varpi - a)^2 + z^2 = 4a^2k^2,$$

so that the radius of the cross-section of the ring is $2ab$, and the distance of the centre of the cross-section from the centre of the ring is a. Thus the volume of the hollow is $8\pi^2a^3b^2$, and the total energy on substituting for Π_∞ from (37) is

$$\begin{aligned} \mathrm{U} &= \tfrac{1}{2}\rho a\mu^2(\mathrm{L}' - 2) + \frac{\mathrm{L}'}{2\pi a}\mathrm{E}^2 + 8\pi^2a^3b^2\,\frac{\mathrm{E}^2 + \pi\rho a^2\mu^2}{32\pi^3a^4b^2} \\ &= \frac{\mathrm{E}^2}{2\pi a}(\mathrm{L}' + \tfrac{1}{2}) + \frac{\rho a\mu^2}{2}(\mathrm{L}' - \tfrac{3}{2}). \qquad (39). \end{aligned}$$

Equations (37), (38), (39) give all desired information about the steady motion of an electrified vortex. The first effect of electrification is to diminish the velocity of the vortex. If $\mathrm{E}^2/\pi\rho a^2\mu^2 = (\mathrm{L}' - \tfrac{1}{2})\,/\,(\mathrm{L}' - \tfrac{3}{2})$, V vanishes, while for values of E greater than that given by this equation, V is negative and the vortex moves backwards. This state of rest of the vortex may evidently be obtained as well by decreasing a as by increasing E. Hence an electrified vortex ring, however small the charge may be, will be reduced to rest if its radius is reduced sufficiently, and if its radius is further reduced, will move backwards, *i.e.*, in a direction opposed to that of the motion of the fluid at the centre of the ring.

Since

$$\frac{d\mathrm{U}}{da} = \frac{\rho\mu^2}{2}(\mathrm{L}' - \tfrac{1}{2}) - \frac{\mathrm{E}^2}{2\pi a^2}(\mathrm{L}' - \tfrac{3}{2}) = 2\pi a\mu\mathrm{V},$$

the energy of the vortex is least when its radius is such that it is stationary. If a is nearly equal to this value the difference between a constant and the energy of the vortex varies as the square of the velocity. This relation is only approximate and does not hold if a differs much from the value that makes $\mathrm{V} = 0$. If a is so large that E^2 can be neglected in comparison with $\pi\rho a^2\mu^2$,

$$\mathrm{U} = \tfrac{1}{2}\rho a\mu^2\,(\mathrm{L}' - \tfrac{3}{2}), \qquad \mathrm{V} = \mu\,(\mathrm{L}' - \tfrac{1}{2})/4\pi a,$$

and as L′ may, if b is very small, be considered a large constant, $\mathrm{U} \propto 1/\mathrm{V}$ in this case. If, however, a is so small that $\pi\rho a^2\mu^2$ can be neglected in comparison with E^2,

$$\mathrm{U} = \mathrm{E}^2 (\mathrm{L}' + \tfrac{1}{2})/2\pi a, \qquad \mathrm{V} = \mathrm{E}^2 (\mathrm{L}' - \tfrac{3}{2})/4\pi^2 \rho a^3 \mu$$

numerically, and hence $\mathrm{U} \propto \sqrt[3]{\mathrm{V}}$. It seems, therefore, that the vortex theory of matter is not much improved in respect of the relations between the velocity and the energy by taking account of the atomic charge, unless the atom is supposed always to have nearly the radius corresponding to minimum energy.

[An electric charge can, of course, have no direct action on the velocity of the vortex. The manner of its action may be illustrated by considering what happens when a vortex ring is suddenly charged. The charge will produce two distinct effects. It will increase the cross-section of the hollow, and also its aperture. The former effect has no further consequences, but the motion of the fluid involved in the latter effect causes the velocity of the fluid at the front of the hollow to decrease, and that at the rear to increase. This causes an increase of pressure in front and a decrease behind, and so causes the ring to go more slowly.—15th August, 1895.]

[It may be desirable to point out that the alteration in the relation between the energy and the aperture of the ring, which is caused by an electric charge, takes place only in the potential part of that energy. The kinetic energy of the fluid is unaltered (to our order of approximation) by the electric charge.—August 29, 1895.]

13. In order to discuss the stability of an electrified vortex, the periods of the small oscillations must be found. For this purpose we must find the surface value of the electric force when the surface of the vortex is given by (11). The electric potential ϕ is now the sum of two terms. The first is the value that it has in the steady motion. The second we shall assume to be of the form

$$\mathrm{G} \frac{{}_m\mathrm{P}_n}{\sqrt{k}} e^{\iota(nv+mw+pt)},$$

so that

$$\phi = \phi_0 \frac{\mathrm{L}}{\mathrm{L}'} + k \left(\frac{b^2}{k^2} - \frac{\mathrm{L}}{\mathrm{L}'} \right) \phi_0 \cos v + \mathrm{G} \frac{{}_m\mathrm{P}_n}{\sqrt{k}} e^{\iota(nv+mw+pt)} \quad . \quad . \quad . \quad . (40).$$

When

$$k = b \{1 + \beta e^{\iota(nv+mw+pt)}\}$$

this is to be constant. Hence

$$\text{const.} = \phi_0 - \frac{\phi_0}{\mathrm{L}'} \beta e^{\iota(nv+mw+pt)} + \mathrm{G} \frac{{}_m\mathrm{P}_n'}{\sqrt{b}} c^{\iota(nv+mw+pt)}$$

giving

$$\mathrm{G} = \frac{\phi_0}{\mathrm{L}'} \frac{{}_m\mathrm{P}_n'}{\sqrt{b}} \beta.$$

Equation (40) now gives

$$\mathrm{F} = \xi k \frac{d\phi}{dk}$$

$$= \frac{1 - 2b \cos v}{2ab} b \left\{ \frac{\phi_0}{\mathrm{L}'} \frac{1}{b} + \phi_0 \left(2 - \frac{1}{\mathrm{L}'} \right) \cos v - \frac{\phi_0}{b\mathrm{L}'} \beta e^{\iota(nv+mw+pt)} \right.$$

$$\left. - \mathrm{G} \frac{d}{db} \left(\frac{{}_m\mathrm{P}_n'}{\sqrt{b}} \right) e^{\iota(nv+mw+pt)} \right\}.$$

The total electric charge is the surface integral of $F/4\pi$.

The terms involving the exponential will evidently vanish on integration, except in the case when m and n are both zero, *i.e.*, except in the case of pulsations.

Hence, excluding pulsations, E has the same value as before, and

$$F = \frac{E}{2\pi a^2 b}\left\{1 + b\,(2L' - 3)\cos v - \left(1 + \frac{1}{\lambda}\right)\beta e^{\iota(nv+mw+pt)}\right\}.$$

The surface condition, given by (31) and (21), is

$$\begin{aligned}\Pi_\infty = {} & \frac{E^2}{32\pi^3 a^4 b^2}\left\{1 + 2b\,(2L' - 3)\cos v - 2\left(1 + \frac{1}{\lambda}\right)\beta e^{\iota(nv+mw+pt)}\right\} \\ & + \frac{\mu^2\rho}{32\pi^2 a^2 b^2}\left\{1 - 4b\left(L' - \tfrac{1}{2} - \frac{4\pi Va}{\mu}\right)\cos v - 2\left(1 + n^2\lambda + \frac{8\pi a^2 b^2 n\lambda p}{\mu}\right)\beta e^{\iota(nv+mw+pt)}\right\} \\ & - p\lambda\rho\left(\frac{\mu n}{2\pi} + 4pa^2b^2\right)\beta e^{\iota(nv+mw+pt)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (41).\end{aligned}$$

In this equation the terms not involving the exponential cancel by (37) and (38). The equation therefore can be satisfied by some value of p, which fact verifies our assumption as to the nature of the disturbance in this case. On simplification, the equation becomes

$$p^2 + 2pn\frac{\mu}{8\pi a^2 b^2} + \left\{n^2 + \frac{1}{\lambda} + \left(\lambda + \frac{1}{\lambda}\right)\frac{E^2}{\pi\rho a^2\mu^2}\right\}\left(\frac{\mu}{8\pi a^2 b^2}\right)^2 = 0 \quad . \; . \quad (42).$$

If $m = n = 0$, this equation does not hold. In this case the equation to the surface is $k = b\,(1 + \beta e^{\iota pt})$. Now, from (35) the force on the surface of the tore $k = b$ is

$$\frac{F^2}{8\pi} = \frac{E^2}{32\pi^3 a^4 b^2}\;\{1 + 2b\,(2L' - 3)\cos v\}.$$

Putting therefore $b + b\beta e^{\iota pt}$ in place of b, the force becomes

$$\frac{E^2}{32\pi^3 a^4 b^2}\{1 + 2b\,(2L' - 3)\cos v - 2\beta e^{\iota pt}\}.$$

Equating this to the pressure, we obtain

$$p^2 + \left(\frac{1}{\lambda} + \frac{1}{\lambda}\frac{E^2}{\pi\rho a^2\mu^2}\right)\left(\frac{\mu}{8\pi a^2 b^2}\right)^2 = 0 \quad . \; . \; . \; . \; . \; . \; . \quad (43).$$

14. Substituting in (42) and (43) the values found in (24), (26), (27) for λ, we have as equations for p;

if m is finite and n not zero,

$$p^2 + 2pn\frac{\mu}{8\pi a^2b^2} + n(n-1)\left\{1 + \frac{\mathrm{E}^2}{\pi\rho a^2\mu^2}\right\}\left(\frac{\mu}{8\pi a^2b^2}\right)^2 = 0 \quad . \quad . \quad . \quad (44);$$

if m is finite and n zero,

$$p^2 = \left\{\frac{1}{\mathrm{L}' - \frac{1}{2}\gamma_m} + \frac{(\mathrm{L}' - \frac{1}{2}\gamma_m)^2 + 1}{\mathrm{L}' - \frac{1}{2}\gamma_m}\,\frac{\mathrm{E}^2}{\pi\rho a^2\mu^2}\right\}\left(\frac{\mu}{8\pi a^2b^2}\right)^2 \quad . \quad . \quad . \quad . \quad (45);$$

if m and n are both zero,

$$p^2 = \frac{1}{\mathrm{L}' - \frac{1}{2}\gamma_m}\left(1 + \frac{\mathrm{E}^2}{\pi\rho a^2\mu^2}\right)\left(\frac{\mu}{8\pi a^2b^2}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (46);$$

if m is large, whatever n may be,

$$p^2 + 2pn\frac{\mu}{8\pi a^2b^2} + \left[n^2 + \frac{2mb\mathrm{K}_n'(2mb)}{\mathrm{K}_n(2mb)}\right.$$
$$\left. + \frac{\{2mb\mathrm{K}_n'(2mb)\}^2 + \{\mathrm{K}_n(2mb)\}^2}{2mb\mathrm{K}_n'(2mb)\,\mathrm{K}_n(2mb)}\,\frac{\mathrm{E}^2}{\pi\rho a^2\mu^2}\right]\left(\frac{\mu}{8\pi a^2b^2}\right)^2 = 0 \quad . \quad . \quad (47).$$

The values of p given by (45) and (46) are always real, but those given by (44) and (47) are imaginary for sufficiently large values of n. Taking (44), if p is to be real, we must have $n - (n^2 - n)\dfrac{\mathrm{E}^2}{\pi\rho a^2\mu^2}$ positive. If $\mathrm{E}^2/\pi\rho a^2\mu^2 = (\mathrm{L}' - \frac{1}{2})/(\mathrm{L}' - \frac{3}{2})$, the condition that $\mathrm{V} = 0$, n must therefore not be greater than 1 for stability, so that all the fluted vibrations of the stationary vortex are unstable.

Thus the presence of an electric charge on the vortex makes it unstable, however small the charge may be. The modification of the vortex-atom theory of matter here considered is therefore not admissible unless some alteration in the hypothesis be made which will convert the instability of the vortex into stability.

XVIII. *On the Magnetic Rotation of the Plane of Polarization of Light in Liquids.—* Part I. *Carbon Bisulphide and Water.*

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THE aim of this investigation is the determination in absolute measure of the magnetic rotation in liquids at different temperatures, the effect of the chemical nature of the liquid on this property, and its correlation with other physical properties.

In the present communication we propose to confine ourselves to a description of our apparatus and method of experiment, and to give the results obtained for carbon bisulphide and water, for sodium light, in a magnetic field of constant intensity, and at different temperatures between 0° and the ordinary boiling point.

We have selected these liquids since they alone have been the subject of absolute measurements, and since one or other has always been the standard in relative measurements of this property.

Absolute determinations have been made in the case of carbon bisulphide by GORDON, BECQUEREL, RAYLEIGH, QUINCKE, and KOEPSEL, and, in the case of water, by QUINCKE and ARONS.

The effect of temperature has only been studied by DE LA RIVE and BICHAT, the latter of whom obtained a parabolic equation representing the effect of temperature on the rotation in carbon bisulphide, and this expression has been exclusively employed to reduce observations on the rotation in this liquid to a standard temperature. RAYLEIGH and KOEPSEL made their determinations at ordinary temperatures, and used 18° as the standard temperature. BECQUEREL made most of his determinations at 0° and reduced the others to this temperature. It is noteworthy that when the results obtained at these two standard temperatures are compared by means of BICHAT'S equation, differences of at least one per cent. are obtained. This difference is eight times the estimated experimental error in the case of the determinations of RAYLEIGH and BECQUEREL, and indicates that the question is worthy of further investigation.

DESCRIPTION OF THE APPARATUS.

Two methods have been used in investigations of this kind, depending upon whether

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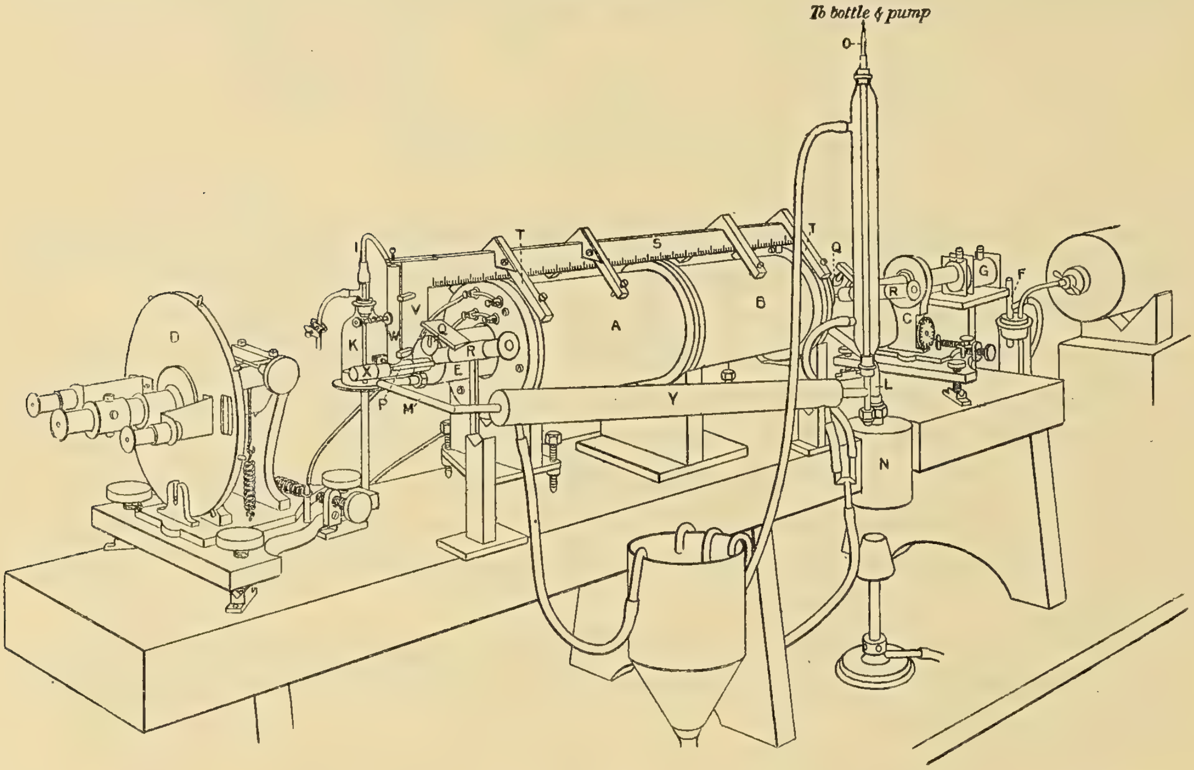
the magnetic field was produced by means of a helix or an electro-magnet. The latter of these methods is ill-suited for absolute determinations on account of the extreme difficulty of measuring the value of the field produced, as it cannot be calculated from the strength of the current exciting the electro-magnet. For this reason we decided to use a helix which consisted of two separate coils placed end to end. These, together with the optical parts of the apparatus, are shown in fig. 1.

Fig. 1.



The flanges of the coils A and B rested in the V-shaped uprights of stands fitted with levelling screws. The stands were supported on a stout wooden bench, carrying also the polarizer C, and the analyzer with its divided circle D; the positions of these being fixed by brass V's screwed to the bench.

The liquid under examination was contained in a glass tube closed by glass plates. This tube was surrounded by a brass jacket E, through which water or vapour could be circulated. In this way observations could be made at different temperatures, and the temperature could be kept constant during a set of observations. By means of studs on its exterior, the jacket was held inside the coils in such a way that the tube and coils were coaxial.

The light used was supplied by a sodium-burner F, and before entering the polarizer was passed through a filtering cell G. Suitable screens, not shown in the figure, were interposed to prevent stray light from the burner reaching the observer.

The current, supplied by a battery of accumulators, was brought up to a reversing switch and then passed through an adjustable carbon resistance, a KELVIN deci-ampere balance, and the coils.

The Helix.

When fixing the dimensions of the helix a variety of points had to be taken into consideration. In the first place it was decided that the capacity of the narrowest tube containing the liquid under examination should not be greater than 30 cub. centims., because larger quantities of sufficiently pure specimens of many of the liquids which it was proposed to examine can not readily be obtained. Experiments having proved that the minimum diameter of tube which allowed the passage of a sufficient quantity of light was 0·6 centim., the length of the tube was therefore fixed at about 62 centims.

Having regard to the unwieldiness and cost of a helix which would project beyond the tube, it was decided to make the tube project beyond the helix.

The internal radius of the helix was fixed by the dimensions of the jacket surrounding the tube, and preliminary experiments showed that for the jacket to be efficient the internal radius of the helix had to be at least 2·5 centims.

From the fact that in most liquids the rotation is much smaller than in carbon bisulphide, in order to obtain sufficiently accurate readings in the case of such liquids, we felt it desirable that our apparatus should be designed to give with carbon bisulphide a double rotation of at least 40°. For the electromotive force and maximum current at our disposal (90 volts and 5 amperes), and for a tube of infinite length, we calculated the number of turns of wire which would give the above rotation, and from considerations of the weight of the helix and the heating effect due to the passage of the current, we chose No. 18 S.W.G. wire as being the most suitable.

Although for electrical reasons it is best to use a long thin helix, in our case the maximum length which could be employed was fixed by the following considerations. Since the rate of change of the difference of magnetic potential between two points on the axis of a helix alters rapidly when one of them moves near the end of the helix, it was essential that the ends of the column of liquid should be at such a distance from the ends of the helix that unavoidable inaccuracies in determining their relative positions would not appreciably affect the result. On calculating the change of the magnetic field near the ends of a helix fulfilling the conditions given above, it was found that, if the projection of the tube was 6 centims., variations in the relative positions of the tube and helix of 2 millims. produced a change in the difference of magnetic potential of about 0·1 per cent. Hence this projection was deemed sufficient, and the length of the helix was fixed at 50 centims.

For convenience in handling, the helix, as already stated, was made in two parts. Adopting Lord RAYLEIGH'S suggestions,* two layers of fine wire (No. 28), the use of which will be evident at a later stage (p. 634), were first wound on the reels, and when winding the thick wire, two wires were wound simultaneously and were thus in contact for their entire length. These wires formed two independent circuits, between which the insulation could be tested from time to time. Although a high value for the insulation obtained in this way does not exclude the possibility of short-circuiting between different layers of the same circuit, nevertheless, it is strong presumptive evidence that no short-circuiting exists.

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* 'Phil. Trans.,' 176, Part 2, 343, 1885.

The coils were wound on turned gun-metal reels. On the inside of the flanges discs of thick drawing paper were fixed by means of shellac, and on the spindles of the reels and over the two layers of fine wire two layers of fine silk cloth were wound. The ends of the different wires were led through small holes in vulcanite plugs passing through the flanges of the reels and were attached to terminals screwed into discs of varnished wood which were fixed to the outside of the flanges.

The wire employed was double-silk-covered and was first wound from the hanks supplied on two large wooden reels, the insulation being carefully examined, and much time spent in removing lumps and improving faulty places in the insulation. We found it impossible to wind the coils uniformly unless these precautions were taken. The ends of the different lengths of wire were scarfed and soldered, using rosin as flux, and carefully bound over with silk.

During the process of winding the gun-metal reels were mounted between the centres of a screw-cutting lathe, the wires being led over a small pulley, carried by the slide-rest, which was made to traverse at the proper speed. Each wire was kept at a uniform tension by a suitable friction brake acting on the circumference of one of the flanges of the wooden reel. This brake was so constructed that if the winding was not satisfactory a few turns could be unwound, the slack being automatically taken up by the wooden reels and the wire kept taut throughout. At the end of each layer the relative position of the wires of the two circuits was interchanged by giving the wires a half-twist before winding the next layer.

The number of turns was determined by a counter attached to the mandrell of the lathe. In addition, after winding each layer a piece of thin paper was laid upon it, and by drawing a pencil along a line parallel to the axis of the coil a permanent record was obtained of the number of turns in the layer. These records served to check the readings of the counter.

The circumference over each layer was measured in three places by a steel tape, and corrections were applied for the thickness of the tape, and the fact that during a measurement the tape formed a helix, the pitch of which was equal to the breadth of the tape.

At the end of each alternate layer the resistance of the wire in each circuit, part of which was on one of the wooden reels and the rest on the gun-metal reel, was determined in order to ascertain if any short-circuiting had occurred.

The coils first wound are referred to as Coils I and II. They had the following dimensions :—

	Coil I.	Coil II.
Length inside flanges	24·866 centims.	25·076 centims.
Internal radius	3·291 ,,	3·285 ,,
External radius	6·396 ,,	6·334 ,,
Number of double turns	2257	2298
Number of layers	26	26

After these coils had been in use for some time, Coil II was re-wound with special precautions to ensure good insulation. Specially-made triple silk-covered wire was used, and the paper lining inside the flanges was replaced by sheet ebonite. The terminals, instead of being attached to varnished wood, were led through separate holes in a block of ebonite, which carried the binding screws and was fixed to the outside of one of the flanges. As a further precaution against short-circuiting a layer of thin white silk cloth was wound between each layer of wire.

The dimensions of this coil, which is referred to as II*b*, were as follows :—

	Coil II*b*.
Length inside flanges	24·923 centims.
Internal radius	3·301 ,,
External radius	6·638 ,,
Number of double turns	2240
Number of layers	26

The Tube.

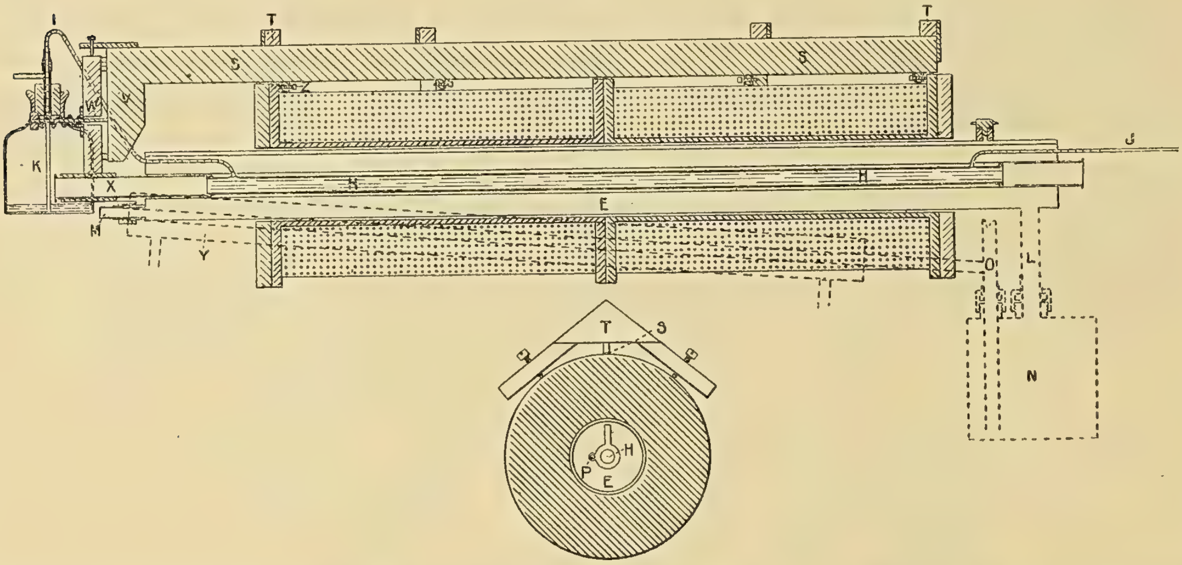
Attempts were first made to devise such a form of tube that the use of cements, which might contaminate the liquid, would be avoided. Tubes were first made having brass sleeves, on which a fine screw-thread had been cut, cemented to the ends by plaster of Paris. By means of brass caps plates of glass were pressed against the ends of the tube, leaden washers being interposed. These tubes answered fairly well, but on account of the difference between the coefficients of expansion of brass and glass they were liable to fracture on heating.

An attempt was next made to use a tube closed by stoppers ground into its ends, the inside and outside surfaces of the stoppers being worked optically—plane and parallel. It was found, however, that if the stoppers were inserted with sufficient firmness to make the tube liquid-tight, they became bi-refringent, and observations were thus rendered impossible.

Eventually we gave up the idea of dispensing with cements, and of those we have used we find that, for all liquids except water, the most convenient is a solution of sodium silicate. In the case of water, a mixture of gold size and red lead may be employed. The ends of the tube were flanged, ground flat, and closed by circles of microscope cover-glass. With this thin glass we find that no correction need be applied to the observed rotation in the liquid for the rotation in the glass ends of the tube.

The tubes had to admit of the expansion of the contained liquid due to rise in temperature, and for this reason, and in order to facilitate filling them with liquid and cleaning them, they had the shape shown in fig. 2. This figure gives sections through the coils, jacket and tube. The transverse section is taken through the right-hand flange of the helix. Near the ends of the main tube, H, are attached two narrow side tubes, I and J, bent in the manner shown. One of these, I, passes through the vertical branch of a small T-piece to the bottom of a small bottle, K, containing the liquid. By forcing air into the bottle through the horizontal branch of the T-piece liquid may be driven over from the bottle to the tube, the air escaping by the other narrow tube, J. When the main tube is full and the liquid extends a short distance along this narrow tube it is closed by a cork.

Fig. 2.



The length of the tube was measured by means of a large pair of slide callipers graduated in millimetres with a vernier reading to tenths.

The Jacket.

The jacket, E, was made of brass and projected 5 centims. beyond the ends of the tube. It is shown in longitudinal and transverse section in fig. 2. It will be seen that the region of the jacket in which the tube is placed corresponds with the space inside the inner tube of an ordinary LIEBIG's condenser. The liquid or vapour which passes through the jacket enters by the tube L and leaves by the tube M which passes through the condenser, Y, shown in the figures.

For temperatures below that of the water-main, the boiler N shown in the figures was removed, and water was kept continuously circulating through the jacket and a cooling spiral by means of a small centrifugal pump driven by a small motor. This spiral was made of composition tubing, and was immersed in a vessel containing pounded ice and water. Temperatures of about 1° could thus be obtained. For temperatures between that of the water-main and 25°, water from the main which had previously passed through a FLETCHER heater was made to traverse the jacket at a uniform rate, a constant pressure-head being employed. Temperatures above 25° were obtained by connecting L, the inlet tube of the jacket, and the end of the LIEBIG's condenser, Y, by means of screw-unions with leather washers to the brass boiler N, shown in figs. 1 and 2. On boiling a liquid placed in this boiler, a continuous stream of vapour passes through the jacket, is condensed by the LIEBIG, and then returns to the boiler. With the same jacketing vapour, different temperatures were obtained by altering the pressure under which the liquid boiled. This was effected by connecting the tube joining the LIEBIG to the boiler with a large glass bottle enclosed in a box and packed round with cotton wool. The air in this bottle could be exhausted by means of a water-pump, the degree of exhaustion being indicated by a mercury manometer. The connection between the jacket and the bottle consisted of a brass tube O (figs. 1 and 2) which passed through a LIEBIG's condenser to prevent liquid distilling over into the bottle, and as a further precaution, a trap was inserted to catch any vapour which might escape condensation in the LIEBIG. The pressure of the gas supply, both to the FLETCHER heater and the burner used to boil the liquid, was kept constant by a regulator. The vapours we have employed are those of carbon bisulphide, methyl alcohol, ethyl alcohol, and water.

The temperature was determined by two thermometers P, one at either end of the jacket. The thermometers were inserted in two recesses, which communicated with that part of the jacket in which the tube was placed; one of these is shown in cross-section in fig. 2. The thermometer scales could be illuminated at will by small two candle-power incandescent lamps, Q (fig. 1), and were read through the microscopes R.

Experiments were made by inserting two other thermometers in the ends of a tube containing liquid, to determine the time during which the temperature of the jacket had to be kept constant in order to insure that readings of the working thermometers represented the true temperature of the liquid. Care was always taken to allow the necessary time to elapse before making observations.

The thermometers were made by CASELLA, and were divided into half-degrees, each degree being about 2 mm. long. The range of one pair was from −20° to 60°, and of the other from 45° to 115°. Each thermometer had a zero mark, and the zero error was determined from time to time. The correction-curves of the thermometers were determined by comparison with a standard from MÜLLER, of Bonn, which had been tested at the Reichsanstalt at Charlottenburg.

The Method of Fixing the Position of the Tube.

It was essential that the position of the tube relative to that of the coils should be known, and for simplicity in calculating the difference of magnetic potential between the ends of the tube the position of one end was kept fixed. This was accomplished as follows :—A graduated brass bar, S (figs. 1 and 2), was supported by four screws, which were inserted in the two cross-pieces, T, shown in the figures ; the points of these screws rested on the flanges of the coils. A stop, Z (fig. 2), attached to this bar, was kept in contact with a face of one of the flanges. This arrangement formed a geometrical clamp, fixing the position of the bar relative to the coils. One end of the bar projected beyond the end of the jacket, and had a vertical piece, V, to which were attached two V's. A vertical brass cylinder, W, was held in these V's by a spring, and carried at its lower end a glass tube, X, which projected inside the jacket. During an observation, the end of the tube containing the liquid was kept in contact with the end of this glass tube, X, in the manner shown in fig. 2. This arrangement could readily be removed to admit of the introduction of the experimental tube into the jacket. A face of one of the flanges was taken as a fixed plane, and the reading on the graduated bar, S, corresponding with this plane, was ascertained. The whole arrangement was then removed from the coils, and, by means of a set-square, the position of the end of the glass tube was projected on to the graduated bar. The distance of the end of the liquid column from the plane of reference was thus obtained.

Besides the cross-pieces, T, two others are shown in the figs. These were used in fixing the position of the tube when observations were being made with one coil only.

To prevent air currents circulating through the inside of the jacket, a circle of microscope cover glass was cemented over the end of the glass tube, X, and a short piece of brass tubing, closed in a similar way, fitted into the other end of the jacket; the spaces where the narrow side pieces of the experimental tube left the jacket were, for the same reason, packed with cotton wool.

The Polarizer.

The polarizer, containing a half-wave-length quartz plate, was taken from a LAURENT half-shadow polarimeter, and was fitted to a stand, C (fig. 1), similar to that carrying the divided circle (see below).

The Analyzer and Divided Circle.

The divided circle, D, made by the Cambridge Scientific Instrument Company, was 19 centims. in diameter, and was graduated on silver in thirds of a degree. Two verniers fixed relatively to the stand could be read by microscopes to minutes. The graduations were so open that readings to the nearest minute could be taken without straining the eyesight of the observer. This is an important point when the eye has also to be used to detect small differences in brightness. The circle was carried by a hollow axle, capable of rotation in V's, and could either be rotated by hand or admitted of a fine adjustment by means of a tangent screw. The analysing Nicol fitted into one end of the hollow axle, and a telescope with cross-wires, magnifying about two diameters, fitted into the other end; either the Nicol or telescope could be separately removed.

The V's supporting the circle were carried by a horizontal plate which was attached to the base plate of the instrument in such a way that it admitted of a rotation through a few degrees about a vertical axis. This adjustment about a vertical axis, together with that about a horizontal axis obtained by means of the levelling screws fixed to the base, enabled the plane of the circle to be set perpendicular to the direction of the incident light.

The verniers could be illuminated by small incandescent lamps, fixed at a distance of several feet from the circle, suitably placed lenses being used to condense the light on the reflectors of the verniers. A key near the observer's hand enabled him to light up the lamps when necessary.

The Source of Light.

In our preliminary experiments we used the form of burner described by Dr. Perkin, Senr.* We found, however, that when the flame was sufficiently intense to be of service in our case, on examination with a spectroscope it gave a faint continuous spectrum and a broad bright band in the region of the D lines. The breadth of the band was more than twice the distance between the D lines. When the light was efficiently filtered, the intensity was too feeble for our purpose.

The burner (F, fig. 1) we finally adopted was one devised by Mr. Boys. Hydrogen direct from a cylinder of compressed gas was allowed to escape through a very small hole in the side of a piece of thick copper tubing, the hole being so small that the requisite amount of gas escaped when the full pressure of the gas in the cylinder was acting. This jet of gas was allowed to play just along the surface of a solution of common salt contained in a small bottle. The hydrogen carrying the fine spray thus produced, passed up the inside tube of a blow-through oxyhydrogen burner, carried by a cork fitted into the neck of the bottle. When the supply of oxygen was excessive, light from this burner, when examined with a spectroscope, also gave a faint continuous spectrum, but by regulating the oxygen supply the continuous spectrum could be almost entirely eliminated, the D lines being very bright and but slightly broadened. If the double rotation to be measured exceeded 20°, it was found advisable to filter this light. The absorption cell (G, fig. 1) we have employed was similar to one described by LIPPICH,* with the exception that the solutions were weaker. In our case the light was made to traverse 10 centims. of a 3 per cent. solution of potassium bichromate and 1·5 centim. of a 4 per cent. solution of uranious sulphate. For double rotations less than 20° there was no advantage in filtering the light. We have found this burner to be more manageable than PERKIN'S burner, and it has the additional advantage that it need not be placed in a fume-chamber.

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* 'Chem. Soc. Trans.,' 45, 421, 1884.

The Switch.

The switch used for reversing the current was so constructed that the coils were automatically short-circuited through a high non-inductive resistance just before breaking the circuit. In this way any chance of damage to the insulation of the coils by the break induced current was avoided. The current after leaving the switch passed through the adjustable carbon resistance, the deci-ampere balance, and the coils in series.

The Balance Constant.

Although the ultimate standard of current was a silver voltameter, the KELVIN balance was used as an intermediate standard. The current employed was always the same, namely, 5 amperes, the movable weight being kept at a fixed point on the scale, and the pointer kept at zero during the observations by altering the carbon resistance. In this way the balance was used merely as a gauge.

It was important that the readings of the balance should not be affected by the field due to the coils. Although the balance and the coils were a considerable distance apart and nearly in the same horizontal plane, we made definite experiments on this point. The balance being disconnected from the coils, a current from a battery was passed through it and the beam brought to the sighted position. A current of 5 amperes from the accumulators was then passed through the coils, but no effect was produced on the balance.

For standardising purposes we used a POGGENDORF silver voltameter. The platinum bowl had a diameter of 17·5 centims., and gave a cathode surface of 250 sq. centims. The anode was a plate of silver 11 centims. square, and the solution employed contained 30 parts of silver nitrate in 100 of water. The voltameter was designed in order that we might reproduce as nearly as possible the conditions under which Lord RAYLEIGH and Mrs. SIDGWICK[*] determined the electro-chemical equivalent of silver. The current was passed through the voltameter for 30 minutes, the time being noted by a chronometer. The rate of the chronometer was determined from the Greenwich time signals.

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* 'Zeit. Instrumentenk.,' 12, 333, 1892.

By a preliminary trial made when the voltameter was replaced by an equivalent wire resistance, the resistance of the circuit was adjusted so that the balance took up its sighted position. It was easy to make this adjustment with such accuracy that after switching on the current when the voltameter was in the circuit, the final adjustment of the balance to its sighted position could be completed in a second or two. During the deposition the current was kept constant by means of the carbon resistance. The basin and the deposited silver were washed with distilled water, dried over a spirit lamp, allowed to cool in a desiccator, and weighed on a BUNGE balance, which, with this load, indicated a change of ·0002 gram. With the amount of deposit obtained, ·001 gram represented one part in 10,000.

On allowing the deposit to soak overnight in water and reweighing, no appreciable alteration was ever obtained.

The errors of the weights employed were determined by comparison with a set which had been tested at the Standards Office.

During the preliminary observations a series of silver depositions were made in which the same solution of silver nitrate was used, the anode being a plate of "pure" silver, obtained from JOHNSON and MATHEY, the fineness of which was found to be 997. From these results it appeared that the balance-constant was steadily increasing, the last value being about 2 parts in 1000 greater than the first. On using a fresh solution of silver nitrate, however, the value obtained was practically the same as that first given by the old solution.

Since a solution of the silver nitrate, supplied by the maker as "pure recrystallized," was very slightly acid, we thought that the low value at first obtained might be due to this cause, inasmuch as the solution became neutral, and eventually slightly alkaline, after having been some time in use. A quantity of silver nitrate was therefore repeatedly recrystallized from hot solutions, and the small crystals thus obtained were dried in a platinum dish over a small flame. Another sample of the nitrate was heated in a platinum dish until it was just completely fused and was kept in this state for some time. Solutions of two other samples were boiled for some hours with finely divided metallic silver, and with freshly precipitated and well washed silver oxide respectively. Several 3-inch voltameters, containing solutions of the above, and also the old solution, were then arranged in series, and the same current passed through them, the current-density being approximately the same as that used when standardising the balance. The whole of the new solutions gave the same weight of deposit, which was smaller than that given by the old solution, the difference being the same as obtained in the large voltameter with the old solution and one freshly prepared.

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[*] 'Phil. Trans.,' 175, Part 2, 411, 1884.

It was evident, therefore, that the old solution gave too heavy a deposit. The balance was, therefore, standardised with the four new solutions, and the values obtained for the current in amperes corresponding with one division of the inspection scale of the balance, were :—

·09989, ·09989, ·09988, ·09990.

In addition to these, the results of four other depositions made from time to time during the observations recorded in this paper, are as follows :—

·09989, ·09989, ·09989, ·09989.

Although the values given by the different solutions were practically identical, it was possible that the mean result might be affected by impurity in the anode supplied by JOHNSON and MATTHEY. To test this point, in the last three depositions we used an anode of fineness 9997, which, through the kindness of Professor ROBERTS-AUSTEN and Dr. ROSE, was made for us at the Mint. It is evident that no error is introduced on this account.

The mean of the above values, ·09989, has been employed in reducing the observations on rotation.

The following are, in chronological order, the results obtained with the old solution :—

·09983, ·09987, ·09990, ·09999, ·09995, ·09993,
·09995, ·10002, ·10005, ·10006, ·10002.

After being used several times the solution had a distinctly blue colour due to the presence of a small quantity of copper dissolved from the anode. It is difficult to account for the increase in the weight of the deposit. Since silver has a higher electro-chemical equivalent than any of the metals likely to be present as impurities in the anode the increase could hardly be due to this cause, and this view is further supported by the fact that on dissolving the heavy deposits in pure nitric acid, silver nitrate was obtained, which, when dissolved in water, gave the same weight of deposit as the fresh solutions. It was at first thought that the increase might be connected with secondary reactions occurring during electrolysis; for, according to certain observers,[*] the greyish deposit which forms on the anode during a deposition is silver oxide, and oxidation could only be the result of secondary decomposition. A quantity of the greyish deposit was therefore detached from the anode by a stream of cold water, and after being well washed in cold water was kept until the weight was constant in a stream of dry air. One portion of the deposit was then heated to redness in a current of dry hydrogen, and another portion was dissolved in nitric acid, and the silver precipitated as chloride. Although in these experiments no extraordinary precautions were taken, the results agree in showing that to 1 part in 1000 the deposit on the anode is pure silver, and it gives, therefore, no support to the idea that secondary reactions have occurred. So far as we know, the only observation of a similar kind to those given above, was made by **Kahle**,* who found that on boiling silver nitrate with silver oxide the amount of electrolytically deposited silver was increased by 5 parts in 10,000. The explanation of these results with the old solution is at present unknown, although most of the facts point to the formation of a sub-salt of silver having a heavier silver ion than an argentic salt.

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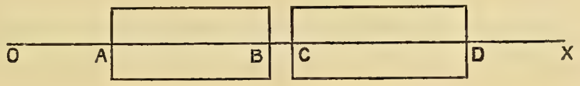
[*] Compare ARONS, 'Wied. Ann.,' 24, 171, 1885.

The Coil Constant.



The difference of magnetic potential between two points O, X, on the axis of the coils produced by a single layer of the coil AB of radius R, containing m turns, when a current C is passing, is

$$\frac{2\pi Cm}{AB}\{\sqrt{R^2+\overline{AX}^2}-\sqrt{R^2+\overline{BX}^2}-\sqrt{R^2-\overline{OA}^2}+\sqrt{R^2+\overline{OB}^2}\}.$$

The difference of magnetic potential due to each layer of both coils was obtained by this formula, and the results added together to give the total difference of potential between the ends of the tube. This was done for three different positions of the point X, distant from one another by 2 millims., because the end O of the liquid column was always kept fixed with respect to the coils, and the position of the end X varied slightly as the different tubes employed had not exactly the same length. By this means the difference of magnetic potential corresponding with any of the tubes used could be obtained by interpolation. A variation in the position of X of 1 millim. corresponded with a change of only 1 part in 5000 in the value of the difference of magnetic potential.

Since it is a matter of some difficulty to decide exactly how to allow for the unavoidable spaces which have to be left at the end of each layer, where the wire is carried up to the next higher layer, it is to a slight extent doubtful what the true length of a coil really is. For this reason, and also to obtain a further check on the perfection of the insulation, we made in the case of carbon bisulphide a series of observations with each of the three coils taken separately, the tube being placed symmetrically with reference to the coil. In these cases the projection was about 20 centims., and the results are not affected by the above uncertainty, and further, the difference of magnetic potential could be calculated by Lord RAYLEIGH's approximate formula. As our coils had a slightly larger diameter than that used by Lord RAYLEIGH, we thought it desirable to calculate another term in his approximation, but found that it produced no appreciable effect. We may state here that the results obtained with each of the single coils agreed with those obtained with two coils taken together.

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* 'Brit. Assoc. Rep.,' Edinburgh, 148, 1892.

METHOD OF OBSERVATION.

After the liquid in the tube had attained a steady temperature, the analysing Nicol was removed and the plane of the circle was set perpendicular to the direction of the incident light, the telescope and cross-wires being used for this purpose.

While one observer attended to the light and kept the current constant the other noted the temperature and made settings with the circle. Each observation consisted of at least three settings with the current in one direction, three when the current was reversed, and two similar sets of three after the circle carrying the analysing Nicol had been rotated through nearly 180°. The temperature at each end of the jacket was read at the beginning, middle, and end of this series of settings. Each of the results given in the tables is therefore the mean of at least twelve such settings and six readings of the temperature. The following is an example of the kind of result thus obtained; it refers to carbon bisulphide, one coil only being used :—

Direction of current.	+		−		−		+	
Vernier.	A.	B.	A.	B.	A.	B.	A.	B.
Circle readings	114° 40′	294° 44′	95° 28′	275° 32′	275° 29′	95° 28′	294° 40′	114° 40′
	114 40	294 43	95 29	275 33	275 30	95 29	294 40	114 39
	114 39	294 43	95 28	275 33	275 30	95 28	294 42	114 41
	..	..	..	..	..	..	294 41	114 40

Temperature	Therm. A.	41·77,	41·80,	41·82.
	„ B.	41·75,	41·77,	41·79.

Differences in consecutive settings as great as 2′ were seldom obtained.

We found that the accuracy of the settings was considerably increased by employing a device first used by Lord RAYLEIGH. As already stated, two layers of fine wire were first wound on the reels when making the coils. By means of a key close to the observer's hand a current from a few dry-cells could be made to pass in-one direction or in the other through these two layers of fine wire. If the intensity of the illumination of the two halves of the field is the same, on making and reversing this current, the one half and then the other is darkened to the same extent. In each setting the position of the analyzer was adjusted until on using the auxiliary current in the manner described, each half of the field appeared to be equally darkened. In this way the eyes do not seem to be strained so much as in the ordinary method of making the settings. Other points which help in the same direction are the mode of illuminating the scales of the verniers and the thermometers to which reference has already been made, and the fact that in a series of observations the observers took turns at making the settings.

With high double rotations, such as 30° or 40°, we were much troubled, to begin with, by bright patches, which appeared in the field of view, and prevented the adjustment of the two halves to equal intensity. This effect was found to be due to light, which, having suffered reflection at the far end of the tube, traversed the liquid a second time, was again reflected from one or other of the glass surfaces which closed the jacket or the cell containing the quartz plate, and finally reached the analyzer, having traversed the liquid three times. Its plane of polarization was therefore rotated through an angle three times as great as that of the light which had only traversed the liquid once, and, consequently, when the analysing Nicol was placed to extinguish the latter, an amount of the former passed through, depending on the magnitude of the rotation. In the case of small rotations, the separation of the planes of polarization of these two beams is so small that the Nicol reduces the intensity of the reflected beam to such an extent that it does not interfere with the settings.

All the glass surfaces which it was possible to alter were inclined at a small angle to the direction of the light, and the ends of the tube were made parallel to one another, but slightly inclined to the axis. By this means the above difficulty was overcome.

RESULTS OBTAINED.

CARBON BISULPHIDE.

Sample No. 1.

A quantity of "pure redistilled" carbon bisulphide from HOPKIN and WILLIAMS was allowed to stand over mercury for some days, the mercury being repeatedly shaken with the liquid. It was then poured off and shaken for 2 hours with a saturated solution of mercuric chloride, separated from the solution, and put over phosphoric anhydride. After standing over the phosphoric anhydride, which was twice renewed, for two days, the liquid was decanted and distilled from a small quantity of anhydride. During the 25 minutes which the distillation lasted the boiling-point was quite constant at

46°·61, bar. 769·4 millims.

Using REGNAULT'S value, dt/dp at the boiling-point = 0°·04 per millimetre, the boiling-point reduced to 760 millims. is 46°·23. A second determination, made after the observations on rotation had been completed, gave

46°·54, bar. 766·8 ; reduced b.p., 46°·26.

The mean value of the b.p. is therefore 46°·25.

At 0° the density of the sample was

1·29271.

This sample is referred to as CS_2, No. 1.

As already mentioned, in order to detect short-circuiting due to leakage in the coils and errors in the calculation of the coil constant, three series of observations were made over the temperature range between 0° and the boiling-point. In two series Coils I. and II. were used separately, and in the other these two coils were used together.

The results are given in the following tables.

As a rule, a set of at least three observations was made at about the same temperature. In the first column is given t, the mean temperature of each observation, in the second column, L, the logarithm of the difference of magnetic potential between the ends of the liquid column, in the third, α, the observed angle of rotation, and in the fourth the mean value of γ, VERDET'S constant at the mean temperature deduced from each set of observations. γ is the angle of rotation in minutes, produced in a column of liquid when the difference between the magnetic potentials at the ends of the column is equal to 1 C.G.S. unit.

Coil I.—CS_2, No. 1.

t.	L.	α.	γ.
7°·73	4·41515	20° 0′·1	·04295 at 7°·8
·78	,,	0·5	
·87	,,	19 59·4	
·92	,,	59·2	
8·39	,,	19 58·7	·04290 at 8°·4
·35	,,	58·7	
·37	,,	58·7	
·44	,,	57·9	
24·04	,,	19 25·5	·04172 at 24°·3
·19	,,	25·6	
·24	,,	25·3	
·32	,,	26·5	
·35	,,	25·1	
·41	,,	25·4	
40·25	,,	18 51·7	·04054 at 40°·3
·24	,,	52·4	
·29	,,	52·5	
·26	,,	53·2	

Coil II.—CS_2, No. 1.

t.	L.	α.	γ.
°		° ′	
0·93	4·15304	20 36·1	·04343 at 1°·0
·93	,,	35·4	
·94	,,	35·8	
1·11	,,	35·3	
·08	,,	35·5	
16·72	,,	20 1·6	·04226 at 16°·8
·77	,,	1·9	
·80	,,	2·5	
·81	,,	2·6	
29·22	,,	19 37·2	·04135 at 29°·3
·23	,,	35·8	
·25	,,	36·5	
·29	,,	36·3	
·34	,,	35·9	
41·44	,,	9 11·0	·04046 at 41°·5
·45	,,	10·3	
·44	,,	11·2	
·49	,,	11·8	

Coils I. and II.—CS_2, No. 1.

t.	L.	α.	γ.
°		° ′	
9·07	4·44299	39 36·6	·04285 at 8°·9
·36	,,	34·6	
·00	,,	36·5	
8·49	,,	37·7	
·60	,,	37·7	
·96	,,	39 35·5	·04281 at 9°·1
9·16	,,	36·7	
·28	,,	34·3	
26·62	4·44255	38 19·5	·04150 at 26°·9
27·14	,,	19·4	
39·93	,,	37 24·4	·04054 at 40°·0
·96	,,	25·8	
40·01	,,	26·1	
·06	,,	27·6	
·08	,,	24·4	

Temperature Variation.

On plotting these values it was found that they practically lay on the same straight line. We, therefore, assumed that the temperature change was linear and on this assumption deduced the equation

$$\gamma_t = \cdot 04351\ (1 - \cdot 001695t)^*.$$

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* The factor ·04351 in this equation is subject to a correction for a defect in the analysing Nicol, see p. 641.

The comparison of the observed and calculated values is given in the following table :—

t.	γ.		Difference.
	Observed.	Calculated.	
1°·0	·04343	·04343	·00000
7·8	·04295	·04293	+ ·00002
8·4	·04290	·04289	+ ·00001
8·9	·04285	·04285	·00000
9·1	·04283	·04284	− ·00001
16·8	·04226	·04227	− ·00001
24·3	·04172	·04172	·00000
26·9	·04150	·04152	− ·00002
29·3	·04135	·04135	·00000
40·0	·04054	·04056	− ·00002
40·3	·04054	·04054	·00000
41·5	·04046	·04045	+ ·00001

The agreement between the observed and calculated values leaves little to be desired, the mean difference being ± 8 in 40,000 ; it also shows that the two coils taken separately as well as when taken together give identical results.

As regards the effect of temperature, it is noticeable that our expression is essentially different from that of BICHAT.* As the data† on which BICHAT's formula is based are not easily accessible, we have reproduced them in the following table, which also gives the agreement between the ratios deduced from his observations and those calculated by his equation

$$\gamma_t = \gamma_0 \left(1 - {}^{\cdot}0_2 104t - {}^{\cdot}0_4 14t^2\right).$$

t.	Rotation.	Ratios.		Difference.
		Observed.	Calculated.	
0°	16° 10′	1·000	1·000	0·000
9	16 2	0·992	0·989	+ 0·003
15	15 50	0·979	0·981	− 0·002
22	15 45	0·974	0·970	+ 0·004
30	15 40	0·969	0·956	+ 0·013
41	15 20	0·949	0·934	+ 0·015
48	14 10	0·876	0·918	− 0·042
0	28 10	1·000	1·000	0·000
30	27 10	0·965	0·956	+ 0·009
40	26 54	0·955	0·936	+ 0·019

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* 'Journ. Phys.,' (1), 8, 204, 1879.

† 'Annales de l'École Normale,' (2), 2, 277, 1873.

A consideration of the differences between the observed and calculated values shows that at high temperatures the agreement is far from satisfactory, and the fact that at 30° and 40° the differences are large and positive, while at 48° the difference is large and negative, proves that the form of the curve has been largely modified by this last point. If this observation be neglected, the linear equation

$$\gamma_t = \gamma_0 (1 - \cdot 00118 t)$$

gives a much better agreement than BICHAT's parabolic expression, the mean difference being only ± 2 parts in 1000. It would, therefore, appear that BICHAT was hardly justified in using his parabolic equation ; indeed, even apart from the fact that the boiling-point of the carbon bisulphide used by him is 2° higher than those obtained by ourselves and the other observers quoted on p. 644, we think that the character of his experimental data scarcely warrants the acceptance of his equation as representing the true temperature variation of the rotation in carbon bisulphide.

Independent evidence in support of this conclusion lies in the fact that if Lord RAYLEIGH's value for the rotation in carbon bisulphide at 18° be reduced to 0° by our expression for the temperature variation, it agrees much better with the determination made at 0° by BECQUEREL than when it is reduced by means of BICHAT's equation.

	Reduced by BICHAT's formula.	Reduced by our linear formula.
RAYLEIGH*	·04302	·04334
BECQUEREL†	·04341	·04341

Absolute Value.

As all observers, except ARONS, who have made absolute measurements of the magnetic rotation in liquids, have used carbon bisulphide, and as the values obtained by them are in all cases lower than ours, we give at this stage a detailed examination of the effects which are likely to influence the absolute value.

These may be due to—

(1.) Errors in the number of turns of wire in the coils, or, what amounts to the same thing, short-circuiting, either partial or entire, of one or more turns.

(2.) Errors in the calculation of the coil constant due to irregularity in the winding where the wire mounts up from one layer to the layer above.

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* *Loc. cit.*

† 'Ann. Chim. Phys.,' (6), 6, 145, 1885.

(3.) Errors in the measurement of the length of the tube or in the determination of its position relative to the coils.

(4.) The light not being monochromatic.

(5.) Inclination of the axis of rotation of the analysing Nicol to the direction of the incident light.

(6.) Defects in the analysing Nicol.

(7.) Errors in the measurement of the current, or errors due to leakage in the leads between the balance and the coils.

(8.) Errors in determining the temperature of the liquid.

(9.) Impurity in the carbon bisulphide.

(1.) May be taken to be inoperative if each of the three coils taken separately give, as they do, the same result.

(2.) Measurements made when the liquid projected more than 20 centims. beyond the coil showed that there is no error due to this cause. (Compare p. 638.)

(3.) Of these, the only error which has not been discussed on p. 633, is that due to inclination of the direction of the light to the lines of force of the magnetic field. Measurements proved that this inclination was always considerably less than 1° of arc, and could therefore produce no appreciable effect on the results. since 1° corresponds with less than 1 part in 5000.

(4.) The quality of the light used has been discussed on p. 629.

(5.) This error is to a first approximation eliminated by reading the Nicol in two positions nearly 180° apart. From the difference in the readings thus obtained McConnel* has shown how to calculate approximately the inclination of the axis of rotation of the Nicol to the direction of the incident light. In our case the difference was in general about 1′·5. which corresponds with an inclination of about 3′. McConnel has further shown that an inclination of 2° would give an error of less than 1′ in a rotation of 60°. It is therefore evident that the first approximation obtained by reading the Nicol in two positions nearly 180° apart is in our case amply sufficient, and it is not necessary to seek a closer approximation by using a bi-prism read in four positions as Lord Rayleigh has suggested.

(6.) Although there seemed to be no reason to expect that different Nicols would give different results, we thought it desirable to test this point, and used four different Nicols, two, A and B, being of the ordinary pattern with the end faces cut inclined to the axis, and two, C and D, with the end faces cut perpendicular to the axis. Of these, A, which was selected on account of its small lateral throw, was used in the experiments given in the preceding tables.

Keeping everything else the same, the following are the rotations obtained with the different Nicols, that of A being taken as unity, each result being the mean of a series of observations :—

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* 'Phil. Mag.,' (5), 19, 317, 1885.

With a double rotation of about 40°—

A	1·0000,
B	·9992,
C	·9992,
D	·9994.

With a double rotation of about 20°—

A	1·0000,
C	·9992,
D	·9992.

It would therefore appear that Nicol A gives a value which is higher by about 8 parts in 10,000 than those given by any of the other Nicols. For this reason, in subsequent observations, we have used Nicol C, and diminished the values obtained with Nicol A by the above amount.

(7.) The question of the accuracy of the current measurement has been dealt with on p. 632. Leakage in the leads, which were thickly insulated with gutta-percha, was tested with a pressure of 107 volts, and the insulation was found to be quite satisfactory.

(8.) This point has been disposed of on p. 627.

(9.) In order to ensure that our results should not be affected by impurity in the liquid, we have made observations on two other samples of carbon bisulphide, both of which were derived from a different source, and one of which was purified in a totally different way from sample No. 1.

Sample No. 2.

The first of these new samples, CS_2, No. 2, was a portion of a fresh stock obtained from Hopkin and Williams. It was purified by the method already given. Its boiling-point was determined on three occasions with the following results :—

46°·01, bar. 754·3 millims. ; reduced b.p. 46°·24.
45°·33, bar. 736·5 millims. ; reduced b.p. 46°·27.
46°·18, bar. 758·0 millims. ; reduced b.p. 46°·26.
Mean b.p. 46°·26.

Two determinations of the density at 0° gave :

1·29277
1·29288
Mean 1·29282

The following table contains the measurements on rotation. In the first column is indicated the coil or coils, C, used in the observations, and in the last column is given γ_0, the mean value of VERDET'S constant at 0° deduced by our linear expression for the temperature variation from each set of observations.

CS_2, No. 2.

C.	t.	L.	α.	γ.	γ_0.
I. and II.*b*.	27°·68	4·43706	38° 46'·3	·04143 at 27°·7	·04347
"	·70	"	47·4		
"	·72	"	45·7		
"	28·65	"	37 42·9	·04136 at 28°·7	·04347
"	·67	"	43·4		
"	·68	"	42·9		
"	30·71	"	37 34·6	·04121 at 30°·7	·04347
"	·72	"	34·7		
"	·73	"	34·6		
				Mean . .	·04347

Sample No. 3

The second sample, CS_2, No. 3, was prepared in the following way. Bromine was added to a quantity of the same stock as employed for No. 2 until the liquid was almost opaque. After standing for two weeks, the liquid was shaken with fresh quantities of a solution of strong potash until it was decolorized, and was then dried over phosphoric anhydride. It was next shaken with fuming nitric acid for two hours,* washed with water, and dried over phosphoric anhydride. The liquid was finally shaken with three fresh lots of strong sulphuric acid, washed with water, and dried over phosphoric anhydride. By this treatment it was hoped that the bisulphide would be freed from likely impurities, which in all probability would be less stable in presence of the above reagents than the bisulphide itself.

Two observations of the boiling-point were made, the liquid being distilled from a small quantity of phosphoric anhydride. During the distillation, which in each case lasted about an hour, the boiling-point was quite constant, and gave—

45·99, bar. 753·5 millims. ; reduced b.p. 46°·25,
45·36, bar. 737·1 millims. ; reduced b.p. 46°·27,
Mean b.p. 46°·26.

Two determinations of the density at 0° gave—

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* Compare FRIEDBURG, 'Ber. Deut. Chem. Gess.,' 8, 1616, 1885.

1·29279
1·29288
Mean . 1·29283

The following table contains the observations on magnetic rotation :—

CS$_2$, No. 3.					
C.	t.	L.	α.	γ.	γ_0.
I. and II.*b*.	10°·22	4·43706	38° 57′·2	·04270 at 10°·3	·04346
”	·21	”	56·8		
”	·27	”	55·9		
”	·36	”	55·2		
”	·42	”	56·3		
I.	29·76	4·14515	19 13·2	·04128 at 29°·8	·04348
”	·79	”	12·9		
”	·83	”	13·3		
”	·85	”	13·2		
II.*b*.	30·42	4·14153	19 2·0	·04124 at 30°·4	·04348
”	·43	”	2·7		
”	·45	”	2·5		
”	·46	”	2·6		
”	41·58	”	18 39·3	·04041 at 41°·6	·04348
”	·60	”	39·7		
				Mean	·04347

In the following table are collected the mean values of the boiling-point, density at 0°, and VERDET'S constant at 0°, given by the three different samples of carbon bisulphide. The value of VERDET'S constant at 0°, in the case of sample No. 1, is derived from the equation given on p. 637, and corrected for the defect in the analysing Nicol dealt with on p. 641 :—

	B.p.	ρ_0.	γ_0.
CS$_2$, No. 1. . .	46°·25	1·29271	·04348
CS$_2$, No. 2. . .	46·26	1·29282	·04347
CS$_2$, No. 3. . .	46·26	1·29283	·04347
Mean values .	46·26	1·2928	·04347

It will be noted that the three different samples give practically identical values for the three physical constants.

For the sake of comparison, the values of the boiling-point and density found by

REGNAULT,[*] THORPE,[†] and PERKIN[‡] are given in the following table :—

	B.p.	ρ_0.
REGNAULT	46°·20	
THORPE	46·04	1·29215
PERKIN	46·23	1·29310

These observations are in satisfactory accord with our own, the agreement being particularly satisfactory in the case of our values and those of PERKIN. The value for ρ_0 assigned to PERKIN is derived from his value for the relative density at 15°, $\rho \frac{15^\circ}{15^\circ} = 1{\cdot}27204$, by means of THORPE's numbers for the thermal expansion.

The whole of our observations, therefore, point to the conclusion that the magnetic rotation of carbon bisulphide between 0° and its boiling-point is expressed by the equation

$$\gamma_t = \cdot 04347\ (1 - \cdot 001696t)$$

or

$$\gamma_t = \cdot 04347 - \cdot 0_4 737t.$$

The values of γ_t at every 10°, as deduced from this equation, are given on p. 654 and the curve which the equation represents is given in fig. 3, p. 651.

Comparison with Results of Previous Observers.

That the values given by the above equation are higher than those obtained by all previous observers, except QUINCKE, is shown in the following table; when necessary the values have been reduced to 0° by our expression for the temperature variation.

	t.	γ_t.	γ_0.
RAYLEIGH§	18°	·04202	·04335
BECQUEREL‖	0	·04341	·04341
QUINCKE¶	21·06	·04409	·04564
KOEPSEL**	18	·04199	·04332
R. and W.	..	..	·04347

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[*] 'Mém de l'Acad.,' 26, 339, 1862.

[†] 'Chem. Soc. Trans.,' 37, 327, 1880.

[‡] See foot-note, p. 652.

[§] *Loc. cit.*

[‖] *Loc. cit.*

[¶] 'Wied. Ann.,' 24, 606, 1885.

[**] 'Wied. Ann.,' 26, 456, 1885.

The only other observer who has made an absolute determination is GORDON,[*] but, as he did not note the temperature of the liquid, and as he used thallium light, it is impossible to include his value in the above comparison.

The very high value obtained by QUINCKE is probably due to an error in estimating the strength of the field between the pole-pieces of the magnet he employed.

In connection with the differences between our value and those of the other observers, it is important to note that defects in the insulation of the coils would tend to give too low a value. In our observations we have used three different coils, and, in the case of one of these, II. *b*, which was used for some of the observations on samples No. 2 and No. 3, as already stated, very special precautions were taken to make the insulation as good as possible. Since these three coils give identical results, we do not think that our results can be affected by errors due to defective insulation.

KOEPSEL does not seem to have concerned himself with this point, although, since his coil was wound double, he may have done so. BECQUEREL used two coils, which gave identical results, in his case, therefore, this error is most probably inoperative; and Lord RAYLEIGH, although he only used one coil, by winding the wire double, was able to apply a severe test of the insulation.

Another likely and important source of error, which Lord RAYLEIGH alone has eliminated, is that due to the inclination of the axis of rotation of the analysing Nicol to the direction of the incident light. BECQUEREL and KOEPSEL appear to have ignored its existence, since they neither took any special pains to ensure that this inclination did not exist, nor did they attempt to eliminate it by taking observations with the Nicol in two positions nearly 180° apart. From McCONNEL'S results it follows that an inclination of only half a degree might be sufficient to account for the difference between our value and that of KOEPSEL.

The accurate determination of the actual temperature of the liquid is another point the importance of which is liable to be under estimated, because, owing to the passage of the current during a set of observations, heat is being continually developed in the coil, and this heat tends to raise the temperature of the liquid. In our apparatus, even although we had a considerable thickness of jacketing liquid or vapour kept in rapid circulation, and the electric current was stopped when reading the verniers and entering the results, after working half an hour we found it necessary to discontinue the observations on account of this heating effect.

Lord RAYLEIGH states that this heating effect was the worst feature in his experiments, because after trying a small thickness of water as a jacketing fluid and finding it unsatisfactory, he merely used a layer of badly conducting material between the coil and the tube to prevent the conduction of heat to the liquid. The temperature of the liquid was taken by a thermometer placed in a small lateral branch attached near the end of the experimental tube, which, it has to be noted, projected about ten inches beyond the coil. Lord RAYLEIGH mentions that in some of his experiments the rise in temperature as indicated by this thermometer during an observation, amounted to as much as 2°. Since the heat which caused this rise was generated at the coil, and had to reach the thermometer by conduction, it is impossible to say what the actual temperature of the liquid near the coil really was, and it is in this part of the liquid that almost the whole of the rotation was produced. It seems to us, therefore, that on this account the temperature may have been under estimated, and hence the value obtained for VERDET'S constant is likely to be too small. The difference between Lord RAYLEIGH'S value and ours amounts to less than 3 parts in 1000, and corresponds with a temperature difference of 1°·6. Lord RAYLEIGH estimates the uncertainty in his temperature determination at 0°·3. Although it is impossible to say what the actual value is, our own experience leads us to think that this is an under-estimation.

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[*] 'Phil. Trans.,' 167, Part 1, 1877.

BECQUEREL gives no particulars regarding his method of determining the temperature. He immersed his coils and tube in a trough filled with pounded ice or water. Inasmuch as the space between the coils and the tube was less than half a centimetre, and as there was no circulation of water in the interspace, in all probability here again the temperature was under estimated.

KOEPSEL used a water jacket which, as far as can be gathered from his paper, could not permit of a greater thickness of jacketing liquid than 0·5 centim. between the tube and the coil, which was wound on a paper reel. Each set of observations lasted about half-an-hour, and he estimated the temperature of the carbon bisulphide by noting the temperature of a sample of the jacketing water which he ran off into a beaker before and after each set. He says he had considerable difficulty in protecting the carbon bisulphide from alteration in the temperature of the room due to the heating effect of the source of light, nevertheless, he makes the remarkable statement that he had no trouble whatever on account of the heating-up of the coils, although in some cases he passed a current of 1·8 amperes for half-an-hour, the resistance being 26 ohms.

KOEPSEL was of opinion that the definition of the division-line of the two halves of the field seen through the liquid was rendered indistinct by very slight differences in temperature, which he considered were less than 0°·1, and he concluded that so long as the definition remained good the temperature of the carbon bisulphide was the same as that of the jacketing liquid. This conclusion is evidently at variance with the experience of Lord RAYLEIGH, who says that in cases like that already referred to, where the temperature of the liquid rose 2° during the observations, the definition still remained satisfactory, although to avoid uncertainty on account of the rise in temperature they were discontinued.

The experiments which we made to determine the time necessary for the liquid to attain the temperature of the jacket support Lord RAYLEIGH'S conclusion.

This matter of temperature, as well as that already mentioned regarding the method of making the settings, are illustrations of what appears to us to be a defect in Koepsel's work, namely, a want of balance in the accuracy with which he attempts to determine the various quantities he observes. Thus, in order to justify the accuracy to which he gives the weight of silver deposited in his voltameter, the time during which the current flowed would have to be known within $\frac{1}{1000}$th of a second, while, as a matter of fact, the current was made and broken by hand and was reversed in the coils several times during a deposition. Again, although his average rotation is about 4° of arc, and his vernier read to only $\frac{1}{100}$th of a degree, the rotations are expressed to 1 part in 250,000.

At the outset we thought that a likely cause of the difference between our value for γ and those of other observers was impurity in the carbon bisulphide used. None of these observers seem to have taken much trouble on this head, nor do they give any physical constant (boiling-point, density, etc.) which can be used as a criterion of the purity of the liquid. Becquerel and Rayleigh adopted the method, which appears to have been first suggested by Cloez in 1869, of allowing the liquid to stand over powdered mercuric chloride for some time and then distilling from grease. Koepsel let the bisulphide *stand* over mercury for some time, dried it with calcium chloride, and then distilled.

In the case of the samples of bisulphide used by Becquerel and by Rayleigh, it seemed to us that water was a likely impurity, and we therefore made experiments to determine its effect. We found, however, that two samples of carbon bisulphide, saturated with water at 18°, gave practically the same rotation as the dry liquid, the values for the wet liquid being the smaller by about 1 part in 3000.

As carbon bisulphide is decomposed on exposure to light, we next examined to what extent the rotation was affected on this account. One sample, contained in a glass tube 1 centim. in diameter, was exposed to daylight for 12 hours. On filtering off the precipitate thus produced, the liquid gave practically the same rotation as it had before exposure. The same result was obtained with a sample which was exposed for 6 days, although a considerable quantity of precipitate was produced and was only removed after repeated filtration. It is, therefore, evident that moderate exposure to light does not appreciably affect the magnetic rotation.

We next tried to ascertain whether the rotation of our bisulphide was altered by treatment with mercuric chloride in the manner employed by Becquerel and Rayleigh. Cloez* first recommended the use of this reagent as a means of removing the evil-smelling oil usually present in carbon bisulphide. Obach† finds, however, that the action of powdered mercurous sulphate is more satisfactory. One sample of our purified liquid was allowed to stand over mercuric chloride and another over mercurous sulphate for three days, the powdered salt being repeatedly shaken with the liquid. The liquid was then decanted and distilled. In neither case was any effect on the rotation produced by this treatment.

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* 'Compt. Rend.,' 69, 1356, 1869.

† 'Jour. Pract. Chem.,' (2), 26, 281, 1882.

Many observers like RAYLEIGH and BECQUEREL have employed grease or fatty matter as a purifying agent. This process also seems to have been due, in the first instance, to CLOEZ, who does not give, however, any reason for its use. So far as we know the only point which can be urged in its support is that fatty matter appears to have the power of removing in some measure the evil-smelling oil when it is present in the liquid, because experience goes to show that after ordinary carbon bisulphide has been used for some time in extracting fat and oils from commercial products it loses its nasty odour. A quantity of olive oil was added to one sample of our purified liquid and a quantity of white wax was dissolved in another. After three weeks the two products were distilled without ebullition from a water bath, but in neither case was any alteration produced in the rotation.

We have gone into this question of the mode of purifying carbon bisulphide in some detail since it has been proposed to utilize observations of the rotation in this liquid as a means of measuring strong currents. The facts go to show that for the purposes of magnetic rotation it is not difficult to obtain carbon bisulphide which is practically pure. Indeed, we found that our stock liquid ("pure redistilled" carbon bisulphide, from HOPKIN and WILLIAMS), without any treatment, although it was somewhat opaque, gave almost the same rotation as it did after treatment. By merely shaking with mercury the opacity can be removed.

It will be evident from the above discussion that the probable sources of error in the measurements tend to give too low a value, and hence, on *à priori* grounds, a high value is more likely to be correct. Although in an absolute determination we do not wish to lay too much stress on the point, we may draw attention to the fact that the mean percentage difference from the mean in the case of our determinations is considerably smaller than that of any of the other observers, a result which is probably due to the large values we obtain for the observed rotation.

WATER.

The water employed was distilled just before use, and boiled in a platinum dish in order to remove dissolved air. The values obtained for the magnetic rotation are given in the following table, coils I. and II.*b*, being employed throughout.

Water.			
t.	L.	α.	γ.
3°·97	4·43707	11° 57′·0	·01309 at 4°·0
·98	"	56·0	
·98	"	56·4	
·92	"	56·5	
4·00	"	56·2	

Water.

t.	L.	α.	γ.
°		° ′	
6·49	4·43703	11 57·2	·01311 at 6°·6
·56	”	57·0	
·62	”	56·7	
·60	”	57·0	
6·75	4·43705	11 57·5	·01311 at 6°·7
·73	”	56·6	
·75	”	56·9	
·71	”	57·2	
20·19	”	11 55·9	·01309 at 20°·2
·21	”	55·9	
·23	”	56·2	
21·25	4·43707	11 56·6	·01309 at 21°·2
·24	”	55·9	
·24	”	56·0	
·26	”	56·1	
32·69	”	11 53·8	·01305 at 32°·7
·71	”	54·4	
·72	”	54·4	
·73	”	54·0	
41·42	4·43707	11 53·3	·01304 at 41°·5
·47	”	53·5	
·50	”	53·8	
·53	”	52·9	
50·22	”	11 51·0	·01299 at 50°·4
·33	”	50·7	
·42	”	50·4	
·52	”	50·5	
60·57	”	11 48·0	·01294 at 60°·7
·63	”	47·5	
·72	”	48·0	
·78	”	48·7	
·87	”	48·4	
·84	”	48·2	
68·81	”	11 45·0	·01289 at 68°·9
·83	”	45·2	
·90	”	45·3	
·97	”	44·9	
78·93	”	11 42·3	·01284 at 78°·0
·93	”	42·5	
·94	”	43·0	
·97	”	43·0	
·97	”	42·9	
88·45	”	11 37·6	·01275 at 88°·5
·45	”	37·9	
·45	”	37·5	
·47	”	38·1	
97·66	4·43703	11 34·0	·01269 at 97°·7
·66	”	33·8	
·67	”	34·7	
·67	”	34·4	
·69	”	34·6	
·69	”	33·9	

On plotting these results it was seen that they no longer lay on a straight line, as in the case of carbon bisulphide, but on a curve, which was concave to the axis of temperature. They are best represented by the following equation :—

$$\gamma_t = {\cdot}01311\ (1 - {\cdot}0_4305t - {\cdot}0_5305t^2),$$

or

$$\gamma_t = {\cdot}01311 - {\cdot}0_64t - {\cdot}0_74t^2.$$

It will be seen from the following table that the agreement between the observed and calculated values is very satisfactory.

t.	γ.		Difference.
	Observed.	Calculated.	
4°0	·01309	·01311	−·00002
6·6	·01311	·01311	·00000
6·7	·01311	·01311	·00000
20·2	·01309	·01309	·00000
21·2	·01309	·01308	+·00001
32·7	·01305	·01305	·00000
41·5	·01304	·01303	+·00001
50·4	·01299	·01299	·00000
60·7	·01294	·01294	·00000
68·9	·01289	·01289	·00000
78·0	·01284	·01284	·00000
88·5	·01275	·01276	−·00001
97·7	·01269	·01269	·00000

The curve corresponding with this equation is given in fig. 3, and the values of γ at every 10° as calculated by the equation, are given in the table on p. 653.

DE LA RIVE[*] is the only previous observer who has attempted to determine the temperature variation of the magnetic rotation in water. The results which he gives are evidently smoothed values, and we find they lie on the straight line represented by

$$\gamma_t = \gamma_0\,(1 - {\cdot}0_378t).$$

The details given in the paper are not sufficient to admit of a discussion of the accuracy of the determinations, although it appears that the settings were read to the nearest 5′; with the rotation obtained a difference of 5′ amounts to 1 per cent.

The only other observers who have made absolute determinations of the magnetic rotation of water are ARONS[†] and QUINCKE[‡]. Their results are given in the following

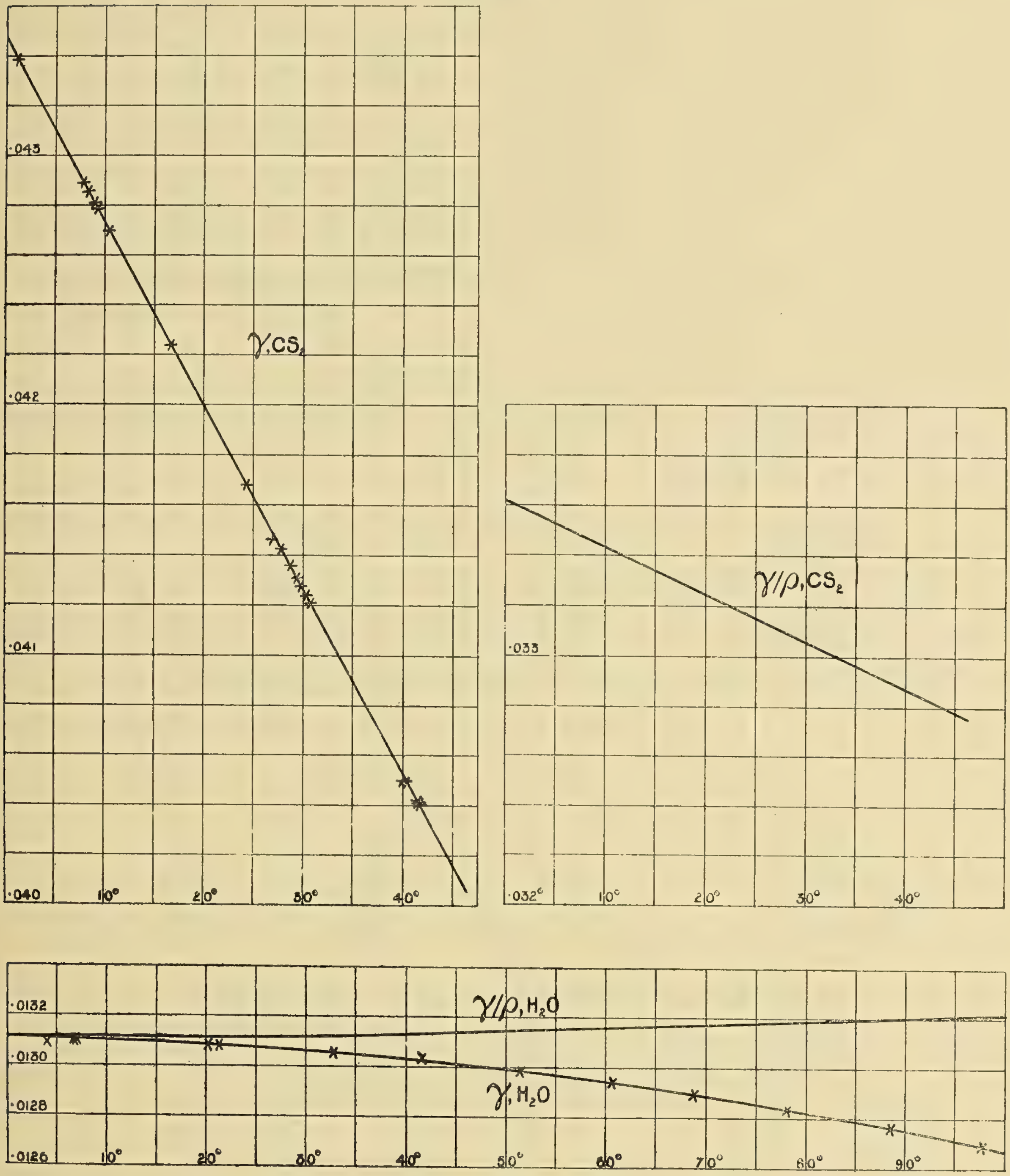
---

[*] 'Ann. Chim. Phys.' (4), 22, 5, 1871.

[†] 'Wied. Ann.,' 24, 161, 1885.

[‡] *Loc. cit.*

Fig. 3.


·043
·042
·041
·040
10°
20°
30°
40°
γ, CS₂
·033
·032°
10°
20°
30°
40°
γ/ρ, CS₂
·0132
·0130
·0128
·0126
γ/ρ, H₂O
γ, H₂O
10°
20°
30°
40°
50°
60°
70°
80°
90°

table, the last column of which contains their values reduced to 0° by our formula for the temperature variation.

	t.	γ_t.	γ_0.
ARONS	23°	·01295	·01298
QUINCKE	21·81	·01414	·01418
R. and W. . . .	..	..	·01311

For the reason already given we do not propose to discuss QUINCKE'S value which, as in the case of CS_2, is very much larger than the others. The difference between our value and that of ARONS amounts to 1 per cent., and is therefore very much greater than the difference between our value for carbon bisulphide and that of any of the other observers. What appears to be weak points in ARONS' methods are, first, that no test was applied of the insulation of the coil; second, that the settings do not seem to have been made with the analysing Nicol in two positions differing by about 180°; and third, that the method of current measurement was complicated, and necessitated a knowledge of the resistance of the coil at the time of observation.

ROTATION IN CARBON BISULPHIDE RELATIVE TO THAT IN WATER.

In addition to the measurements already considered, several observers have determined the rotation in carbon bisulphide in terms of the rotation in water. These results are collected in the following table. In those cases where the temperatures of observation are stated the values we obtain at those temperatures are given in the last column.

	t.	$\gamma_{CS_2}/\gamma_{H_2O}$.	R. and W.
DE LA RIVE*	° ..	3·16	..
BECQUEREL†	15	3·25	3·235
QUINCKE‡	21·06	3·22	3·204
WACHSMUTH§	..	2·6989	..
PERKIN‖	12·3	3·250	3·249

Of the results obtained by previous observers, that of Dr. PERKIN is no doubt the most accurate, and it is noteworthy that our value is practically identical with his. This affords additional evidence of the view that ARONS' value for water is much too low, since the difference between our value for water and that of ARONS is three times greater than the difference between our value for carbon bisulphide and that, say, of Lord RAYLEIGH.

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* *Loc. cit.*

† 'Ann. Chim. Phys.,' (5), 12, 5, 1877.

‡ *Loc. cit.*

§ 'WIED. Ann.,' 44, 377, 1891.

‖ This value has not yet been published, but Dr. PERKIN had the kindness to send it to us, together with the values of the density and boiling-point of his carbon bisulphide.

The following table contains our values for the relative rotation at every five degrees between 0° and 40°.

t.	$\gamma_{CS_2}/\gamma_{H_2O}$.
0°	3·316
5	3·288
10	3·261
15	3·235
20	3·209
25	3·184
30	3·159
35	3·135
40	3·111
45	3·087

RELATION BETWEEN MAGNETIC ROTATION AND DENSITY.

DE LA RIVE,[*] in 1871, investigated the effect of temperature on the value of the quotient, rotation divided by density, in the case of a few liquids. He found that for ethyl iodide, ethyl alcohol, and amyl alcohol, the quotient was approximately constant, while for water and sulphuric acid it diminished as the temperature rose. Since his time no definite attempt has been made to make a careful study of this question.

In the following table we give the values of this quotient for water and carbon bisulphide. The values of the density employed, in the case of water, were taken from ROSETTI's[†] tables, and in that of carbon bisulphide were deduced from our value at 0° and THORPE's[‡] numbers for the thermal expansion.

Water.		
t.	γ.	γ/ρ.
0°	·01311	·01311
10	·01310	·01311
20	·01309	·01311
30	·01306	·01312
40	·01303	·01313
50	·01299	·01315
60	·01294	·01316
70	·01289	·01318
80	·01282	·01319
90	·01275	·01321
100	·01267	·01322

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[*] *Loc. cit.*

[†] 'Ann. Chim. Phys.,' (4), 17, 370.

[‡] *Loc. cit.*

Carbon bisulphide.		
t.	γ.	γ/ρ.
0°	·04347	·03362
10	·04273	·03344
20	·04200	·03325
30	·04126	·03307
40	·04053	·03208

The quotient for water is practically constant up to 20°, it then very slowly *increases,* the rate of increase between 20° and 100° being practically constant.

For carbon bisulphide the quotient decreases at a constant rate as the temperature rises. It will be noticed, as is also evident from the curve (fig. 3), that the rate of decrease in the case of carbon bisulphide is very much greater than the rate of increase in the case of water.

The detailed discussion of this question cannot be profitably undertaken until the data for a larger number of liquids have been determined. Evidence on one important point can, however, be obtained from the numbers given above. In investigations dealing with the connection between the magnetic rotation and the chemical nature of liquids, it has been the usual custom to employ the expression

$$(\mathrm{M}\gamma/\rho)_{\text{substance}}/(\mathrm{M}\gamma/\rho)_{\text{water}},$$

as representing the measure of the molecular rotation, where M is the molecular weight. It is evident that the value of this expression will vary with the temperature unless the quotient γ/ρ in the case of the substance is affected to the same extent as it is in the case of water. From the above tables it follows that this does not hold good for water and carbon bisulphide. The values of this measure of the molecular rotation for carbon bisulphide at every 10° between 0° and 40° are given below.

Carbon bisulphide.		
t.	$(\gamma/\rho)_{CS_2}/(\gamma/\rho)_{H_2O}$.	$(\mathrm{M}\gamma/\rho)_{CS_2}/(\mathrm{M}\gamma/\rho)_{H_2O}$.
0°	2·564	10·838
10	2·551	10·783
20	2·536	10·720
30	2·521	10·656
40	2·504	10·584

It is therefore evident that before precise relationships can be established between magnetic rotation and chemical nature, the effect of temperature must be more fully investigated.

A knowledge of this effect is of especial importance when dealing with magnetic rotation as a means of detecting tautomeric change. From the large variation in the value of the ordinary measure of the molecular rotation PERKIN* concluded that the structure of certain substances altered as the temperature rose. In a recent paper BRUHL† has used measurements of the dispersion for the same purpose, and the results he obtained differ to a marked extent from those of PERKIN. It is not possible at present to say which of these properties is best suited to the end in view; it is evident, however, that the conclusions drawn from magnetic rotation will always be uncertain until more has been ascertained regarding the effect of temperature in the case of stable liquids like carbon bisulphide, where no change in structure is involved.

It will have been seen that in the ordinary method of treating the results of observations on magnetic rotation when dealing with questions of chemical constitution values are employed which do not refer to the substance alone, but which are affected by the properties of water. The only justification for the use of water in relative observations is the elimination of variations in the strength of the magnetic field in which the observations are made. If the temperature of observation is always the same this can readily be done; if, on the other hand, the temperature varies, it is essential to know how the rotation of water alters with the temperature. In the past, this alteration was unknown, and the arbitrary measure of the molecular rotation above referred to has come into use. Since an expression for the temperature variation has now been obtained it is to be hoped that observers will employ a measure of the molecular rotation which does not involve the properties of water. Indeed, other considerations make such a measure all the more desirable. Up till now we have made observations on eight liquids besides water and carbon bisulphide, and in all cases, except that of water, the relation between rotation and temperature is linear, and the quotient, rotation divided by density, diminishes as the temperature rises. It is highly probable, therefore, that as regards magnetic rotation, as in the case of so many other properties, the behaviour of water is exceptional, and hence it is particularly ill-suited for the use to which it has been put. Again, on account of the smallness of the rotation in water, the unavoidable inaccuracies in determining the rotation, and thus estimating the strength of the magnetic field, produce a larger percentage error in the results than if a liquid, such as benzene, having a considerably higher rotation than water, were used for this purpose.

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[*] 'Chem. Soc. Trans.,' 61, 800, 1892; 65, 815, 1894.

[†] 'Jour. Pract. Chem.,' (2), 50, 119, 1894.

XIX. *On the Occlusion of Oxygen and Hydrogen by Platinum Black.*—Part I.

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I. *Introduction.*

THE word "*occlusion*" was first used by GRAHAM,[*] to signify the absorption of, or shutting up, of gases in solid substances. Many porous bodies, such as charcoal, possess this property, and it is also to be met with among the metals, pre-eminently among the metals belonging to the eighth group of the Periodic Table, and especially among the platinum metals.

Platinum in the coherent state, and also in the state of sponge, has been very fully investigated by GRAHAM, in his classical researches. A large number of observations on platinum black are also on record,[†] but these, for the most part, have been made with impure material, and the results are often contradictory.

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[*] GRAHAM, 'Phil. Trans.,' 1866, 399; see also 'Chem. and Phys. Researches,' 1876, p. 263.

[†] *Cf.* the Dictionaries by GMELIN-KRAUT, LADENBURG, DAMMER, &c.

Our object has been to prepare platinum black in as pure a state as possible, and to subject it to careful examination. As is well known, platinum black is the most active form of platinum, it is a powerful oxidising agent, readily inducing so-called "catalytic actions." Discovered in 1820, by EDMUND DAVY,[*] it was thought by him to be platinous nitrite. DÖBEREINER,[†] BERZELIUS,[‡] and ZEISE[§] considered it a sub-oxide of platinum, whilst LIEBIG[||] pronounced it to be nothing but the very finely divided metal. How far LIEBIG was right will be seen during the course of this investigation, the results of which we now beg to lay before the Society. In modern times BERTHELOT and others have worked with platinum black, but a discussion of their work is reserved for a later part of this communication.

II. *Repetition of Graham's Experiments.*

Before proceeding to the description of our experiments on platinum black, we may state that a few experiments were made with platinum foil and platinum sponge. The foil which we used was obtained from Messrs. JOHNSON, MATTHEY and Co., and was said to be pure. It weighed 51 grms. and was about $\frac{1}{1000}$ inch in thickness. After being rolled up into the shape of a cylinder it was introduced into an experimental tube of hard glass, somewhat similar to that represented in fig. 2A. The temperature and pressure of the air in the experimental tube were noted, and from a subsequent determination of the capacity of the tube it was found, on exhausting the tube at the ordinary temperature by means of a SPRENGEL pump, that the volume of the air pumped out corresponded exactly to the amount of air contained originally in the tube. In other words, the platinum foil gave off no measurable quantity of gas when exhausted at the ordinary temperature. On heating the foil to a dull red heat, however, 1·04 cub. centim. of gas was extracted. Taking the density of the platinum foil as 21·5, this amount of gas corresponds to 0·4 vol. of gas given off, the volume of the platinum being taken as unity. The composition of the gas was as follows:—

	cub. centims.		vol.
CO_2	0·76	or	0·3
O_2	trace		trace
Unabsorbed by alkaline pyrogallate .	0·28		0·1
	1·04		0·4

Pure dry hydrogen was then admitted to the platinum foil and left in contact with it for three days. On connecting the tube with the pump again it was found that no hydrogen was given off from the foil at the ordinary temperature, and only a trace at a red heat. The platinum foil was next kept in contact with hydrogen for three hours at a red heat, and then allowed to cool down gradually in an atmosphere of the gas, but the same result was obtained. It was then charged twice with pure dry oxygen, once at the ordinary temperature and again at a red heat, but no appreciable quantity (0·1 cub. centim.) of gas could be pumped off.

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[*] ED. DAVY, 'Phil. Trans.,' 1820, p. 108; 'Schweigg. Journ.,' 31, p. 340.

[†] DÖBEREINER, 'LIEBIG'S Annalen,' 14, p. 10.

[‡] BERZELIUS, 'Schweigg. Journ.,' 34, 81.

[§] ZEISE, 'Kongl. Danske Vidensk. Selsk. Forh.,' 1825, 26, 13.

[||] LIEBIG, 'Pogg. Annalen,' 17, 102.

GRAHAM found that the amount of hydrogen occluded by platinum depended largely on the texture of the sample employed. The following volume of hydrogen he found were occluded by unit volume of platinum.

Platinum wire (from fused platinum) .	0·128 — 0·207	vols.
Wrought platinum (old crucible) . .	3·83 — 5·53	,,
,, ,, (old tube) . . .	2·28 — 2·80	,,
Platinum sponge	1·48	,,

It will be seen from these numbers that the least absorptive form of platinum is the wire made from fused metal, but its absorptive power is greater than the foil examined by us. It is also very remarkable that wrought platinum occludes about three times as much hydrogen as platinum sponge. As it was conceivable that the texture of our platinum foil might be altered by alternately charging it with hydrogen and oxygen electrolytically, it was placed in dilute sulphuric acid and made to serve as the negative and positive pole alternately during the electrolysis of the acid. It was finally charged with hydrogen, washed with distilled water, dried in the steam bath, and then replaced in the experimental tube. No gas was given off *in vacuo* at the ordinary temperature, but at a red heat 1·20 cub. centim., equal to 0·5 vol., were extracted. This gas suffered no diminution of volume on being treated with caustic potash and alkaline pyrogallate, and caught fire on applying a small flame. It was probably hydrogen, and shows that platinum foil charged with hydrogen may be exposed to the air and heated in a steam bath without the whole of the hydrogen being oxidised to water by the oxygen of the air. The former experiments were now repeated, but it was found that the occlusive power of the platinum foil had not altered by alternately charging it with oxygen and hydrogen electrolytically.

A similar set of experiments was now made with platinum sponge, 44·2 grms. = 2·06 cub. centims. of platinum sponge, prepared by igniting platinum black at a red heat, were introduced into the experimental tube. After charging with oxygen and hydrogen practically no gas was given off *in vacuo* at the ordinary temperature. The following results were obtained :—

Oxygen.

	Oxygen extracted *in vacuo* at a dull red heat.
Charged with O_2 at a red heat for several hours, and allowed to cool down gradually	4·6 cub. centims. = 2·1 vols.
Charged a second time	4·9 ,, = 2·4 ,,

Hydrogen.

	Hydrogen extracted *in vacuo* at a dull red heat.
Charged with H_2, 3 days at ordinary temperature	1·4 cub. centims. = 0·7 vols.
,, ,, for one night	0·9 ,, = 0·4 ,,
,, ,, at a red heat, and cooled gradually	6·5 ,, = 3·1 ,,
,, ,, ditto ditto.	5·2 ,, = 2·5 ,,

The results with hydrogen are rather surprising, inasmuch as platinum sponge occludes more hydrogen when it is heated in the gas and then allowed to cool down gradually than when it is simply allowed to remain in contact with the gas at the ordinary temperature.

It will be seen later on, that platinum black behaves in precisely the opposite manner. If the sponge is left in contact with the hydrogen for a sufficiently long time it may, however, as the experiments seem to indicate, take up the same quantity of hydrogen as when it is heated. The action of the heat may thus be simply to increase the rate at which absorption takes place.

Another series of experiments was made with sponge prepared by igniting ammonium platinichloride.

The sponge weighed 9·15 grms. and its volume was taken as 0·44 cub. centim. As will be seen from the following numbers, its occlusive power for both oxygen and hydrogen was less than that of the first sample of sponge. Initially it was found to be charged with 0·39 cub. centim. = 0·9 vol. of a gas which was not further investigated.

	Gas exhausted *in vacuo* at a red heat.
Charged with H_2 at a red heat	0·3 cub. centims. = 0·7 vol.
,, ,, at ordinary temperature	0·2 ,, = 0·4 ,,
,, O_2 at a red heat	0·2 ,, = 0·4 ,,
,, ,, at ordinary temperature	0·25 ,, = 0·5 ,,

GRAHAM also made some experiments with the view to determine the influence of temperature on the absorption of hydrogen by platinum. Thus he found that a sample of platinum foil charged with hydrogen for three hours at 230° C., contained 1·45 vol. of hydrogen, whilst the same sample left in contact with hydrogen for the same time at 100° C., had only occluded 0·76 vol. of hydrogen. This agrees with what we have also found for platinum sponge.

III. *Preparation of Platinum Black.*

The best method of preparing platinum black is as follows:—100 grms. of hydrogen platinichloride are dissolved in about two litres of distilled water and boiled. After neutralizing with sodium carbonate, the boiling solution is slowly poured into a boiling solution of about 100 grams of sodium formate in two or three litres of water. Violent effervescence takes place, and the platinum is precipitated as a black powder, which is then thoroughly boiled out about eight or ten times with distilled water, the water in each case being separated by decantation. On one occasion, the platinum black was boiled out with dilute acids and alkalis. This process is, however, not to be recommended, as the dried substance on heating *in vacuo* gave off corrosive vapours which attacked the mercury of the pump and consequently had to be redissolved and reprecipitated in the usual way. The platinum black drained as free as possible from water is then dried either in a steam bath at 100° C. or over calcium chloride at the ordinary temperature.

Platinum black prepared in this way is a fine, black, impalpable powder and can be poured from one vessel into another like precipitated silica. It ought to be carefully preserved from all traces of grease and volatile matter. In all the experiments which we have made with it, ordinary greased taps and india-rubber joints have been avoided as much as possible. Taps can be lubricated with slightly deliquesced phosphorus pentoxide, and after a little practice glass tubes can be joined together before the hand blow-pipe about as easily, and much more securely, than by means of rubber tubing and wire.

IV. *The Water retained by Platinum Black.*

LIEBEG stated that platinum black was nothing else than finely divided platinum. Qualitatively speaking, this may be quite true, but we have never yet succeeded in preparing platinum black free from, firstly, water; and secondly, oxygen and traces of carbon dioxide. It is impossible, even by means of phosphorus pentoxide *in vacuo*, to completely remove the water below a temperature of at least 400° C., about which temperature the platinum black begins to pass into platinum sponge.

In order to see whether the amount of water given off, or the amount of water retained, by platinum black *in vacuo* at different temperatures were constant, the following series of water determinations was made. The platinum black, dried at 100° C., was placed in a hard glass tube connected with a weighed U-tube, containing phosphorus pentoxide, which was attached to the pump. A vacuum was then produced in the apparatus whilst the platinum black was kept heated at the required temperature until all the water had been taken up by the phosphorus pentoxide.

The increase in weight of the tube gave the amount of water directly. The following table gives the percentage amount of water given off by the platinum black at the different temperatures. Three different preparations of platinum black were used, and it will be seen that they contained about 0·5 per cent. of water, and that about one half of the total quantity of water was given off *in vacuo* at 130° C.

Temperature.	1st preparation.	2nd preparation.	3rd preparation.		
	I.	II.	III.	IV.	V.
°	per cent.	per cent.	per cent.	per cent.	per cent.
130	..	0·26	0·21	..	..
237	..	..	0·33	..	..
280	..	..	..	0·40	0·38
320	..	..	..	..	0·43
330	..	..	..	0·46	..
360	..	..	..	..	0·47
410	..	..	..	0·49	..
440	..	..	0·43	..	..
650 (?)	0·54	0·46	0·45	0·52	0·49

From these figures it would be fair to assume that at any given temperature the amount of water retained by platinum black is constant.

V. *The Density of Platinum Black.*

During the course of this investigation it was frequently necessary to know the volume occupied by a given weight of platinum black. A determination of its density was therefore necessary. This was made in the usual way with a pyknometer specially constructed for the purpose. After the platinum black was introduced into the pyknometer it was half filled with distilled water and connected with a pump. The water, with the platinum black suspended in it, was then boiled, in order to get rid of all traces of air adhering to the powder. After the pyknometer had cooled down to the ordinary temperature again it was filled up to the mark with boiled distilled water. The accurate adjustment of the water level to the mark was made after the temperature 25° C. had been attained by plunging the pyknometer into a thermostat at 25° C.

Weight of platinum black taken	11·4811 grms.
,, pyknometer and water, at 25° C. . . .	16·3099 ,,
,, ,, ,, + platinum black .	27·1989 ,,

The loss of weight of the platinum black in water is thus 11·4811 + 16·3099 − 27·1989 = 0·5921 grm., and hence the density of platinum black at 25° C., compared with water at the same temperature, is 11·4811 ÷ 0·5921 = 19·4.

The platinum black, however, contained 0·552 per cent. of water, and if we calculate the density of dry platinum black, which still, however, contains oxygen, as will be seen later on, we get for the loss of weight in water

$$11{\cdot}4117 + 16{\cdot}3099 - 27{\cdot}1989 = 0{\cdot}5287 \text{ grm.},$$

and for the density 25°/25°

$$11{\cdot}4177 \div 0{\cdot}5287 = 21{\cdot}6,$$

or, compared with water at 4° C., 21·5.

As it is only necessary to know the density in order to translate the results of actual experiments into volumes of gas given off or occluded by unit volume of platinum black, the density 21, which is sufficiently near for the purpose, has been used throughout to effect the comparisons.

VI. *The Absorption of Oxygen by Platinum Black.*

As has already been stated, platinum black invariably contains a considerable quantity of oxygen which has no doubt been absorbed or occluded from the air during the process of washing or drying. Platinum black prepared in the way described in Section III. takes up no more oxygen on being exposed to an atmosphere of this gas for several days. The object of the following series of experiments was to determine the total quantity of oxygen and carbon dioxide contained in platinum black, and also to ascertain how much of these gases was given off *in vacuo* at different temperatures. For this purpose the platinum black dried at 100° C. was heated *in vacuo* in a hard glass tube A (fig. 2, p. 668), at first to moderate temperatures by means of the vapour jacket F, and finally to a red heat. The experimental tube A was attached to the pump and the rest of the apparatus by a ground glass joint B, lubricated with phosphoric acid, and further protected by a little mercury poured into the thistle-shaped enlargement at the top. It is bent as shown in the figure, and is supposed to lie in front of the plane of the paper in order to prevent the upward current of hot air from acting prejudicially on the ground joint and the taps. The washing and drying tubes shown in the figure, with the exception of the tube E, which was filled with phosphorus pentoxide to dry the gas evolved, may be left out of consideration. Before beginning an experiment, the capacity of the experimental tube, together with that of the narrow bore tubing between the taps C and D, was first of all determined by pumping out the air and measuring it at the same temperature and pressure as that which obtained when the taps were shut previous to exhausting the apparatus up to the tap D. After introducing the platinum black, the true capacity of the experimental tube was then found by subtracting the volume of the platinum black from the capacity of the tube as determined in the above way. On now exhausting the tube A, containing a known quantity of platinum, it was found that no measurable amount of gas was given off from the platinum black at the ordinary temperature *in vacuo.* The platinum black was then heated successively at the different temperatures given in the following tables until no more gas was given off. It was found that the gas consisted chiefly of oxygen. At low temperatures (up to 237°), almost pure carbon dioxide along with a trace of nitrogen or some other gas not absorbed by alkaline pyrogallate was however evolved. Frequently, a slight explosion* or sudden swelling up of the platinum black was noticed in the neighbourhood of 300° C., but the cause of this is unknown, and in no case was the explosion accompanied by a large evolution of gas. In the following tables the temperatures are contained in the first column; the second and third columns contain the amount, in cubic centimetres, of carbon dioxide and oxygen respectively given off. In the last column are tabulated the number of volumes of oxygen given off by unit volume of platinum black. All the gas measurements contained in this communication are reduced to 0° C. and 760 millims., unless otherwise stated.

I. Platinum Black taken, 5·1187 grms. = 0·244 cub. centim.

Temperature.	CO_2.	O_2.	O_2.
°	cub. centims.	cub. centims.	(vols. Pt. = 1).
17 ⎧0·0 ⎫			
100 ⎪0·0 ⎪			
184 ⎨0·74 ⎬†	0·67	2·97	12·2
237 ⎪0·83 ⎪			
280 ⎩2·07 ⎭			
340	..	15·07	61·8
400	..	2·30	9·4
650 (?)	..	0·55	2·2
		Total . . .	85·6

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* *Cf.* Descotils, 'Gilb. Annalen,' 27, 231.

† In this experiment the total gas given off up to the temperature 280° C. was analysed all together.

II. Platinum Black taken, 5·2380 grms. = 0·249 cub. centim.

Temperature.	CO_2.	O_2.	O_2.
°	cub. centims.	cub. centims.	(Vols. Pt. = 1).
18	0·0	0·0	0·0
100	0·82	0·02	0·09
184	2·73	0·06	0·24
237	0·08	0·53	2·13
280	..	1·44	5·78
360	..	8·19	32·89
440	..	5·15	20·68
650 (?)	..	2·56	10·28
		Total . . .	72·09

III. Platinum Black taken, 5·0754 grms. = 0·242 cub. centim.

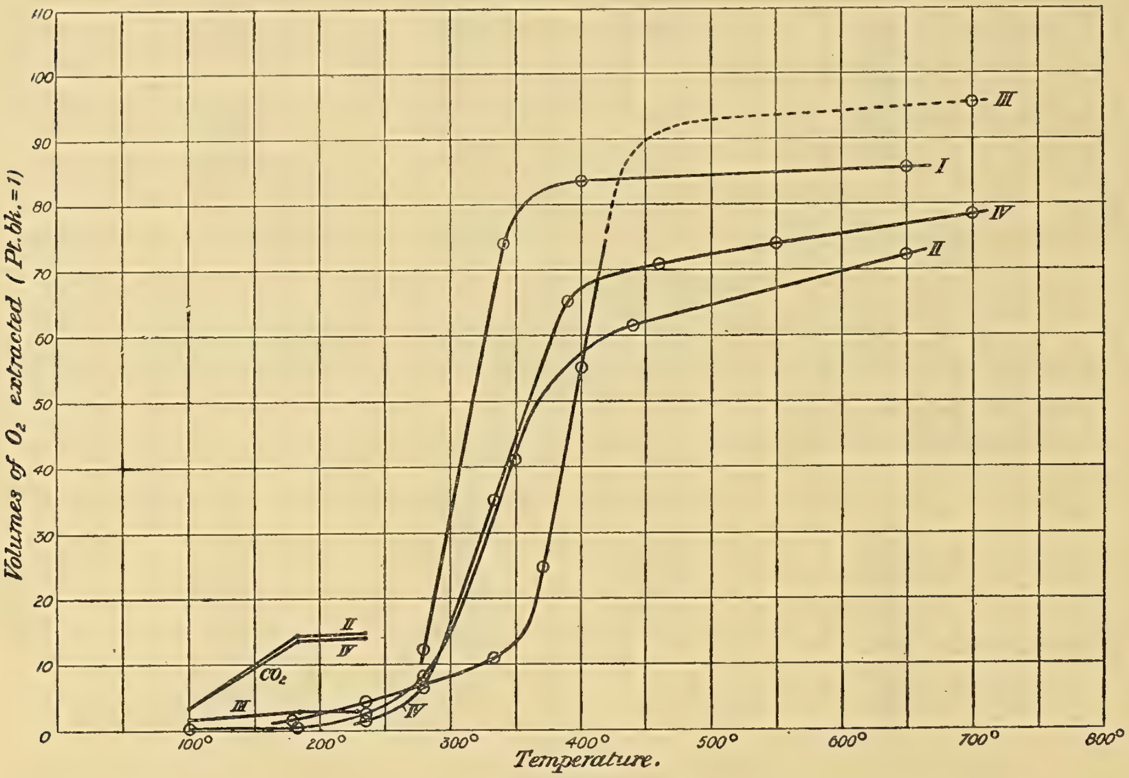
Temperature.	CO_2.	O_2.	O_2.
°	cub. centims.	cub. centims.	(Vols. Pt. = 1).
18	0·0	0·0	0·0
100	0·40	0·0	0·0
184	0·32	0·37	1·53
237	0·05	0·65	2·69
280	..	0·75	3·10
333	..	0·88	3·64
370	..	3·36	13·88
400	..	7·48	30·91
700 (?)	..	9·63	39·79
		Total . . .	95·54

IV. Platinum Black taken, 5·8069 grms. = 0·277 cub. centim.

Temperature.	CO_2.	O_2.	O_2.
°	cub. centims.	cub. centims.	(Vols. Pt. = 1).
18	0·0	0·0	0·0
100	0·94	0·0	0·0
184	2·74	0·0	0·0
237	0·24	0·41	1·48
280	..	1·30	4·69
335	..	7·92	28·59
390	..	8·46	30·55
460	..	1·37	4·95
550	..	1·03	3·72
700 (?)	..	1·22	4·40
		Total . . .	78·38

The results of the above experiments are plotted graphically in the accompanying curves, fig. 1, in which the ordinates represent the number of volumes of oxygen or carbon dioxide given off by unit volume of platinum black, and the abscissæ the temperatures. A glance at the above tables or curves shows that the oxygen begins to come off about 200° C., that the evolution is very rapid between 300° and 400°, and that the bulk of the oxygen is given off a little above the latter temperature.

Fig. 1.



The following short table gives the total volumes of carbon dioxide and oxygen given off per unit volume of platinum black.

	CO_2.	O_2.	CO_2+O_2.
I.	2·7	85·6	88·3
II.	14·18	72·09	86·27
III.	3·18	95·54	98·72
IV.	14·15	78·38	92·53

The numbers vary considerably, although the platinum black used in all four cases was taken from the same preparation, but it is very remarkable that when the amount of oxygen obtained is small (II. and IV.) the amount of carbon dioxide is large.

The curves representing the evolution of carbon dioxide are also shown in fig. 1, and if the amount of carbon dioxide is added to the quantity of oxygen extracted, curves are obtained which more nearly coincide. It is difficult to account for this. If the platinum black had got contaminated with traces of organic matter in any conceivable way, then possibly the carbon might annex some of the oxygen, and this would afterwards reappear as carbon dioxide. Carbon dioxide formed in this way would occupy the same volume as the original oxygen, and some weight is lent to this hypothesis by the fact that the sum of the oxygen and carbon dioxide is more nearly constant.

Platinum black, dried at 100° C., thus contains rather less than 100 volumes, or about 0·66 per cent. by weight of oxygen. The amount of oxygen contained in platinum black varies, however, in different samples, and seems also to depend largely on the temperature at which it is dried.

As regards the carbon dioxide contained in the four specimens of the same preparation of platinum black which have been examined, its amount seems to vary between about 3 and 14 volumes; but, as has already been suggested, the apparently larger quantities of carbon dioxide may possibly correspond to an accidental introduction of some carbonaceous impurity. The numbers representing the total absorption of oxygen by platinum black have been confirmed in a later section of this communication, using a different sample of platinum and an indirect method.

VII. *The Absorption of Hydrogen by Platinum Black.*

Platinum black, as is well known, possesses, in common with palladium, the power of absorbing or occluding considerable quantities of hydrogen. In view of the fact, however, that platinum black always contains a certain amount of oxygen, it is important, in determining its absorptive power, to distinguish between the amount of hydrogen which is really absorbed by the platinum black *per se*, and that which is simply burnt up to form water by the oxygen already contained in platinum black. In making experiments with a view to ascertain the absorptive power of platinum black, prepared in the usual way, for hydrogen, this has been steadily borne in mind.

The apparatus which was used was essentially the same as that used for the determination of the oxygen in platinum black, with the addition of the washing and drying tubes shown on the left of fig. 2.

Hydrogen, prepared to begin with from pure zinc and pure sulphuric acid, was supplied by means of a KIPP'S generating apparatus to the first washing tube, G (RICHARDSON'S pattern),[*] containing a solution of potassium permanganate; it then passed through the next washing apparatus, H, charged with a solution of potassium hydroxide. Finally, the hydrogen was thoroughly dried by traversing the U-tubes,

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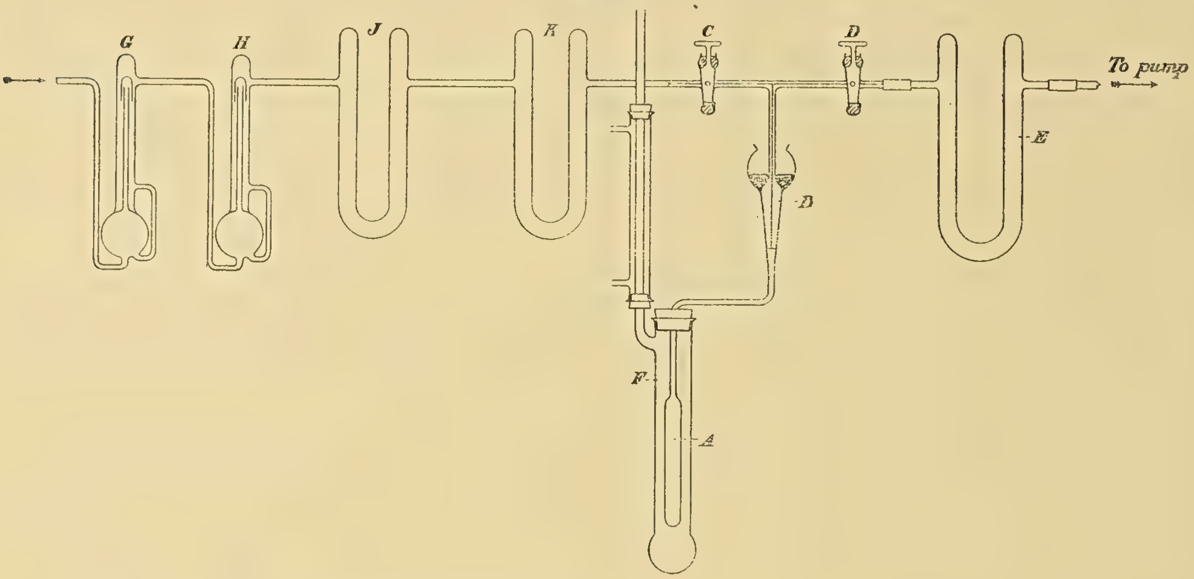
[*] RICHARDSON, 'Trans. Chem. Soc.,' 1894, 469.

J and K, filled with fragments of solid potassium hydroxide and phosphorus pentoxide respectively.

Fig. 2.



The platinum black under investigation was placed in the experimental tube A, and the water which was pumped off was collected and weighed in the U-tube E. The tube E was attached to the apparatus by stout rubber tubing joints, which were securely wired. The hydrogen pumped off by the Sprengel pump at the different temperatures, and finally at a red heat, was collected and measured dry over mercury. Knowing, then, the amount of water originally contained in the platinum black, and the amount of water collected in E after charging with hydrogen, it was easy, from the difference of the two, to calculate both the amount of hydrogen which had gone to form water with the oxygen pre-existing in the platinum black, and also this amount of oxygen itself. As a check on these results, however, the total amount of hydrogen absorbed by the platinum black (independent of that required to fill the experimental tube A, of known capacity) was determined by introducing a gas burette, B, shown in fig. 11, p. 687, between the stopcock C, and the U-tube K.

In performing an experiment, the platinum black was first introduced into the experimental tube A, which was then rendered vacuous by means of the pump. The stopcock D was now shut, and the pure dry hydrogen slowly admitted in to the experimental tube A, through the stopcock C, until no more absorption took place. The bulk of the hydrogen was absorbed almost instantaneously, but, on allowing the stopcock to remain open for a few hours, a very slight additional amount of hydrogen was absorbed. The quantity of hydrogen filling the experimental tube at the ordinary temperature and pressure could easily be calculated from the capacity of the tube diminished by the volume occupied by the platinum black itself.

The following four tables contain the results obtained in four experiments made with the same preparation of platinum black, dried at 100° C. It will be noticed that platinum black charged with hydrogen behaves in a different manner from platinum black charged with oxygen, inasmuch as part of the hydrogen can be pumped off *in vacuo* at the ordinary temperature. The first column in the tables gives the temperatures at which the platinum black was heated. The lower temperatures were maintained by means of vapour jackets, whilst for the higher temperatures an air-bath was used, and the temperatures were determined either by a BALY and CHORLEY thermometer or by a LE CHATELIER thermo-electric junction. The second and third columns contain the total quantities of hydrogen given off, in cubic centimetres and volumes (platinum black = 1 vol.) respectively.

A. PLATINUM Black used, 5·3194 grms. = 0·253 cub. centim. Total hydrogen absorbed, 67·9 cub. centims.

Temperature.	Hydrogen exhausted up to temp. *t*.	
°	cub. centims.	Vols. (Pt. bk. = 1).
15	4·30	17·0
100	9·25	36·6
150	13·23	52·3
225	15·11	59·7
410	17·09	67·6
580	18·03	71·3

Water found, 0·0742 grm.

B. PLATINUM Black used, 5·3067 grms. = 0·253 cub. centim. Total hydrogen absorbed, 78·34 cub. centims.

Temperature.	Hydrogen exhausted up to temp. *t*.	
°	cub. centims.	Vols. (Pt. bk. = 1).
15	6·19	24·5
100	13·75	54·5
200	21·86	86·6
300	23·87	94·5
400	25·01	99·0
550	26·25	104·0

C. PLATINUM Black used, 5·1763 grms. = 0·246 cub. centim. Total hydrogen absorbed, 76·01 cub. centims.

Temperature.	Hydrogen exhausted up to temp. *t*.	
°	cub. centims.	Vols. (Pt. bk. = 1).
17	6·66	27·0
100	10·85	44·0
184	19·28	78·2
237	20·96	85·0
370	23·01	93·4
550	25·23	102·4

D. PLATINUM Black used, 5·1362 grms. = 0·245 cub. centim. Total hydrogen absorbed, 76·69 cub. centims.

Temperature.	Hydrogen exhausted up to temp. *t*.	
°	cub. centims.	Vols. (Pt. bk. = 1).
18	6·62	26·4
100	11·82	48·2
184	20·57	84·0
237	22·72	92·7
280	24·02	98·0
400	25·12	102·5
600	26·82	109·5

Water found, 0·0737 grm.

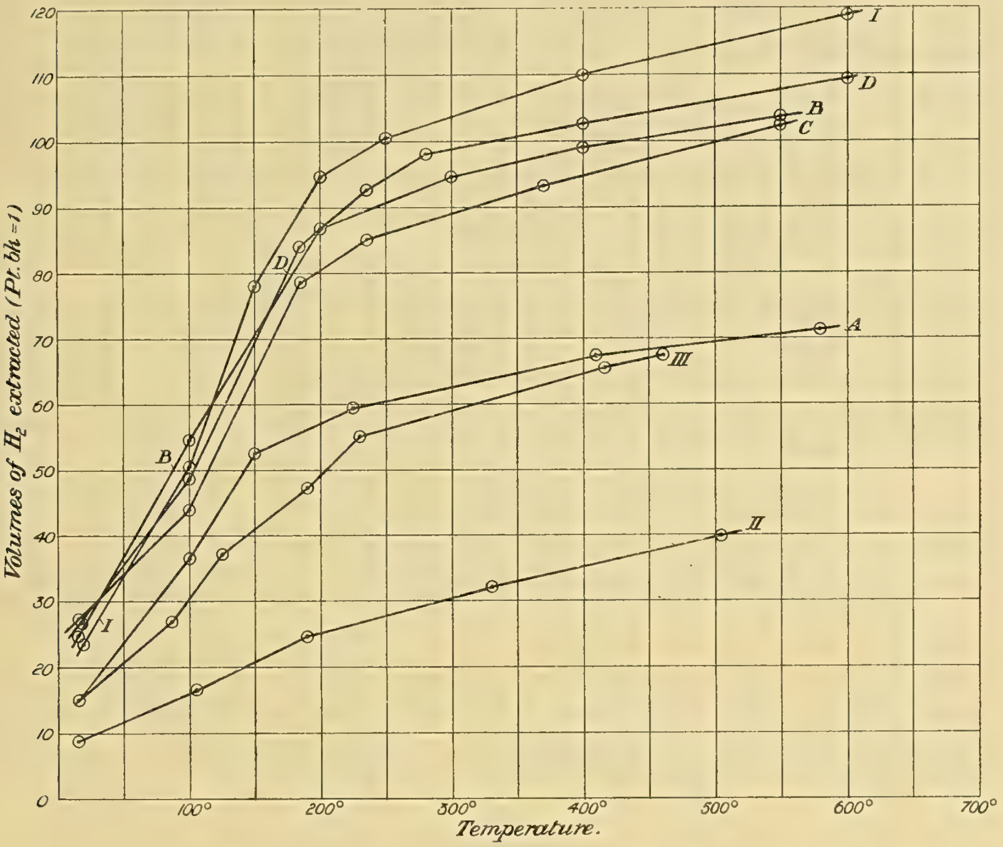
These results, together with three others, marked I., II., and III., are plotted graphically in the accompanying plate of curves. The ordinates represent the total number of volumes of hydrogen given off by unit volume of platinum black, whilst the abscissæ give the temperatures.

The Curves I., II., and III. were obtained from a different preparation of platinum black, and a slightly different method was used. Curve A is evidently too low, although the experiments were made as nearly as possible under the same conditions. In general, however, it is evident from a consideration of the curves that the hydrogen is more easily pumped off than oxygen, and that about five-sixths of the total quantity can be extracted about 300° C. It was also noticed that platinum black which had been charged with hydrogen was converted into platinum sponge at a lower temperature than platinum black which had not been so charged. The Curves B, C, and D are fairly concordant with each other and also with I. where a different sample of platinum black was used.

Neglecting A, the total amount of hydrogen absorbed in Experiments B, C, and D is :—

B.	C.	D.
309·7 vols.	309·1 vols.	313·0 vols.

Fig. 3.



Part of the total hydrogen absorbed, namely, that portion which has not gone to form water with the oxygen originally contained in the platinum black, is again evolved on heating to a dull red heat *in vacuo*. Assuming that all the rest has been converted into water in this way, an assumption which is justified by the results, we may construct the following table, which explains itself:—

Experiment.	Platinum black used.	Total hydrogen absorbed.	Hydrogen extracted *in vacuo* at a red heat.	Difference between last two.	Water corresponding to difference.	Water originally in platinum black.	Sum of the last two.	Total water found.	Difference (experimental error).
	grms.	cub. centims.	cub. centims	cub. centims.	grms.	grms.	grms.	grms.	grms.
A	5·3194	67·9	18·03	49·87	0·0404	0·0287	0·0691	0·0742	+0·0051
B	5·3067	78·34	26·25	52·09	.. ..	.. ..	.. ..	.. ..	.. ..
C	5·1763	76·01	25·23	50·78	.. ..	.. ..	.. ..	.. ..	.. ..
D	5·1362	76·69	26·82	49·87	0·0404	0·0277	0·0681	0·0737	+0·0056
I.	5·133	.. ..	29·15	.. ..	.. ..	.. ..	.. ..	.. ..	.. ..
II.	5·236	59·2	9·90	49·30	0·0399	0·0242	0·0641	0·0601	−0·0040
III.	5·190	67·2	16·68	50·52	0·0410	0·0240	0·0650	0·0689	+0·0039

From the table it is evident that no matter what the absolute amounts of hydrogen absorbed and hydrogen evolved may be, the differences between the two are in all cases about 50 cub. centims. The slight variations in the weight of platinum black used do not appreciably interfere with this coincidence.

This 50 cub. centims. of hydrogen, even in the case of the apparently bad experiments A, II., and III., probably gets about 25 cub. centims. of oxygen from the platinum black to form water, and the amount of water capable of being formed in this way is given in the table. If to this amount of water we add the water originally contained in the platinum black, we get a number which corresponds as closely as can be expected with the total amount of water found.

Of the total absorption of hydrogen by platinum black, about 310 volumes, only about 110 (the maximum is 119) volumes are really occluded by the platinum *per se.*

The amount of oxygen originally contained in six of the above samples of platinum black, viz., A, B, C, D, II., and III., can be calculated in the following way.

The difference between the total hydrogen absorbed and the hydrogen finally extracted, tabulated in the fifth column of the above table, represents the amount of hydrogen which has been burnt up to water by the oxygen contained in the platinum black.

If we call this quantity n, then $n/2$ represents the quantity of oxygen contained by the platinum black, and $n/2v$, where v is the volume of the platinum black, the number of volumes of oxygen occluded by unit volume of platinum. The following values are thus obtained :—

A	contained	98·6	vols.	of oxygen.
B	„	103·0	„	„
C	„	103·2	„	„
D	„	101·8	„	„
II.	„	99·0	„	„
III.	„	102·3	„	„

The direct determinations described in the previous section showed that platinum black contained rather less than 100 volumes of oxygen, so that the results obtained by the two different methods agree very well.

VIII. *The Influence of Diminished Pressure on the Absorption of Hydrogen by Platinum Black.*

The platinum black used for the following experiments had not such a high absorptive power for hydrogen as some of the samples previously examined. In the course of some other experiments it had been charged with hydrogen and exhausted at 130° C. It was then again charged with hydrogen at atmospheric temperature and pressure and exhausted at ordinary temperature. In this condition it still retained a large quantity of hydrogen. In order to see the effect of diminished pressure on the absorption of the rest of the hydrogen, the gas was admitted from a gas burette into the apparatus similar to that represented in fig. 2 until the required pressure was registered by means of a manometer which replaced the U-tube E, in fig. 2. The time allowed for charging ranged from 20 to 45 hours. The hydrogen contained in the manometer and pump was first pumped out and measured, then the gas filling the experimental tube A at the given temperature and pressure together with the hydrogen given off by the platinum black was extracted and measured. From these measurements and the amount of gas which had disappeared from the gas burette the following table was drawn up.

Column 1 gives the pressure at which the platinum black was charged with hydrogen.

Column 2, the hydrogen used as measured by the gas burette.

Column 3, the hydrogen contained in the experimental tube (calculated from the capacity of the tube and the temperature and pressure of the gas).

Column 4, the hydrogen contained in the manometer and pump. This gas was pumped out separately and measured directly.

Column 5, the amount of hydrogen actually absorbed, *i.e.*, the total hydrogen used, minus the sum of the volumes of the hydrogen contained in the experimental tube and pump.

Column 6 gives the amount of hydrogen extracted from the platinum black after subtracting the hydrogen contained in the experimental tube.

All the experiments were made at the ordinary temperature (about 19° C.) and the weight of the platinum black used was 7·96 grms. = 0·379 cub. centim.

Pressure.	Total hydrogen used.	Hydrogen contained in experimental tube (calculated).	Hydrogen contained in manometer and pump (found).	Hydrogen.	
				Absorbed.	Exhausted.
millims.	cub. centims.	cub. centims.	cub. centims.	cub. centims.	cub. centims.
767·0	38·05	17·94	17·12	2·99	3·05
371·4	17·68	8·69	6·08	2·91	2·90
176·5	9·42	4·14	2·51	2·77	2·78
30·9	3·46	0·72	0·42	2·32	2·31
11·1	2·26	0·26	0·22	1·78	1·80

As a check on these numbers, the platinum black was again charged with hydrogen, at ordinary temperature and pressure, and the gas pumped out fractionally at reduced pressures. 3·20 cub. centims. were required to charge the platinum black fully, and the following quantities were pumped off at reduced pressures.

Pressure.	Hydrogen extracted.
millims.	cub. centims.
416·9	0·00
264·5	0·59 (?)
125·7	0·38
59·5	0·21
22·4	0·39
0·0	1·89
	Total . 3·46

The total hydrogen extracted exceeds that required for charging by 0·26 cub. centim.

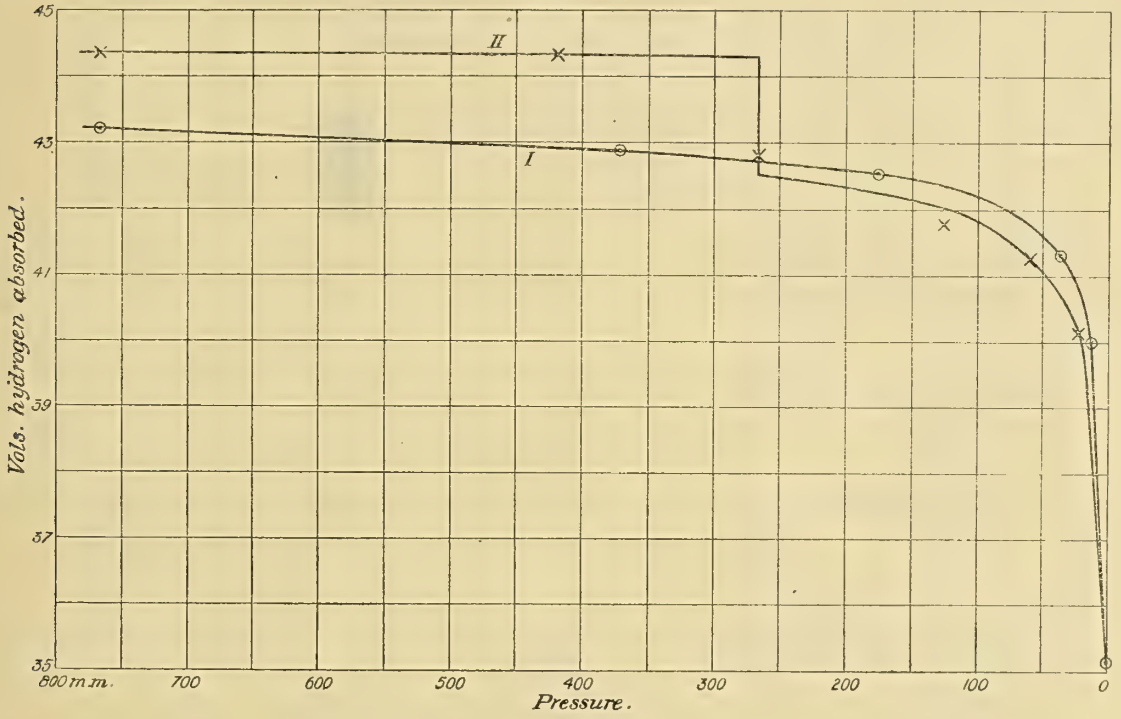
This is due to the fact that a slight leakage into the pump occurred at the pressure 264·5, so that the quantity pumped off at this pressure (0·59) is certainly too high.

The platinum black exhausted at the ordinary temperature gave off 13·14 cub. centims. = 35·13 volumes of hydrogen when heated to dull redness. We are, therefore, in a position to draw up the two following tables, which show the effect of diminished pressure on the absorption of hydrogen by platinum black. In the first series of experiments the mean of the hydrogen absorbed and hydrogen exhausted is used.

I.			II.		
Pressure.	Hydrogen absorbed.		Pressure.	Hydrogen absorbed.	
millims.	cub. centims.	vols.	millims.	cub. centims.	vols.
0·0	13·14	35·13	0·0	13·14	35·13
11·1	14·94	39·95	22·4	15·03	40·19
30·9	15·45	41·31	59·5	15·42	41·23
176·5	15·92	42·56	125·7	15·63	41·79
371·4	16·04	42·89	264·5	16·01	42·81
767·0	16·16	43·20	416·9	16·60	44·38
			(767·0)	16·60	44·38

The results of these determinations are plotted graphically in fig. 4. It will be seen that diminished pressure has a certain but not very pronounced effect on the absorption of hydrogen by platinum black at pressures above 200 or 300 millims. Below this pressure, however, the absorption rapidly falls off, and then remains nearly constant again. In other words, platinum black *in vacuo* absorbs a certain amount of hydrogen. This particular sample absorbed 35 volumes of hydrogen, but if a better sample had been employed the curves would probably have been displaced upwards parallel to themselves or nearly so, until 35 had become about 75, as in the case of the best samples of platinum black. On increasing the pressure of the hydrogen up to about 300 millims. a further quantity is absorbed (with the best samples of platinum black this would be about 25 or 30 volumes). An increase of pressure now has no appreciable effect up to pressures slightly above the atmospheric pressure. The effect of higher pressures is reserved for the next section, but it would seem from the run of the curves that this would also have little or no effect on the amount of hydrogen absorbed. The break in curve II. corresponds to the leakage into the pump which has been already mentioned. As no oxygen can be pumped off platinum black charged with oxygen at the ordinary temperature, it was impossible to make a corresponding series of experiments on the influence of diminished pressure on the absorption of oxygen. The effect of increased pressure has, however, been observed and recorded in a subsequent section.

Fig. 4.



IX. *The Influence of Increased Pressure on the Absorption of Hydrogen by Platinum Black.*

In determining the influence of increased pressure on the absorption of hydrogen by platinum black, the method adopted was to enclose platinum black saturated with hydrogen at atmospheric temperature and pressure in a graduated tube containing excess of hydrogen, and then to increase its pressure by means of a column of mercury. If no further absorption takes place, then the product of the pressure and volume (PV) of the gas should remain constant. On the other hand, if PV diminishes with increased pressure, then it would be possible to calculate the amount of absorption from the diminution of PV.

The apparatus which we used for this purpose is represented in figs. 5, 6, and 7. Fig. 5 shows the apparatus ready for an experiment. The vessel A, to contain the platinum black, is sealed to a graduated and carefully calibrated glass tube B, which is connected with the tube C, into which mercury can be poured in order to raise the

Fig. 5.

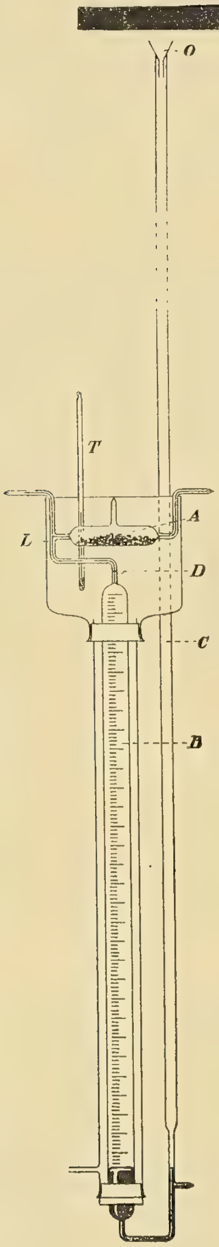


Fig. 6.

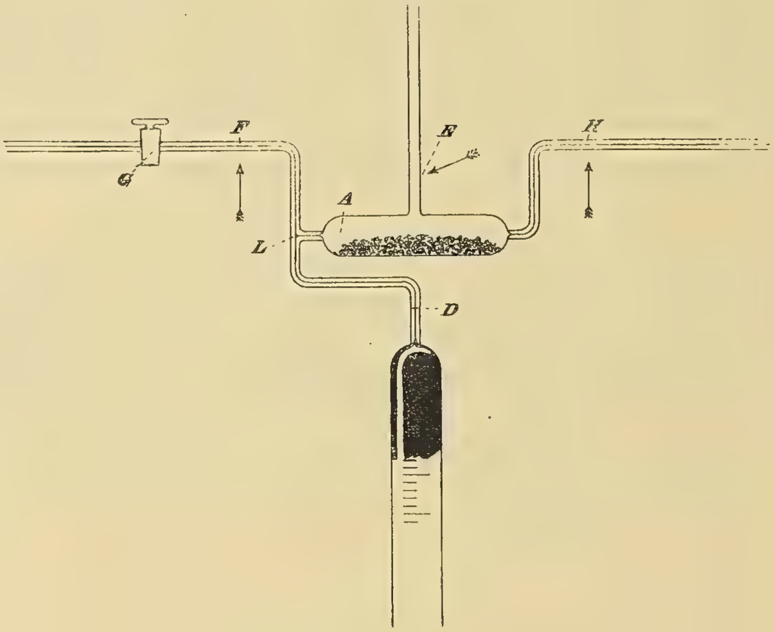
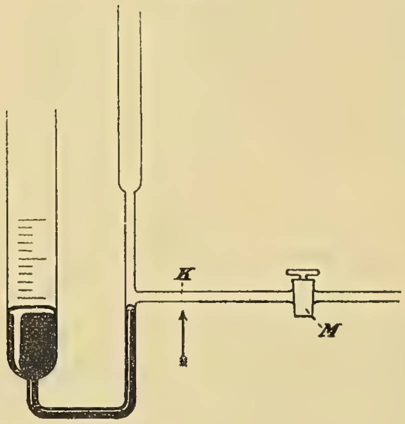


Fig. 7.



pressure. The graduations of the measuring tube or burette begin at the point D. After the experiments were at an end, the vessel A containing the platinum black was cut off from the graduated tube at the point D and its capacity separately determined. Knowing, then, the total volume of the hydrogen confined in the tube, its temperature being kept nearly constant by means of the water-jacket shown in the figure, and the height of the mercury column, it was easy to observe the product PV at the different pressures.

After the apparatus was fitted up and thoroughly dried, the platinum black was introduced through the tube E, fig. 6, which was then sealed off at the point E (indicated by an arrow).

The stopcock G being shut, some mercury was poured into the apparatus through the funnel O, fig. 5, and the tube H connected with a pump in order to produce a partial vacuum, so that no explosion would occur on allowing hydrogen to enter. When a sufficient vacuum was produced, pure dry hydrogen was passed into the apparatus through G. The whole apparatus was then filled with mercury, as shown in fig. 6, almost up to the point L.

The hydrogen thus swept out all the air from the vessel A, and in order to get rid of the excess of water formed, the current of hydrogen was continued for four hours. The exit tube was then sealed off at the point H. At this stage the stopcock M, fig. 7, was opened and the mercury allowed to flow out until it attained the level shown in fig. 7. The measuring tube was thus filled with pure hydrogen. As it was doubtful whether the stopcock M would withstand the high pressure (three-and-a-half additional atmospheres), it was sealed off at the point K, a long capillary being left in order to allow the mercury to flow out slowly after the experiments were completed.

The stopcock G was then shut and the apparatus allowed to stand overnight. As no further contraction in the volume of the hydrogen occurred, the apparatus furnishing hydrogen was sealed off at the point F.

The apparatus, with its water-jacket, then presented the appearance as shown in fig. 5. Successive quantities of mercury were now poured in through the funnel O and the height of the mercury column (measured with a steel scale and reduced to 0° C.), the volume of the gas and its temperature noted. When no further contraction occurred, another quantity of mercury was introduced and the corresponding readings taken. In this way the numbers recorded in the first three columns of the following table were obtained.

Temperature.	Pressure, millims. Hg. (0°).	Burette reading in millims.	Volume of hydrogen in cub. centims.	PV.	Volume of hydrogen in cub. centims. (0° and 760 millims.).
19°·0	765·8	682·0	136·8	97950	128·88
18·8	1222·7	388·9	85·5	97810	128·69
18·1	1664·5	255·0	62·7	97880	128·79
18·0	2507·1	131·3	41·6	97840	128·74
18·0	3415·2	66·3	30·5	97720	128·58

The next column contains the burette readings in millims., reduced to cub. centims. and increased by 14·6 cub. centims., the capacity of the vessel A down to the mark D. In the fifth column are tabulated the products of pressure and volume PV. The last column contains the volume of the hydrogen reduced to 0° C. and 760 millims. of mercury.

A glance at the table suffices to show that the product PV remains very nearly constant, and exhibits only a slight falling off (about 0·2 per cent.).

It is obvious, from the last column of the table, that by increasing the pressure of the hydrogen from one atmosphere to between four and five atmospheres, there is practically no change in the absorption of hydrogen by platinum black, since the reduced volume of the hydrogen at high pressure is only 0·3 cub. centim. less than at atmospheric pressure. As the platinum black used in these experiments weighed 5·972 grms., or had a volume of 0·284 cub. centim., it follows that only one additional volume of hydrogen was absorbed by unit volume of platinum black at the highest pressure.

X. *The Influence of Increased Pressure on the Absorption of Oxygen by Platinum Black.*

A corresponding series of experiments to those described in last section was now performed with oxygen. The apparatus and the method of charging it remained precisely the same. The quantity of platinum black employed was 5·979 grms. = 0·284 cub. centim., and the oxygen, made from pure potassium chlorate, was passed successively through sulphuric acid containing a little chromic acid, potassium hydroxide, and phosphorus pentoxide, before being introduced into the apparatus.

The results are given in the following table :—

Temperature.	Pressure, millims. Hg. (0°).	Burette reading in millims.	Volume of Oxygen in cub. centims.	PV.	Volume of Oxygen in cub. centims. (0° and 760 millims.).
15°·0	819·3	616·9	125·4	99658	128·1
,,	1165·7	400·0	87·4	96577	127·1
,,	1642·4	252·6	62·0	96526	127·0
,,	2112·6	171·4	48·1	96323	126·7
,,	2739·0	105·6	37·0	96064	126·4
,,	3417·0	61·6	29·5	95553	125·7

It will be seen that the falling off in the values of PV is much more marked in the case of oxygen than in the former experiments with hydrogen. At the highest pressure the volume of oxygen is less than the original volume by about 2 per cent.

Altogether 2·4 cub. centims. of oxygen have disappeared, and this corresponds to an additional absorption of oxygen by the platinum black of eight-and-a-half volumes, by increasing the pressure from one atmosphere to four-and-a-half atmospheres.

Platinum black charged with oxygen is thus much more influenced by changes in the pressure of the gas than platinum black charged with hydrogen. In the latter case, as we have already seen, the influence of pressure is so slight, at least between the atmospheric pressure and four-and-a-half atmospheres, as to be almost negligable.

XI. *The Influence of Temperature on the Absorption of Hydrogen by Platinum Black.*

Graham determined the absorptive power of platinum foil for hydrogen at a red heat, at 230° C. and at 100° C. and found that the higher the temperature the greater was the absorptive power of the foil. We found this statement to be true for platinum sponge also, but it is obvious that it cannot hold good for platinum black, since platinum black at a high temperature is converted into sponge, and this we know absorbs at most only a few volumes of hydrogen, while platinum black absorbs over 100 volumes at ordinary temperature.

The investigation of the absorptive power of platinum black for hydrogen at temperatures above the atmospheric, but still below the temperature at which it is converted into sponge, thus became a matter of great interest.

The sample of platinum black to be examined was charged with hydrogen at the ordinary temperature and pressure and then sealed up in an atmosphere of hydrogen in the manner shown in fig. 8. The volume of the hydrogen having been ascertained, heat was applied to the platinum black by jacketting with a convenient vapour. The new volume of the hydrogen, always kept approximately at atmospheric pressure, was then read off. If it were found that the new volume, on reduction to 0° C. and 760 millims. pressure, were less than or greater than the original volume when similarly reduced, then the difference between the two would represent the amount of absorption or expulsion of hydrogen directly.

The apparatus shown in fig. 8 was sealed on, by means of the dotted tube C, to an apparatus for producing pure dry hydrogen. The stopcock on the dotted tube C (not shown in the figure) was then shut and a weighed quantity of platinuum black introduced into the bulb A through the tube D. Previous to the introduction of the platinum black, however, a weighed quantity of glass wool was placed in the bottom of A in order to prevent the finely divided platinum from falling through the capillary tube into the graduated tube B. Since the whole of the tube B, together with the bulb A and a portion of the stem of D up to the mark F, had been carefully calibrated, the weight of each substance introduced into A was carefully noted, so that the necessary corrections for the volume of each could be made.

The level of the mercury having been lowered beneath the junction of the tube C with B, the stopcock already mentioned was opened and pure dry hydrogen passed through the apparatus for five or six hours, until the moisture which had condensed in the upper part of A and in D had been removed completely. A weighed glass thimble tube containing a weighed quantity of phosphorus pentoxide was then dropped

Fig. 8.

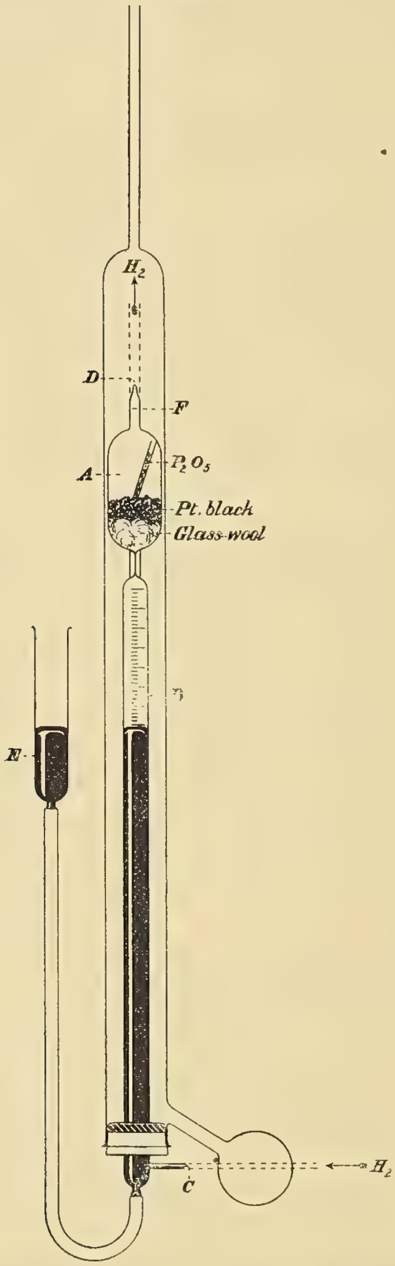
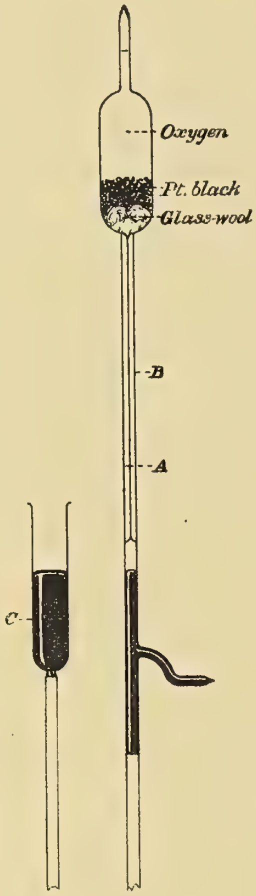


Fig. 9.



on the surface of the platinum black, so that any moisture given off on heating would be immediately absorbed and not interfere with the volume of the hydrogen. The open end of D was now connected with a narrow tube dipping under the surface of mercury and the current of hydrogen continued for another hour. At the end of this time the stopcock was shut and the side tube drawn off and sealed at the point C. When C had sufficiently cooled the mercury reservoir E was raised until the mercury level had nearly reached the top of the tube B. The upper tube D (the open end of which was still dipping into mercury) was then drawn off and sealed at the point D, a little way above the mark F. When the experiment was at an end the conical part of the tube above F was cut off, and its capacity determined separately.

The whole apparatus was then allowed to stand at atmospheric temperature and pressure for twenty-four hours in order to make sure that the absorption of hydrogen was complete. The volume of the hydrogen having been determined, the experimental tube was surrounded with a vapour jacket as shown in the figure. The volume, pressure, and temperature of the hydrogen was then noted for each different vapour-jacket. Each jacketting process had to be continued for at least a day before the volume became constant. Duplicate determinations, as will be seen in the following tables, were made at the temperature of boiling aniline in order to determine approximately the extent of the experimental error.

After each heating process the apparatus was allowed to stand over-night and a fresh observation taken in the morning in order to see how much of the hydrogen, if any, was re-absorbed on cooling.

The density of the phosphorus pentoxide, which does not seem to have been previously determined, was ascertained in the usual way by weighing in benzene. It was found to be 2·27, and it is further assumed that the deliquesced portion does not appreciably affect its volume. As the correction for the volume of the phosphorus pentoxide was itself very small we may consider this assumption justified. The total amount of correction of the capacity of A from all sources was 0·73 cub. centim., the initial reduced volume of the hydrogen being 33·40 cub. centims. The weight of platinum black was 4·6918 grms. = 0·22 cub. centim. The pressures are reduced to 0° C., and where necessary corrected for the vapour pressure of mercury and for the difference of temperature of the column of mercury heated in the vapour jacket.

The results are contained in the following table, those obtained after cooling being marked with an asterisk (*).

Temperature.	Volume.	Pressure.	Volume reduced to 0° C. and 760 millims. Hg.	Total hydrogen given off on heating.	
°	cub. centims.	millims.	cub. centims.	cub. centims.	volumes.
17·8	36·03	750·4	33·40	0·00	0·0
78·0	45·09	753·0	34·75	1·35	6·0
*18·1	36·13	753·5	33·60	0·20	0·9
131·7	58·26	802·2	41·50	8·10	36·3
*18·0	42·99	760·7	40·37	6·97	31·3
184·5	57·11	1027·3	46·07	12·67	56·7
..	50·11	1137·1	46·14	12·74	57·1
*19·0	48·62	760·0	45·46	12·06	54·1

The comparatively small variations in some of the above pressures from atmospheric pressure can have, as we saw during the investigation of the influence of increased pressure on the absorption of hydrogen by platinum black, no appreciable effect on the final result.

The table shows distinctly that on heating platinum black charged with hydrogen at ordinary temperature and pressure, a large proportion of the absorbed hydrogen is expelled, and that on cooling only a small fraction of this is re-absorbed, also that the higher the temperature at which the platinum black has been heated, the less is the amount of re-absorption.

It was impossible to determine the total quantity of hydrogen initially in this particular sample of platinum black experimented upon, but if we assume that it contained 110 volumes of hydrogen (this is about the average amount of hydrogen contained by the best samples of platinum black), then the quantity of hydrogen retained by the platinum black when heated in an atmosphere of hydrogen at different temperatures is as follows :—

Temperature.	Volumes of hydrogen retained by the platinum back.
°	volumes.
17·8	[110]
78·0	104
131·7	73·7
184·5	53·3

These results are plotted as Curve I, along with the results obtained with oxygen, in fig. 10, p. 685.

Arguing from these results we might expect to find that platinum black absorbs a still larger quantity of hydrogen when it is cooled down below the ordinary temperature.

As yet no experiments in this direction have been made, but we hope in a subsequent paper to be able to communicate the results of such experiments to the Society.

XII. *The Influence of Temperature on the Absorption of Oxygen by Platinum Black.*

Platinum black, charged with oxygen, was next heated in an atmosphere of oxygen. The experiments were conducted in precisely the same way as those just described, and the results are given in the following table :—

Temperature.	Volume.	Pressure.	Volume reduced to 0° and 760 millims.	Oxygen *absorbed* on heating.	
°	cub. centims.	millims.	cub. centims.	cub. centims.	vols.
17·0	31·32	751·8	29·17	0·0	0·0
78·0	37·87	755·0	29·27	−0·1	
* 16·0	30·78	750·3	28·71	0·5	2·2
131·6	39·40	753·0	26·34	2·9	12·5
* 17·8	30·02	698·3	25·90	3·3	14·2
182·9	39·41	728·2	22·62	6·6	28·4
* 15·5	30·13	590·7	22·16	7·0	30·2
237·1	36·77	708·9	18·36	10·8	46·5
* 18·0	30·87	487·8	18·59	10·6	45·7

A glance at the table suffices to show that the behaviour of platinum black when heated in an atmosphere of oxygen is totally different from the behaviour of platinum black charged with hydrogen and heated in an atmosphere of hydrogen. The latter gas is expelled on heating, whilst the former is absorbed.

The numbers are not quite so satisfactory as could be desired, for the platinum black on cooling (those experiments marked with an *) seems to absorb more gas (except in the last case). We believe, however, that this is only apparent. The phosphorus pentoxide employed to keep the gas dry got all used up about 78°, so that the gas was in all probability slightly damp, although nothing like saturated with aqueous vapour. At the high temperatures, therefore, the aqueous vapour would behave like a perfect gas, and an error would be thus introduced, that is, the true or reduced volume of the oxygen as calculated from the measurements at high temperature is probably too great on account of the presence of the aqueous vapour. At ordinary temperature the effect of the water vapour will in great measure have disappeared, and as it is extremely unlikely that more oxygen is absorbed on cooling, the volume of the gas measured at the ordinary temperature, after cooling down from the high temperature, is taken as representing more nearly the correct volume of the gas. The difference between the two, however, does not materially affect the general significance of the set of experiments.

The process of heating could not be carried farther then 237° C., because of the fact that at this temperature combination had just begun to take place between the hot mercury and the oxygen, a perfectly distinct ring of red oxide being visible within the graduated tube.

In carrying the heating to a higher temperature it was therefore necessary to slightly modify the apparatus.

At this stage the platinum black was transferred to a tube of hard glass, and the contents of the tube exhausted with the SPRENGEL pump. No gas was given off at the ordinary temperature, but at a red heat 111 volumes were liberated. Potash produced no diminution in the volume of the gas obtained, and it was practically all absorbed by alkaline pyrogallate. Of the 111 volumes given off, 46·5 volumes were absorbed on heating from ordinary temperature up to 237° C., hence the platinum black used in these experiments originally contained 111 — 46·5 = 64·5 volumes of oxygen. The low initial absorptive power of this sample of platinum black is accounted for by the fact that it had previously been used for another set of experiments, and had been charged alternately with hydrogen and oxygen.

Starting with platinum black containing 64·5 volumes of oxygen at the ordinary temperature (17°) the following table shows the amount of oxygen absorbed on heating at different temperatures in an atmosphere of oxygen.

Temperature.	Volumes of oxygen contained in platinum black.
° 17·0	64·5
78·0	66·7
131·6	78·7
182·9	94·7
237·1	111·0

These numbers are represented graphically in Curve II., fig. 10. The Curves I. and II. show very clearly the difference in behaviour of platinum black towards hydrogen and oxygen.

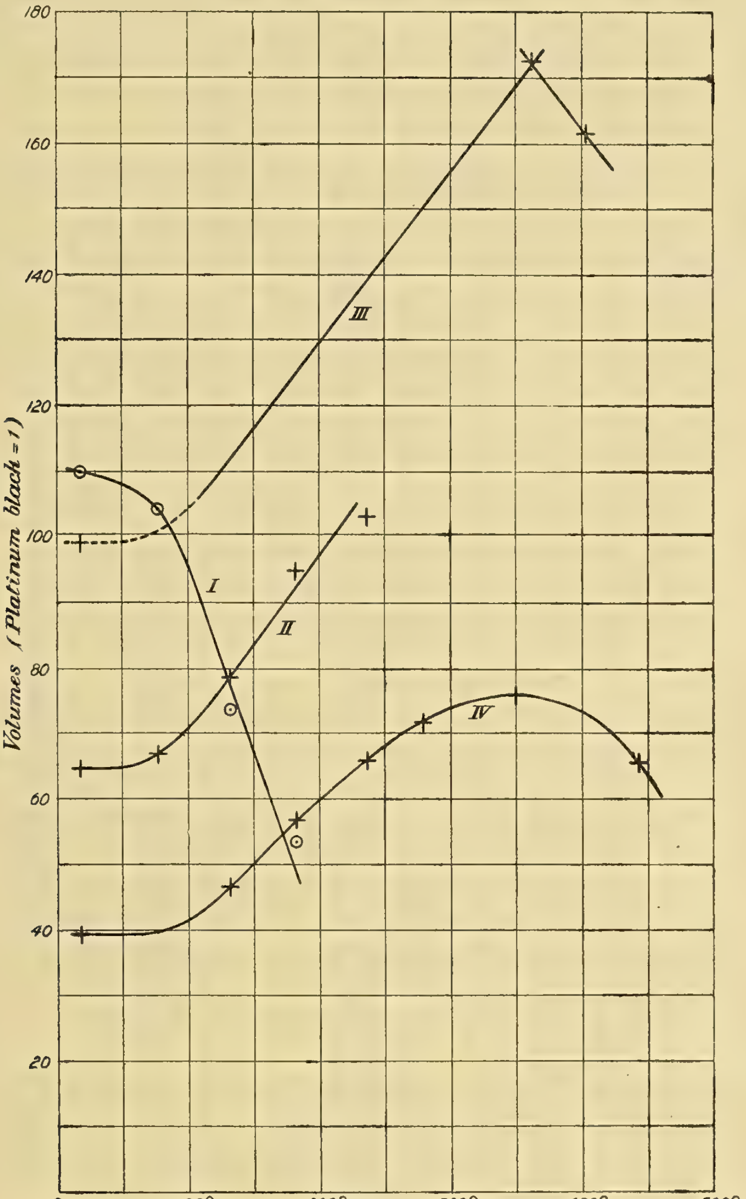
Knowing that platinum black is converted at a high temperature into platinum sponge, and that platinum sponge has a very low absorptive power for oxygen, we would expect to find a turning point in Curve II., or in other words it became important to determine the temperature at which there was a maximum absorption of oxygen.

For reasons which have been already alluded to, the apparatus in use was valueless for this purpose. The apparatus shown in fig. 9, p. 680, somewhat after the nature of a constant volume air thermometer, and constructed entirely of hard glass, was therefore made. In most respects it was similar to the apparatus last employed, but in place of the graduated glass tube B of fig. 8, the hard glass capillary tube AB was substituted. This tube had a mark A engraved on its stem, and the volume of oxygen contained in the bulb was always kept at constant volume by raising or lowering the mercury cistern C.

In the first experiments a hard glass thimble tube containing phosphorus pentoxide was also introduced into the bulb of the apparatus to keep the gas dry. This had soon to be abandoned, however, for reasons which will appear immediately.

In the first experiment a sample of platinum black containing initially 98·8 volumes of oxygen was employed. Heated in the presence of oxygen for a day at 362° C., 73·3 additional volumes of oxygen were absorbed. On heating for another day at 403° C., 9·2 volumes were *expelled.* Above this temperature, however, more oxygen was absorbed, and it was noticed that phosphorus pentoxide had distilled out of the small tube, and the results then became very irregular. Up to 403° C., the results are plotted roughly in Curve III., fig. 10, below.

Fig. 10.



Above this temperature they are no longer trustworthy, and on examination it was found that the platinum black, now converted into sponge, was contaminated with yellow brittle patches. It therefore occurred to us, that the phosphorus pentoxide had combined in some way with the platinum sponge, and that the compound then went on absorbing further quantities of oxygen, even up to a temperature of about 600° C.

Our suspicions were verified in the following manner. The platinum sponge containing the yellow patches was thoroughly boiled out with distilled water in order to remove all the free phosphoric acid. On fusing the residue with a mixture of potassium and sodium carbonates and a little potassium nitrate, and dissolving the fused mass in water, large quantities of phosphoric acid were found in the filtrate. Drying the gas with phosphorus pentoxide was thus rendered impossible, and subsequent experiments had to be conducted without it.

In the section of this paper dealing with the amount of water retained by platinum black, it was incidentally mentioned that platinum black is converted into platinum sponge at a temperature of about 400° C. We were therefore of opinion that the temperature of maximum absorption of oxygen could not differ much from this temperature, or at any rate could not exceed it.

Curve III. of fig. 10 indicates that the required temperature is less than 400° C., but, to make quite sure of the upper limit of temperature, a sample of platinum black was heated in an atmosphere of oxygen, kept at atmospheric pressure and in absence of phosphorus pentoxide, at 444° C., the temperature of boiling sulphur.

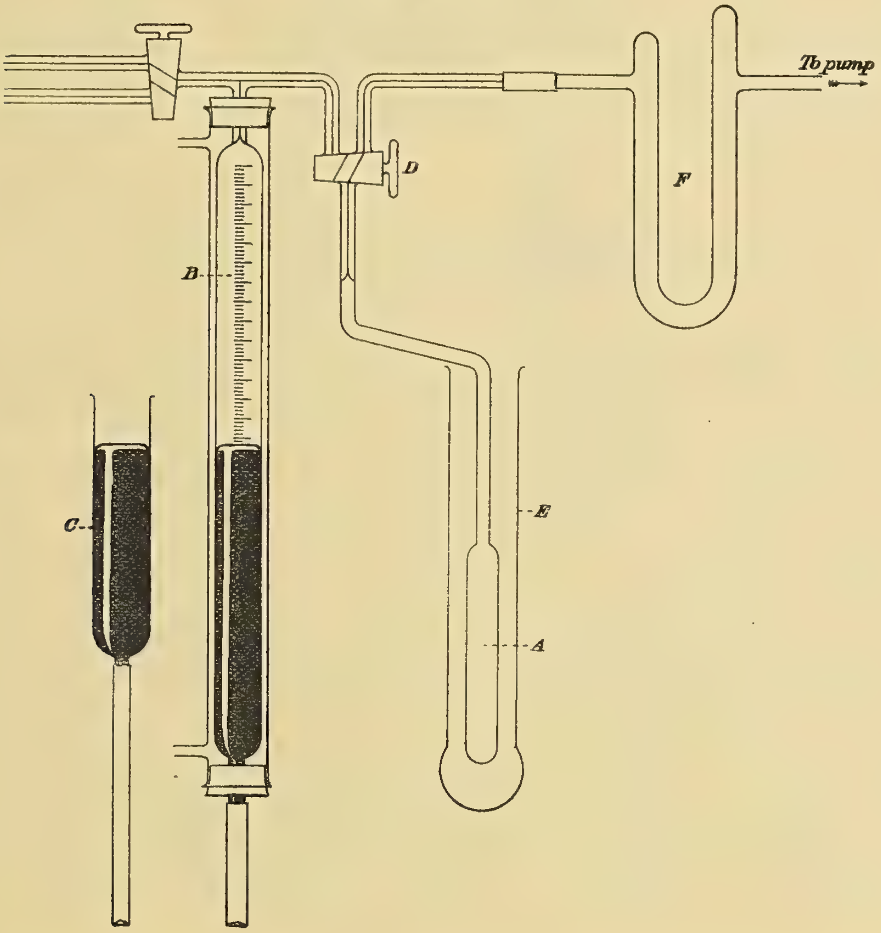
5·2766 grms. = 0·251 cub. centim. of platinum black, containing 98·8 volumes of oxygen, were placed in the experimental tube A, which was then sealed on to the gas burette B, as shown in fig. 11, p. 687. The platinum was then placed in communication with the pump and the phosphorus pentoxide tube F, by means of the three-way stopcock D, and heated in a water-bath at 100° C., in order to get rid of as much water as possible. When the experimental tube was cold and vacuous, pure dry oxygen was admitted from the gas burette B, and the total volume of oxygen in burette and tube noted. The platinum was now heated in the sulphur-bath E, until the volume of oxygen in the gas burette remained constant, the pressure being always maintained approximately atmospheric by raising or lowering the reservoir C. After cooling, another measurement was made, and it was found that the oxygen in the burette and experimental tube had expanded 11·4 cub. centims. (0° and 760 millims.), *i.e.*, the platinum black had *given off* 45·6 volumes of oxygen at the temperature of boiling sulphur (444° C.).

The experimental tube of known capacity was then again connected with the pump. On exhausting at ordinary temperature, no gas was given off from the platinum black, but, on careful heating to dull redness, 15·1 cub. centims. = 60·4 volumes of oxygen, were extracted.

The results are given in tabular form below, and it will be noted that the sum of the volumes of oxygen given off at 444° C. and at a dull red heat amounts to 106 volumes, which is in fair agreement with 98·8, the quantity of oxygen originally contained in the platinum black, as determined by a direct experiment.

	Cub. centims.	Volumes.
O_2 evolved on heating platinum black in an atmosphere of O_2 under atmospheric pressure at 444° C.	11·4	45·6
O_2 still retained and given off at a red heat	15·1	60·4
Total	26·5	106·0

Fig. 11.

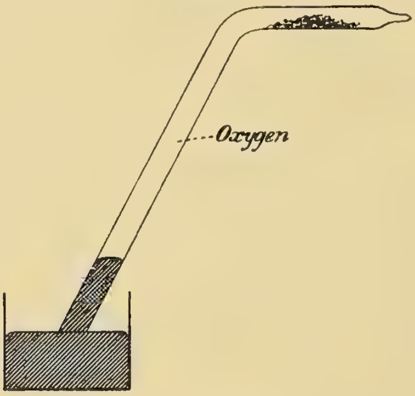


It was thus satisfactorily proved that the temperature of maximum absorption of oxygen was below 444° C., but, in order to determine *directly* whether the formation of a phosphorus compound of platinum was answerable for the abnormal absorption of oxygen already referred to, the following experiments were made :—

(1.) The experiment which has just been described may be regarded as the first of this series. It shows that, in absence of phosphorus pentoxide, platinum black does not go on absorbing oxygen indefinitely above 444° C., but in reality gives off at this temperature about 46 volumes of the oxygen originally absorbed.

(2.) As much phosphorus pentoxide as was used in the previous experiments (0·3 to 0·4 gram) was heated in an atmosphere of pure dry oxygen in the apparatus represented in fig. 12, to a temperature approaching dull redness. Only a minute fraction of a cubic centimetre was absorbed, hence the presence of a lower oxide of phosphorus in the phosphorus pentoxide has nothing to do with the abnormal absorption.

Fig. 12.



(3.) A similar experiment to the last was made, but the phosphorus pentoxide was replaced by a mixture of platinum black and phosphorus pentoxide. Almost the whole of the oxygen contained in the tube was rapidly absorbed at a dull red heat, the temperature being probably about 50° to 100° C. higher than the boiling-point of sulphur. Our suspicions regarding the formation of a phosphorus compound of platinum, perhaps phosphate or pyrophosphate, were thus fully confirmed.*

A new set of experiments, but with a different sample of platinum black, was therefore made without employing any phosphorus pentoxide to keep the gas dry. The platinum black was taken from an old preparation, as it was the only specimen available at the time, and subsequently it was found that it was rather an inferior one, containing only 39·4 volumes of oxygen. Although the absolute amount of gas absorbed is so low, this should in no way affect the turning point or temperature of maximum absorption of oxygen, as it has already been seen that the temperature at which practically all the oxygen or hydrogen (whether large or small in amount) is given off *in vacuo* from platinum black is always the same.

The experiments were made in the apparatus shown in fig. 9, p. 680.

Vapour jackets were used for heating up to the temperature 280° C., after which the capillary tube was bent at right angles at the point B, and the bulb then heated, first in the air bath, and afterwards in a bath of boiling sulphur, the vertical stem of the capillary tube being temporarily inclined in order to allow of the bulb being plunged in the sulphur bath.

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* Since these experiments were made, Mr. BARNETT ('Chem. Soc. Trans.,' 1895, p. 513), working in Professor TILDEN's laboratory, has prepared platinic pyrophosphate, PtP_2O_7, by distilling phosphorus pentoxide over spongy platinum in a current of oxygen.—[Sept. 23, 1895.]

The results obtained are tabulated below.

I. PLATINUM black used, 5·6465 grams = 0·269 cub. centim.
Capacity of apparatus corrected, 18·76 cub. centims.

Temperature of heating.	Temperature of measurement.	Pressure corrected.	Volume reduced to 0° C. and 760 millims.	Oxygen absorbed.	
°	°	millims.	cub. centims.	cub. centims.	vols.
..	14·2	795·3	18·66	..	..
132	14·5	718·4	16·84	1·82	6·8
184	15·0	597·6	13·98	4·68	17·4
238	14·8	493·8	11·56	7·10	26·4
280	16·2	433·9	10·11	8·55	31·8
350	16·8	384·1	8·93	9·73	36·2
444	18·0	500·5	11·57	7·09	26·4

II. PLATINUM black used, 5·137 grams = 0·246 cub. centim.
Capacity of apparatus corrected, 17·85 cub. centims.

Temperature of heating.	Temperature of measurement.	Pressure corrected.	Volume reduced to 0° C. and 760 millims.	Oxygen absorbed.	
°	°	millims.	cub. centims.	cub. centims.	vols.
..	16·0	779·8	17·30	..	..
330	16·2	414·2	9·18	8·12	32·9
444	15·0	476·0	10·59	6·71	27·2

It is evident from these tables that absorption of oxygen takes place up to *at least* 350°, and that the temperature of maximum absorption lies between 350° and 444° C.

The original amount of oxygen contained in these samples of platinum black was 39·4 volumes, hence the total quantity of oxygen contained in the platinum black after heating in an atmosphere of oxygen at the undermentioned temperatures is as follows :—

I.

Temperature.	Oxygen contained in the platinum black.
°	vols.
18	39·4
132	46·2
184	56·8
238	65·8
280	71·2
350	75·6
444	65·8

II.

Temperature.	Oxygen contained in the platinum black.
°	vols.
18	39·4
330	72·3
444	66·6

The results of Experiment I. are almost identical with Experiment II., and have been plotted graphically in Curve IV., fig. 10, p. 685. A glance at the curve shows that the temperature at which platinum black absorbs the maximum quantity of oxygen is about 350° to 360° C. It is also interesting to note that this is the temperature at which most of the oxygen can be extracted from platinum black *in vacuo.*

The residual platinum black or sponge of Experiment II. was transferred to a tube of hard glass and heated to redness in vacuo. 15·78 cub. centims = 64·1 volumes of oxygen were given off, hence we have

	vols.
Oxygen originally in platinum black . . .	39·4
,, absorbed on heating to 444° . . .	27·2
Total	66·6

The total 66·6 vols. agrees fairly well with 64·1 vols. directly determined.

XIII. *Résumé and General Discussion of the Results.*

As far as our experiments on platinum foil and platinum sponge go, they may be said in general to confirm Graham's observations. At most, only a few volumes of oxygen or hydrogen are occluded by these substances.

The slight differences that do occur are no doubt due, as Graham himself pointed out, to differences in the material experimented with.

Of recent work on the occlusion of gases by platinum foil, the most remarkable is that of Berliner,* who arrived at results totally different from those obtained by Graham and ourselves. According to Berliner, ordinary platinum foil contains as much as from 71 to 227 vols. of occluded gas, and foil which has been deprived of this gas (not analysed) can afterwards absorb from 204 to 271 vols. of hydrogen and 100 vols. of oxygen. Berliner invariably states the amount of gas absorbed or expelled directly in volumes referred to platinum as unity, and carefully avoids stating the amount of gas actually measured in cubic centimetres. From the table given on p. 807 of his paper, together with statements made on p. 803, we arrive at the following conclusions :—

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* Berliner, 'Wied. Annal.,' 1888, vol. 35, p. 791.

(1.) The volume occupied by the *largest* sample of platinum foil with which he worked was 0·0043 cub. centim. ; (2.) The *largest* volume of occluded or expelled gas which he measured never exceeded, and was usually much less, than one cubic centimetre ; (3.) The graduated tube in which he measured his gas was not jacketed, and its temperature was only determined by thermometers suspended outside it. Although these thermometers were graduated in tenths of a degree C. and could probably be *read* to hundredths, yet we have no guarantee that they really indicated the temperature of the gas. A variation of 1° C. in his gas corresponded to a change in its volume of 0·116 cub. centim. and since the volume of the platinum foil was in general about 0·003 cub. centim., an error of one degree in the temperature of the gas means a difference in the occlusive power of 40 *volumes more or less ;* (4.) An error of one millimetre in the measurement of the pressure of the gas similarly corresponds to an apparent occlusion of 14 *volumes more or less.*

Although working on a small scale with an apparatus such as BERLINER used has its advantages, *e.g.*, the surface of the platinum can be made and kept quite clean, and it can be heated to a very high temperature by means of an electric current, yet it has also very great disadvantages, as we have just seen, but until his results are confirmed by a less objectionable method, we must look upon them with great distrust.

In now passing on to the occlusion of gases by platinum black, we are obliged to neglect almost all the earliest observations, because they took no account of the oxygen it already contained, and mostly worked with an impure material, insufficiently washed free from accidental impurities, such as sugar, glycerol, alcohol, nitroso and chlorine compounds, &c. The former would give off large quantities of carbon dioxide, whilst traces of the latter give off corrosive vapours which attack the mercury of the pump. It has long been known that platinum black contained oxygen, but probably the first accurate determination was made by NEUMANN,[*] who found it to contain from 63 to 77 volumes. Most of the specimens which we examined, and which were dried at 100° C., contained approximately 100 volumes of oxygen, provided they had not been used previously for other experiments. Its density we found to be 19·4, or allowing for the water contained in the black, and which can only be got rid of by heating in vacuo to a temperature at which it no longer is platinum black but sponge, 21·5. LIEBIG estimated its density at 15·78, 16·32, and 17·572.

On placing the platinum black in an atmosphere of oxygen, and increasing the pressure of the gas to four-and-a-half atmospheres, only eight-and-a-half additional volumes were absorbed. Platinum black heated in an atmosphere of oxygen absorbs a further quantity of oxygen up to the temperature 360° C. At higher temperatures, and under atmospheric pressure, the gas is again expelled. Most of the oxygen contained in platinum black can be extracted at about 400° C. in vacuo, but it requires to be heated to a red heat for its complete removal.

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[*] NEUMANN, 'Monatsh.,' vol. 13, p. 40.

The amount of hydrogen occluded by platinum black independent of that required to form water with the oxygen already contained in it, is about 110 volumes for the best samples of black, the total amount of hydrogen absorbed being about 310 vols. This amount, however, is very largely influenced by the presence of traces of accidental impurities, probably grease, &c. An increase of pressure, up to four-and-a-half atmospheres, on platinum black already charged with hydrogen, only causes it to absorb one additional volume of hydrogen. A portion of the hydrogen occluded by platinum black can be removed at the ordinary temperature by means of the pump alone, but not until the pressure has decreased to about 300 millims. The bulk of the hydrogen can be extracted in vacuo at about 300° C., but a red heat is necessary for its complete removal. On heating platinum black charged with hydrogen in an atmosphere of hydrogen, a portion of the gas is immediately liberated, and in this respect it differs entirely from platinum black charged with oxygen.

We are not yet in a position to form an opinion as to whether we are dealing with true chemical compounds of these gases with platinum black, as maintained by BERTHELOT, or whether we are simply dealing with cases of absorption or of solid solution as suggested by VAN'T HOFF,[*] but we are at present engaged on experiments which we hope will throw some light on this subject.

BERTHELOT[†] is very definite in his statement that true chemical compounds of hydrogen and platinum having the formula $Pt_{30}H_2$ and $Pt_{30}H_3$ exist, and quite recently he again emphasised this opinion in an appendix to a paper by CAILLETET and COLLARDEAU.[‡] According to these gentlemen a gas battery working under a pressure of about 600 atmospheres gives a much higher electromotive force and has a greater capacity than a similar battery working under ordinary pressure. Whether the increased efficiency is due to an increased absorption of gas under the high pressure was not, however, directly determined.

The evidence cited by BERTHELOT not only for the existence of definite chemical compounds of platinum with oxygen and hydrogen, but in particular for the compounds with hydrogen, having the extraordinary formulæ mentioned above is, in our opinion, insufficient to decide the question.

From the peculiar behaviour of the platinum black employed by BERTHELOT we conclude that it was by no means quite pure. For example, his moist platinum black gave off 21 per cent. of water at 100° C., and on heating to nearly 600° C *in vacuo* liberated nearly 200 volumes of oxygen and *less than one volume of carbon oxide,* besides nitrogen and oxides of nitrogen. The same sample after drying at 150° C. is said to have lost half its original quantity of oxygen. It then contained 90 volumes of oxygen and as much as 31 *volumes of carbon dioxide,* along with several other gases. Now we have already seen that platinum black absorbs more oxygen on heating, provided the temperature does not rise above 360° C., and we are unable to explain why BERTHELOT'S platinum black lost oxygen, more especially as we found that even *in vacuo* the oxygen does not begin to come off until the temperature of about 300° C. is reached.

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[*] VAN'T HOFF, 'Zeits. f. physikal. Chem.,' vol. 5, p. 322 (1890).

[†] BERTHELOT, 'Ann. de Chim. et de Phys.,' 1883, vol. 30, p. 519.

[‡] CAILLETET and COLLARDEAU, 'Comptes Rend.,' vol. 119, p. 830.

The heat which is evolved on the absorption of hydrogen by platinum black has been measured by BERTHELOT[*] and by FAVRE,[†] and has been used by BERTHELOT as an additional argument in favour of his formulæ. In view of the fact, however, that the heat due to the combination of the oxygen pre-existing in the platinum black with hydrogen is also included in these measurements, no safe conclusion can be drawn from these numbers. Until a sample of platinum black can be prepared free from oxygen and other impurities, or until the amount of oxygen contained in it has been properly allowed for, the numbers supposed to represent the heat of absorption of hydrogen by platinum black are of little value.

If a chemical compound of platinum and hydrogen exists at all we would from analogy prefer to look for a compound similar to Pd_2H, but as the existence of even this substance has been recently called in question,[‡] we prefer for the present to leave the matter as it stands until sufficient data have been accumulated for an adequate enquiry.

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[*] BERTHELOT, *loc. cit.*

[†] FAVRE, 'Comptes. Rend.,' vol. 77, p. 649; and vol. 78, p. 1257.

[‡] HOITSEMA, 'Zeitsch. f. physikal. Chem.,' vol. 17, p. 1.

XX. *A Dynamical Theory of the Electric and Luminiferous Medium.*—Part II. *Theory of Electrons.*

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1. In a previous paper the concrete representation of electrical and optical phenomena by means of a rotationally elastic fluid æther has been discussed.* In an Appendix it has been shown that whenever there is direct question of interaction between the molecules of matter and the æther, whether it be in the phenomena of magnetism or in the optical phenomena of dispersion and moving material media, the consideration of groups of electrons or permanent strain-centres in the æther, which form a part of, or possibly the whole of, the constitution of the atoms of matter, suffices to lead to a correlation of the various modes of activity; while this scheme seems to be free from the chief difficulties which have pressed on other methods of representation.

The present paper is chiefly concerned with the further development of the molecular aspect of this theory. As a preliminary, it is maintained that a dynamical theory of electric currents, based on the ordinary conception of a current-element, must lead to expressions for the electrodynamic forces which are at variance with the facts. On the other hand, a theory which considers moving electrons to be the essential elements of the true currents in material media, gives a definite account of the genesis and the mutual relations of both types of forcive, the electromotive and the ponderomotive, and gives formulæ for them, which correspond in the main with those originally deduced by Maxwell from consideration of the properties of his concrete model of the electric field, though they are not substantiated by his later abstract theory based on current-elements. That theory is held to be defective, in the first place on account of the discrepancy with experiment above mentioned, and in the second place, because it is not competent to describe the mode of genesis of a conduction current by electrical separation produced in the element of volume of the conductor under the influence of the field of force; it is thus an incomplete formulation of the phenomena.

The application of the method of electrons to vibrational phenomena leads to formulæ for optical dispersion, and optical propagation in metals, which are in general agreement with experimental knowledge. These subjects have already been treated from a similar point of view by VON HELMHOLTZ in 1892, but his equations, which are arrived at in a more abstract manner, are fundamentally at variance with those of the present theory. They lead, however, to the same type of dispersion formula when the medium is transparent, a type which has been generally accepted as in good agreement with exact measurements of dispersion.

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* 'Phil. Trans.,' 1894, A., pp. 719–822.

The application to the optical properties of moving media leads to FRESNEL'S well-known formula, as had already been shown in the previous paper. If the theory of the constitution of matter which is suggested in that paper is allowed, it also leads to an explanation of the null result of the well-known second-order experiment of MICHELSON and MORLEY, of which previous theoretical discussions have quite failed to take cognizance.

As the statistical processes connected with molecular theory are of a very delicate character, and subject to suspicion especially when pushed to the second order of small quantities, the formulæ are derived by two independent methods, which supplement and illuminate each other but are analytically of very different types. One of these methods, which lies nearer to recent procedure in electrodynamics, and has been used by LORENTZ in a comprehensive discussion of the subject of moving media, published at the beginning of the present year, is to adapt the fluxes and forces occurring in the fundamental circuital equations of the free æther, so as to make similar equations apply to the æther in ponderable media. The other method is to investigate the forcive acting on a single electron, and then to build up the electric and optical phenomena out of the various types of movements of electrons that can occur in dielectric and conducting media. It is only in this latter way that a rational detailed account of the ponderomotive forces in media which are transmitting currents, or are electrically or magnetically polarized, can be derived ; as a special example, a formula for the ponderomotive pressure due to radiation is obtained.

The discussion of the topics above mentioned, with the exception of the MICHELSON-MORLEY experiment, is quite independent of any speculation as to the nature of electrons or the relation of æther to matter. It may be founded directly on MAXWELL'S equations of the electric field in free æther, as now experimentally confirmed.

But when we go further, it seems to be a strong argument in favour of a rotational æther, or at any rate of an æther whose actual properties are represented with close fidelity by the scheme of a rotationally elastic fluid medium, that in this way we derive an actual physical structure for an electron, and that by the orbital motions of electrons in the atom we derive a representation of an atom as a fluid vortex, an idea which has always been present to physical speculation from LEUCIPPUS, through DESCARTES, down to the recent definite dynamical conceptions of vortices. To justify tentative adhesion to such a view, it is not necessary to be able to produce an explanation of the fundamental properties of mass and gravitation in matter : in particular these are not in necessary connection with the electrical and optical phenomena. But it is easy to see that a rotational æther is not inadequate to including such properties among its relations: if the nuclei of the electrons are supposed small enough, the inertia of matter would be definitely represented by the electric inertia of the electrons; and as the electrons may then have vacuous nuclei and have each a free period of radial vibration in the fluid æther, which is not subject to damping by radiation (but subject, however, to a certain instability unless the free spherical form of surface is fortified by some kind of constraint), the gravitation between them may be represented or illustrated by the hydrodynamical pulsatory theory of Bjerknes and Hicks.

Examination of Theories involving the Electrodynamic Potential Function.

2. Every scheme for reducing the phenomena of electric currents to a purely dynamical basis must start ultimately from the formula for the electrodynamic potential of a system of ordinary currents, discovered and first extensively applied by F. E. Neumann.

In a preliminary tentative discussion of this formula, the only course open appears to be to take the current flowing in an element of volume, and the coordinates of the element, as the independent variables of a continuous analysis extended over the field of currents.

If the function is treated merely as a potential of the forces of the field, we may employ the principle of Neumann, afterwards elaborated and extended by von Helmholtz, but still hypothetical in its dynamical aspect, that a variation of the potential, due to alteration of the positions of the conductors, but without any variation of the strength of the current flowing along any linear element of a tube of flow, leads to the ponderomotive forces tending to alter their positions: while a variation due to alteration of the strengths of the currents, the conductors being fixed or moving, leads to electromotive forces tending to change the strengths of the said currents, but not to alter the motion of the bodies.

The dynamical theory of Maxwell aims at going further: the potential function with sign changed is assumed to be the kinetic energy residing in the latent or impalpable active medium which is associated with the current system. The induced electric forces are now taken to be reversed kinetic reactions corresponding to the currents considered as generalized electric velocity components, in accordance with the Lagrangian formula $-\left(\frac{\delta}{dt}\frac{d\mathrm{T}}{d\dot{\theta}} - \frac{d\mathrm{T}}{d\theta}\right)$; and these together with the applied electromotive forces make up the total ones that drive the current in conformity with Ohm's law. In the same way the ponderomotive forces acting on the conductors are the reversed kinetic reactions corresponding to change of position of the material system, which require to be compensated by equal and opposite applied forces if mechanical equilibrium of the conductors is to subsist. It was one of the discoveries of MAXWELL that, for a system of *circuital* conduction currents, the potential function can actually be formally represented as the kinetic energy of a latent moving system, coupled with the palpable conductors and thereby influencing them—of which system the currents are to be treated as generalized velocity components corresponding to electric coordinates which do not appear themselves in the function—without thereby introducing any discrepancy into the general scheme, as already experimentally determined, of the equations of the electric field.

Whichever of these methods of development of the potential function is essayed, it is a necessary preliminary with a view to a complete analysis to take the strength of the current flowing in each element of volume, or it may be in a linear element of a tube of flow, as dynamically a separate electric entity. The object of the following discussion is to consider how far this is consistent with a more detailed examination of the forcives thus derivable, particularly with the nature of that part of the internal stress in a conductor carrying a current which tends to alter its shape but not to produce motion of translation of the conductor as a whole.

3. According then to the type of theory which considers a current system to be built up of physical current-elements of the form $(u, v, w)\,\delta\tau$, the energy associated with an element of volume $\delta\tau$, as existing in the surrounding field and controlled by the element, is

$$\mathrm{T} = (\mathrm{F}u + \mathrm{G}v + \mathrm{H}w)\,\delta\tau.$$

The ponderomotive force acting on the element will be derived from a potential energy function $-\mathrm{T}$, by varying the coordinates of the material framework: it must in fact consist, per unit volume, of a force

$$\left(u\frac{d\mathrm{F}}{dx} + v\frac{d\mathrm{G}}{dx} + w\frac{d\mathrm{H}}{dx},\ u\frac{d\mathrm{F}}{dy} + v\frac{d\mathrm{G}}{dy} + w\frac{d\mathrm{H}}{dy},\ u\frac{d\mathrm{F}}{dz} + v\frac{d\mathrm{G}}{dz} + w\frac{d\mathrm{H}}{dz}\right),$$

and a couple

$$(v\mathrm{H} - w\mathrm{G},\ w\mathrm{F} - u\mathrm{H},\ u\mathrm{G} - v\mathrm{F}),$$

the former being derived from a translational, the latter from a rotational virtual displacement of the element.* We may simplify these expressions by taking the axis of z parallel to the current in the element $\delta\tau$, so that u and v become null; then we have

$$\text{a force}\left(w\frac{d\mathrm{H}}{dx},\ w\frac{d\mathrm{H}}{dy},\ w\frac{d\mathrm{H}}{dz}\right)\text{ and a couple }(-w\mathrm{G},\ w\mathrm{F},\ 0).$$

According to the AMPÈRE-MAXWELL formula, there should be simply a force at right angles to the current, specified by the general formula

$$(vc - wb,\ wa - uc,\ ub - va),$$

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* In the previous paper, § 120, the couple was omitted.

which becomes for the present special axes of coordinates

$$\left\{-w\left(\frac{dF}{dz}-\frac{dH}{dx}\right),\ w\left(\frac{dH}{dy}-\frac{dG}{dz}\right),\ 0\right\}.$$

The forcive at which we have here arrived thus differs from the AMPÈRE-MAXWELL one by

$$\text{a force}\left(u\frac{dF}{dz},\ v\frac{dG}{dz},\ w\frac{dH}{dz}\right)\text{ and a couple }(-wG,\ wF,\ 0)\ ;$$

these are equivalent to forces acting on the ends of each linear current element, equal at each end numerically to (wF, wG, wH) per unit of cross section, positive at the front end and negative at the rear end. They are thus of the nature of an internal stress in the medium, and are self-equilibrating for each circuital current and so do not disturb the resultant forcive on the conductor as a whole due to the field in which it is situated. From MAXWELL's stress standpoint they would form an equilibrating addition to the stress-specification in the conductor which is the formal equivalent of the electrodynamic forcive.

According to the AMPÈRE-MAXWELL formula, the forcive on an element of a linear conductor carrying a current is at right angles to it, so that the tension along the conductor is constant so far as that forcive is concerned. The traction in the direction of the current, arising from the above additional stress, would introduce an additional tension, equal to the current multiplied by the component of the vector potential in its direction, which is not usually constant along the circuit, and so may be made the subject of experimental test with liquid conductors, as it would introduce differences of fluid pressure. There will also be an additional transverse shearing stress which should reveal itself in experiments on solid conductors with sliding contacts.

In particular these additional forces should reveal themselves in the space surrounding a closed magnetic circuit, where the ordinary Amperean force vanishes because the magnetic field is null; in that case (F, G, H) may be interpreted as the total impulsive electric force induced at any point by the making of the circuit. Professor G. F. FITZGERALD has devised an experiment in which the behaviour of a thread of mercury carrying a strong current and linked with a complete magnetic circuit was closely observed when the circuit was made and broken. No movement was detected, whereas, when the magnetic circuit was incomplete, the ordinary Amperean forces were very prominent. According to the above analysis, the two types of forcive should be of the same order of magnitude in such a case: the result of the experiment is therefore against this theory. A like negative result has also attended an experiment by Professor O. J. LODGE, in which he proposed to detect minute changes of level along the upper surface of a uniform mercury thread by an interference arrangement on the principle of NEWTON's rings: when the current was turned on, the section of the thread became more nearly circular owing to the mutual attractions of the different filaments of the current, but there was no alteration in the direction of its length.

This experimental evidence, combined with the fact (*infra*) that a theory of moving electrons, which are certainly independent physical entities, leads simply to the AMPÈRE-MAXWELL forcive, seems to justify the conclusion that either the above analysis is wrong, or else the ordinary treatment of electrodynamics in terms of a specification by current elements is physically untenable. As tending to the exclusion of the first alternative, and also as of independent critical interest, the following discussion by aid of the more usual analytical method employed by AMPÈRE, NEUMANN, and VON HELMHOLTZ is given.

4. Assuming that current elements serve as a sufficient physical specification, it is known that the mutual energy (kinetic) of two such elements, $\iota\delta s$ and $\iota'\delta s'$, must be

$$\iota\delta s \,.\, \iota'\delta s' \left(\frac{1}{r}\frac{dr}{ds}\frac{dr}{ds'} + \frac{d^2\phi(r)}{ds\,ds'}\right),$$

where $\phi(r)$ is some function of their distance apart.

The variation of this energy, due to mutual displacement of the elements, is

$$\iota\delta s \,.\, \iota'\delta s' \left(-\frac{\delta r}{r^2}\frac{dr}{ds}\frac{dr}{ds'} + \frac{1}{r}\frac{d\delta r}{ds}\frac{dr}{ds'} + \frac{1}{r}\frac{dr}{ds}\frac{d\delta r}{ds'} + \frac{d^2\delta\phi(r)}{ds\,ds'}\right).$$

On substituting

$$\frac{d}{ds}\left(\frac{1}{r}\frac{dr}{ds'}\delta r\right) - \frac{d}{ds}\left(\frac{1}{r}\frac{dr}{ds'}\right)\delta r \quad \text{for} \quad \frac{1}{r}\frac{d\delta r}{ds}\frac{dr}{ds'},$$

and similarly for the following term, the variation becomes

$$\iota'\delta s' \,.\, \iota\delta s \frac{d}{ds}\left(\frac{1}{r}\frac{dr}{ds'}\delta r\right) + \iota\delta s \,.\, \iota'\delta s' \frac{d}{ds'}\left(\frac{1}{r}\frac{dr}{ds}\delta r\right) + \iota\delta s \,.\, \iota'\delta s' \frac{d^2\delta\phi(r)}{ds\,ds'}$$
$$+ \iota\delta s \,.\, \iota'\delta s' \left(\frac{1}{r^2}\frac{dr}{ds}\frac{dr}{ds'} - \frac{2}{r}\frac{d^2 r}{ds\,ds'}\right)\delta r.$$

If we take the aggregate for a series of elements $\iota'\delta s'$ which form a circuital current, the second and third terms vanish ; and the forcive on the element $\iota\delta s$, due to a circuital current system, is thus compounded of

$$\iota\delta s \,.\, \iota'\delta s' \left(\frac{1}{r^2}\frac{dr}{ds}\frac{dr}{ds'} - \frac{2}{r}\frac{d^2 r}{ds\,ds'}\right)$$

towards each element $\iota'\delta s'$ of the influencing current system, which is the ordinary forcive of AMPÈRE, together with a force acting on each end of $\iota\delta s$, positive on the forward end and negative on the rearward end, and for each end compounded of

$\iota'\delta s' \,.\, \iota r^{-1}\, dr/ds'$ acting towards each element $\iota'\delta s'$ of the influencing system. The components of the aggregate of this latter force, by which the present result differs from the Amperean one, are

$$\iota\int\iota'\frac{x}{r^2}\frac{dr}{ds'}ds', \quad \iota\int\iota'\frac{y}{r^2}\frac{dr}{ds'}ds', \quad \iota\int\iota'\frac{z}{r^2}\frac{dr}{ds'}ds',$$

where (x, y, z) are the coordinates of the element $\iota'\delta s'$ with reference to an origin situated at the element under consideration $\iota\delta s$. On integration by parts round the complete current circuit these become

$$\iota\int\frac{\iota'}{r}\frac{d}{dx}\left(r\frac{dr}{ds'}\right)ds', \quad \iota\int\frac{\iota'}{r}\frac{d}{dy}\left(r\frac{dr}{ds'}\right)ds', \quad \iota\int\frac{\iota'}{r}\frac{d}{dz}\left(r\frac{dr}{ds'}\right)ds',$$

that is, by definition,

$$\iota\mathrm{F}, \quad \iota\mathrm{G}, \quad \iota\mathrm{H},$$

which is the same expression as we had previously arrived at.

Apart from detailed explanation, the previous mode of variation may be cast into the briefer analytical form

$$\delta\iota\,(\mathrm{F}\,dx + \mathrm{G}\,dy + \mathrm{H}\,dz)$$

$$= \iota\left(\frac{d\mathrm{F}}{dx}dx + \frac{d\mathrm{G}}{dx}dy + \frac{d\mathrm{H}}{dz}dz\right)\delta x + \ldots + \ldots + \iota\,(\mathrm{F}\,d\,\delta x + \mathrm{G}\,d\,\delta y + \mathrm{H}\,d\,\delta z)$$

of which the last term

$$= \iota\,|\,\mathrm{F}\,\delta x + \mathrm{G}\,\delta y + \mathrm{H}\,\delta z\,| - \iota\,\delta x\left(\frac{d\mathrm{F}}{dx}dx + \frac{d\mathrm{F}}{dy}dy + \frac{d\mathrm{F}}{dz}dz\right) - \ldots - \ldots,$$

so that the whole

$$= \left\{\left(\frac{d\mathrm{G}}{dx} - \frac{d\mathrm{F}}{dy}\right)\iota\,dy - \left(\frac{d\mathrm{F}}{dz} - \frac{d\mathrm{H}}{dx}\right)\iota\,dx\right\}\delta x + \ldots + \ldots + |\,\iota\mathrm{F}\,\delta x + \iota\mathrm{G}\,\delta y + \iota\mathrm{H}\,\delta z\,|,$$

which represents the work of the Amperean forcive on the linear element together with a traction $\iota\,(\mathrm{F}, \mathrm{G}, \mathrm{H})$ on its ends.

Discrimination between Velocities and Momenta in Generalized Dynamics.

5. In the dynamics of systems whose internal connexions are only partially known, it is essential to have a clear view of the circumstances which determine whether the various quantities which enter into the specification of the energy are to be classed as coordinates, velocities, or momenta. For example, in determining the forcives between cores in problems of cyclic motion, the circulations must be treated as generalized momenta, while in the Maxwellian electrodynamics of complete circuits

the currents flowing in them are rather to be classed as velocity components. The basis of the distinction between these two classes of quantities is of course fundamental; and it is to be found in the way in which they occur in the Hamiltonian analysis. The essential property of a velocity is that it is a perfect differential coefficient, with respect to the time; any function involving rate of change of configuration, which enjoys this property so that its time-integral is a function of position only, may be taken to be a velocity, provided we, if need be, contemplate also a corresponding forcive. On the other hand, any such function of the rate of change of configuration, even though it be a perfect differential with respect to the time, must be treated as a momentum if it is known to remain constant with the time, while no applied forcive controls it; for if it were a velocity, linked up with other velocities, its constancy in the free motion could not usually fit in with the analytical theory.

In the theory of cyclic fluid motion, the circulations, being constant, must thus be taken as momenta; and when the energy is expressed in terms of them, it must be modified before the forcives can be derived from it in the manner of LAGRANGE and HAMILTON. In the theory of electrodynamics the currents are not unalterable with the time; and as they are differential coefficients with respect to the time of definite physical quantities, the charges of electricity, they may be taken as velocities, provided we recognize the play of corresponding forcives. In the electrodynamics of complete circuits, after MAXWELL, there is no reason, in that theory taken by itself, why the functions designated as the electrokinetic momenta should not be taken as velocities instead, if so desired; for they satisfy all the above conditions, though of course the corresponding forcives would be of quite different types from the usual ones. This remark is in illustration of the fact that the distinction between momenta and velocities is to a certain extent one of convenience.

A theory which would regard the current flowing in a complete circuit as a generalized momentum of the latent æthereal motion labours under many difficulties. Such a hypothesis seems tempting at first sight, and, for example, lends itself easily to the inclusion of permanent magnetism in a dynamical system: it has accordingly been developed, amongst others, by VON HELMHOLTZ, who formulated the equations on this basis in his Memoir on Least Action. The electrodynamic potential function would then represent the kinetic energy modified in ROUTH's manner by the substitution of electric momenta for velocities, which in the present case simply changes its sign: and this would seem to require that the kinetic energy of the electric field should in ordinary circumstances be negative, which could hardly be correct.

In the analogous problem of cyclic irrotational motion in fluids, with the circulation guided by rigid ring-cores, the circulations which correspond formally to the electric currents remain constant in the free motion, and so must be analytically momenta, and not velocities. The energy in this vortex problem is given by the same formula as in the electric problem, but it must be modified by change of its sign before the forcives are derived from it; and thus the circumstance, first emphasized by Lord KELVIN, that the forcives between such rigid ring-cores are equal but opposite to the forcives between the analogous rigid conductors carrying electric currents, must find its explanation in the fact that the electric currents are of the type of velocities, while the fluid circulations are of the type of momenta.

6. In the dynamics of the circulation of a fluid through ring-shaped solids, the expression for the energy in terms of vortex filaments,

$$T = \tfrac{1}{2}\Sigma\iint\sigma_1\sigma_2 r^{-1}\cos\epsilon\, ds_1\, ds_2,$$

represents its proper distribution with respect to each solid as a whole, but does not represent its distribution as regards each element of a vortex filament. Thus, when the solids are treated as rigid bodies, and are referred to the appropriate number of coordinates (six for each), this expression for the energy is competent to determine the aggregate forcive on each solid. But it is not competent to express the actual forcive on an element of the solid, because, in its formation by integration from the actual distribution of kinetic energy throughout the elements of volume of the fluid, a process of integration round the apertures of the solids has been employed, which supposes that each element of a vortex filament is connected with other elements so as to be part of a complete circuit.

In any such case the distribution of the resultant forcive throughout the solid has an undetermined part which is of the nature of an internal stress. In the example of cyclic fluid motion there is however only needed, in addition to this expression for the energy, the property that the forcive on the solids is distributed as a fluid pressure over their surfaces, in order to obtain from this form of the energy a quite definite result. The distribution of forcive on a flexible core is thus at each point at right angles to its axis, and the tension in such a core is therefore constant all along it.

If we attempt to deduce electrodynamic forcives, in cases of circuital electric flow, from the cognate expression for the energy, with current in place of vorticity but with different sign, the applicability of the method is limited in like manner. An additional hypothesis, or experimental principle, that the electrodynamic forcive on an element of a conductor is at right angles to the current which it carries, will, however, make the problem definite.

But if we were to remove the restriction that the flow of true electricity (*i.e.*, of electrons) must be circuital, and to treat for each element of the conductor the strength of the current as dynamically an electric velocity corresponding to a single definite electric coordinate which does not explicitly appear, the energy function would now be adequate by itself to determine the distribution of the forcive, and the result would be as given above, § 3. The discrepancy between this result and experiment thus seems to show that it is not legitimate to consider the current as a generalized velocity distributed over the elements of the conductor and independent for each of them. Nor is this in any way surprising, when we bear in mind that the velocity of a *single* electron is certainly of the type required for a generalized dynamical velocity component, and when we infer, as below (§ 23), that the forcive on a current considered as made up statistically from the moving electrons, is different from what would be obtained by taking the current-element itself as a generalized velocity component. Our conclusion is, then, that the only proper basis for the dynamical analysis of the phenomena of currents flowing in conductors, in fact, of all cases of the flow of true electricity, is to treat the currents as the statistical aggregates of the movements of the electrons.

Two different Methods of Analysis.

7. With a view to the analytical formulation of the properties of an æther pervaded by electrons, it is necessary in the first place to take a survey of the various ways in which these electrons are distributed. In statical circumstances the great bulk of them are grouped together into polar molecules, which may be either in totally irregular orientation as in an unpolarized dielectric, or may possess features of regularity which can be represented in a statistical manner by the type of theory first developed by POISSON with respect to induced magnetism. If the element of volume possesses an electric charge, it must also contain free electrons not so grouped ; but their number is excessively small compared with the former class.

Proceeding now to kinetics, the motions of these various classes of electrons constitute true currents of various types. A drift of free electrons constitutes an ordinary conduction current, and FARADAY'S fundamental law of electrolysis shows that when there is an electrolyte (or a dielectric) in the circuit of a steady current, the current must be made up half by a drift of positive electrons, and half by a drift of negative ones. The simplicity of the relations of a steady circuital current is due to the fact that it involves circulation, but not strain, in the surrounding æther. As a sub-class there is the so-called convection current, due to the transfer of a charge of electrons along with a material body, in its motion through the æther. There is also the current of displacement when the polar dielectric is excited, consisting of a drift within small range of the opposed positive and negative electrons in each molecule, owing to its orientation by the electric field. There is also a current due to convection of an excited dielectric, which in the case of uniform velocity may be represented as formally equivalent to a magnetization of the medium. Then again there is the molecular current, due to the orbital motions of the electrons in the molecule ; so long as we avoid the dynamics of molecular structure we can only consider the time average of this current, and that is most conveniently represented as the magnetic polarity of the molecule ; the displacement current in a dielectric above mentioned involves a similar process of averaging with respect to time. There is also the current of convection of electrons which rearranges the electric charge of a conductor. Of this last practically nothing is known ; the subject has wholly eluded experimental enquiry, except in the case of electrolytes, where the modern theories of solution and electrolysis, in the hands of HITTORF, KOHLRAUSCH, ARRHENIUS, and NERNST, have yielded at any rate the first approximation to an analytical theory. As a matter of fact, the empirical and variable character of conductivity as depending on physical state, and its overwhelming importance as compared with the other phenomena in conductors of metallic type, restricts within narrow limits the problems in which a knowledge of their relations of convection and polarization would be of practical import.

Finally, there is one case in which an experimentally recognized result in the medium involves the dynamics of the molecule ; when the magnetic field is increased the orbital motions in the molecule are altered so as to oppose the change, and this induced alteration in the molecular current constitutes diamagnetism.

8. In the face of all these various types of motion of electrons that have to be taken into account, there are two distinct modes of procedure open to us ; and they have both been used by MAXWELL, without perhaps that sharp demarcation between essentially different methods which is necessary to the perfect grasp of either of them.

We may start from the dynamical equations of the free æther, which are known, and which apply exactly to a sub-element of volume which does not contain any electrons ; we can make use of the various forces and fluxes which have proved useful in forming a representation of electrical phenomena, and by their help we can replace these dynamical equations, which are of the second order in differentiations, by the two circuital relations which are each of the first order. The advantages, both for analysis and for explanation, of a transformation of this kind have long been recognized in pure dynamics. Now we can pass, by integration of these relations, from the sub-element of volume which contains no electrons, to the effective element of volume which contains a number of electrons large enough to enable us to smooth out their individual peculiarities and so retain a continuous differential analysis. The transformation is most conveniently effected so that the circuital relations shall remain unaltered in form, while the meanings of the various quantities that enter into them are modified by taking account of the presence of the distribution of electrons in the enlarged element of volume. This method of smoothing out the molecular discreteness of the æther is the one that has been most in favour in recent years ;* but when we come to the consideration of moving media and convection currents, and particularly of ponderomotive forces, it will be necessary to supplement it by the more direct procedure now to be described.

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* [It appears to have escaped notice that the expression of the equations of the electric field in the form of the two circuital relations, together with linear equations (involving the constitution of the medium) which connect the corresponding forces and fluxes in these relations, had been given by MAXWELL himself. At the end of the Memoir "On a Method of Making a Direct Comparison of Electrostatic with Electromagnetic Force . . . ;" 'Phil. Trans.,' 1868, he alludes to the difficulty that had been felt in grasping the basis on which the electric theory of light is founded ; and he proceeds to attempt to diminish this difficulty by setting down the formally simplest foundation of hypothesis which

The other method of analysis involves the utilization of the ideas of the electrodynamic potential in an amended form. It is concerned primarily with the dynamics of a single electron in an electric field given as regards motion and strain, or kinetic and potential energy; it forms the kinetic energy T, and the potential energy W, of the electron, and thence its fundamental Lagrangian function or electrodynamic potential T − W. The relations between the motion and the forcives of the electron are now to be deduced by dynamical methods; and there only remains a process of summation in order to determine the equations that are appropriate to the various modes of coordinated groupings and movements of electrons that have been enumerated above. This process has a fundamental theoretical superiority over the previous one, in that it includes in its scope the ponderomotive forces which act on the bodies in the field. The various discussions in MAXWELL'S "Treatise," which proceed by use of the vector potential of electrodynamic induction, are to be classed as essentially related to this point of view.

Method of averaged Forces and Fluxes: the Circuital Relations: Optical Dispersion.

9. The æther is to be regarded as containing a distribution of electrons, that is of intrinsic centres or nuclei from each of which a configuration of rotational strain spreads out into the surrounding space. An additional strain may be considered as imparted to this medium in two distinct ways, (i) as the result of tractions applied over an outer boundary surface, the electrons being supposed held fixed, and (ii) as the result of movement of the electrons, each of which will carry its atmosphere of strain along with it, practically without alteration unless the velocity of the electron is so great as to approximate to the velocity of radiation. A rotational strain imparted in the first manner is circuital from its nature, as depending on absolute rotation of the element of æther. But the circuital character of a strain of the second kind is vitiated by the existence of the nuclei or intrinsic singularities that belong to it; the surface integral of the rotation, taken over any interface, remains constant and therefore keeps its initial zero value only so long as no electrons cross that interface. But we can retain the circuital property for this kind of strain also, if we associate an ideal rotational displacement with the movement of each electron, in such manner that the integral of this rotation taken (vectorially) throughout a small volume including the initial and final positions of the electron is equal to the strength of the electron multiplied by its linear displacement. This is expressed electrically by saying

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will lead to the theory at which he had previously arrived in a more dynamical and inductive manner. The form which he thus gives to the theory is precisely the one into which its equations have more recently been thrown by HEAVISIDE and HERTZ. Aug. 25.]

that a moving electron constitutes a convection current, which must be added to the flux of elastic rotation in the æther in order to obtain a total current which shall possess the circuital property. But this convection current is, so far, only a kinematic fiction; it allows us to retain the circuital relation between the total electric current and the velocity of the æther, but it is not to be counted in estimating the stress in the æther, from which the static dynamical effects result.

In ordinary electrodynamics, what we may call the *true current*, whether of convection or conduction, is measured in this way, per unit volume, by the aggregate directed drift of the electrons in the element of volume. It is completed or rendered circuital by the *displacement current*, or rate of change of rotational strain, in the æther; but in most cases of ordinary occurrence the magnitude of this displacement current is so very small compared with that of the true current, that it is allowable to neglect the former, and therefore make the true current flow in complete circuits. The *total current* of Maxwell, which consists of both parts taken together, is exactly circuital in all cases.

In circumstances of conduction, though the electric displacement (*i.e.*, rotational strain) in the medium is absolutely negligible, yet the drift of the electrons which constitutes the true current causes an irrotational flow of the medium (the magnetic field) which is related to the current in precisely the same way as the flow in a perfect fluid is related to the vortex filaments which suffice to specify it; the energy of the current system is thus the kinetic energy of this irrotational flow, the rotational flow arising from strain being in comparison inappreciable. As here primarily introduced, the true current was a fiction, so far as elastic stress in the medium is concerned; but it has now acquired an objective meaning as the mathematical quantity that serves to completely specify the energy of the flow of the medium which is associated with movement of electrons in bulk, that is, the energy of the magnetic field. The ordinary electrodynamics of conduction currents is a dynamical problem of the æther in which the kinetic energy is a function of the true current, and the potential energy, when there is such in the field, is a function of the rotational strain of the medium, that is, of the æthereal displacement current only.

10. We proceed to apply these ideas to the comparatively simple circumstances of the mode of transmission of regular vibrations by a medium thus constituted. Using the ordinary vector notation for brevity, let $\mathfrak{D}$ denote the actual rotational strain in the æther, so that $d\mathfrak{D}/dt$ is the displacement current; let $d\mathfrak{D}'/dt$ denote the true current due to movement of electrons, and let $\mathfrak{B}$ denote the velocity of the flow of the æther, that is, the magnetic induction. Each of these quantities is supposed to have its averaged value per unit volume, the irregularity of distribution due to the presence of electrons in the element of volume under consideration having been smoothed out in the analysis. The kinematic relation, which the introduction of the true current was here primarily intended to conserve, is

$$\text{curl}\ \mathfrak{B} = 4\pi \frac{d}{dt} (\mathfrak{D} + \mathfrak{D}').$$

Again, as the electric force at any point in the æther is $4\pi c^2\mathfrak{D}$, c being the velocity of radiation, the ordinary dynamical equation of the free æther,

$$-\frac{d\mathfrak{B}}{dt} = 4\pi c^2\ \text{curl}\ \mathfrak{D},$$

applies exactly throughout any sub-element of volume which does not contain electrons. On summing up for all the sub-elements which go to make up the ordinary element with its contained electrons, we derive the same equation in which $\mathfrak{B}$ and $\mathfrak{D}$ are now defined as averages taken over the volume of the element, as in the kinematic equation above.

It remains to specify the relation between $\mathfrak{D}$ and $\mathfrak{D}'$. For the case of a dielectric medium in which the positive and negative electrons are combined into polar molecules so that in each molecule they exactly compensate each other, the statical effect of an applied electric force, $4\pi\mathfrak{D}$, is to induce a polarity, $\mathfrak{D}'$, so that the total circuital electric displacement is $\mathfrak{D} + \mathfrak{D}'$; thus

$$\mathfrak{D} + \mathfrak{D}' = K/4\pi . 4\pi\mathfrak{D} = K\mathfrak{D},$$

where K is a quantity which enters in electrostatics as the inductivity of the complex medium; we take it to be a constant independent of $\mathfrak{D}$, for sufficient reason *à priori*, at any rate for small values of $\mathfrak{D}$, such as occur in ordinary electric vibrations. If we suppose that the periods of the free oscillations of the electric polarity of a molecule are very high compared with the periods of the vibrations of the medium that are under consideration, this equation will still hold for the problem of a vibrating medium, as an equilibrium theory will apply to the molecules. But when the medium shows dispersive quality, its vibrations must excite sensibly the independent vibrations of the molecules, and the equation of equilibrium for the induced polarity of the molecules must involve the kinetic reaction of their vibrations. Thus

$$\mathfrak{D} - \frac{\mathfrak{D} + \mathfrak{D}'}{K} = \frac{i}{K} \frac{d^2\mathfrak{D}'}{dt^2},$$

where i/K is a coefficient of inertia, of a kind which we do not need at present to further particularize. As the element of volume resists rotation, the uncompensated statical forcive tends to produce absolute rotation,—not relative strain as in the case of an elastic solid,—and is used up in accelerating it.

This is the general form of the relation between $\mathfrak{D}$ and $\mathfrak{D}'$ when vibrations are transmitted across a dielectric medium which contains a large number of molecules to the wave length; the actual number for light-waves, about 10^3 for solids and liquids and 10^2 for gases, is amply sufficient to justify the process of averaging which has been employed. To retain the present simple form, the analysis must however be restricted to molecules with a single efficient free period. And the hypothesis implicitly involved, that no portions of free æther (supposed perfectly continuous, *i.e.*, not itself molecular) are so effectively enclosed by surrounding molecules as to have efficient free periods of their own, involves an assumption that the volume occupied by the nuclei of electrons is small compared with the whole space. If this were not the case, the relation connecting $\mathfrak{D}$ and $\mathfrak{D}'$ would contain second differential coefficients of $\mathfrak{D}$ with respect to time, as well as of $\mathfrak{D}'$, so that in the formula for K' in § 11 (*infra*), the first term would no longer be unity. First differential coefficients of $\mathfrak{D}$ are always excluded because they would imply viscosity in the free æther.

As yet the true current is a current of dielectric molecular displacement which is perfectly reversible, so there is no dissipation of energy such as would cause absorption of the vibrations in the medium and consequent opacity. This will, however, occur should the paired electrons of the polar molecules occasionally get separated from each other when under the action of the electric force; such electrons would then travel different ways across the field and thus give rise to a conduction current. It is definitely known that this is what happens in the case of electrolytes under experimental conditions; and it is very probable (§ 23 *infra*) that the transfer of electricity by metals is of a similar nature. It is therefore necessary that we should include a similar agency in the case of light propagation; only it must be borne in mind that under a force which reverses with such tremendous rapidity, there are not the same opportunities for diffusion of momentum by action between contiguous molecules as there are in cases of steady force, so that the coefficient of opacity must be expected to be very much smaller than the conductivity of the substance as ordinarily measured by aid of Ohm's law.

Propagation in Metals.

11. In circumstances of statical strain, the relation between the total strain $\mathfrak{D}$ communicated to a given portion of the æther from its surroundings, and the polarization $\mathfrak{D}'$ of the molecules that are excited by it, is $\mathfrak{D} + \mathfrak{D}' = K\mathfrak{D}$.

Under vibrational circumstances there will be a kinetic reaction, $\mathrm{i}\,d^2\mathfrak{D}'/dt^2$, which will take part in this equilibrium, and the corresponding equation is

$$\mathrm{i}\,\frac{d^2\mathfrak{D}'}{dt^2} + \mathfrak{D}' = (K - 1)\,\mathfrak{D}.$$

When there are free electrons in the medium, as well as polar molecules, there will also be a conduction current; if m be the translational inertia of a free electron e, the electric force being $c^2\mathfrak{D}/4\pi$, its equation of motion will be $m\ddot{x} + \sigma'\dot{x} = e.4\pi c^2\mathfrak{D}$, not indeed in the literal sense that there is a frictional force $\sigma'\dot{x}$ acting on each electron, but in the sense that on the average of a large number of electrons the part of the electric force that is not used up in accelerating their motion, is spent in maintaining their steady drift, σ' being thus a coefficient of resistance to migration through the medium, whose value may however be dependent on the period; the conduction current $\mathfrak{C}$ will therefore be of the type

$$\mathfrak{C} = k\,(md/dt + \sigma')^{-1}\,\mathfrak{D}.$$

The circuital relations which express the properties of the æther will be

$$\frac{1}{4\pi}\,\mathrm{curl}\,\mathfrak{B} = \frac{d}{dt}\,(\mathfrak{D} + \mathfrak{D}') + \mathfrak{C}$$

$$-\frac{d\mathfrak{B}}{dt} = 4\pi\mathrm{C}^2\,\mathrm{curl}\,\mathfrak{D}.$$

It is thus only the first of these relations that is modified; but we may reduce it back again to the standard form which obtains for free æther. Eliminating $\mathfrak{D}'$ from it, and writing $-p^2$ for d^2/dt^2, so that $2\pi/p$ represents the period of the vibrations, we have

$$\frac{1}{4\pi}\,\mathrm{curl}\,\mathfrak{B} = \frac{d}{dt}\left(1 + \frac{\mathrm{K}-1}{-\mathrm{i}p^2+1}\right)\mathfrak{D} + k\,\frac{-md/dt+\sigma'}{m^2p^2+\sigma'^2}\,\mathfrak{D},$$

which is now of the standard form

$$\frac{1}{4\pi}\,\mathrm{curl}\,\mathfrak{B} = \mathrm{K}'\,\frac{d\mathfrak{D}}{dt},$$

provided

$$\mathrm{K}' = 1 + \frac{\mathrm{K}-1}{1-\mathrm{i}p^2} - \frac{km}{m^2p^2+\sigma'^2} - \frac{kp^{-1}\sigma'\iota}{m^2p^2+\sigma'^2},$$

being thus a complex quantity.

The interfacial conditions are, as usual, that the tangential components of $\mathfrak{D}$ and $\mathfrak{B}$ are continuous (the medium being non-magnetic), so that this theory differs from the one applicable to transparent media simply in the complex value of K′.

When the medium is perfectly transparent the square of the refractive index assumes the form $1 + \mathrm{A}/(\beta^2 - p^2)$, which agrees with a formula given by VON HELMHOLTZ and generally recognized to be a good representation of the dispersive properties of ordinary media; as that author has remarked, it may be helped out of any outstanding discrepancy by assuming slight non-selective opacity due to ordinary conduction, such as would not sensibly affect FRESNEL'S laws of reflexion. In this theory there is only one absorption band in the ultra violet, corresponding to $p = \beta$; if however the molecule has several free periods, the square of the index must still, irrespective of special theory, be a rational function of p^2, and therefore when expressed in partial fractions must assume the form $1 + \Sigma A_r/(\beta_r^2 - p^2)$; while in the case of opaque media the constants typified by A will be complex, and other partial fractions not corresponding to absorption bands will also enter.

In the case of opaque media, a peculiarity of this analysis, as compared with that of VON HELMHOLTZ* and others, is that the conduction current is derived from the electric displacement by the inverse operator $k\,(md/dt + \sigma')^{-1}$ instead of a direct operator involving a coefficient of conductivity in place of the coefficient of resistance σ'. Notwithstanding that there are as many as four adjustable constants, it is possible, to some extent, to crucially compare the resulting formula for K′ with the experimental facts for metallic media. It is known that the broad type of formal theory which simply assigns to the metals a complex index of refraction $K'^{\frac{1}{2}}$ is confirmed by the general agreement between results deduced from calculations relating to reflexion experiments by BEER, VOIGT, DRUDE and others, and those obtained directly from deviation experiments with thin metallic prisms by KUNDT.† It turns out however that the real part of K′ is invariably negative for metallic media; and this is a fundamental difficulty in ordinary elastic theories, as it implies instability of the optical medium. On the present theory it implies that k is sufficiently large to allow the third term of K′ to outweigh the first two terms. It is also found (by both methods) that if $K'^{\frac{1}{2}} = n\,(1 - \iota\kappa)$, then for the better conducting metals, silver, gold, copper, n is less than unity, involving velocity of propagation greater than in a vacuum, while κ is a considerable number; ‡ this implies that the fourth term in the formula for K′ is in these cases small compared with the third, which is just what is to be expected from the smaller value of the resistance coefficient σ'. This point is confirmatory of the present scheme ; for if the conduction current were derived from the displacement $\mathfrak{D}$ by a direct operator involving conductivity instead of an inverse one involving resistance as here, the opposite effect would be indicated.

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* VON HELMHOLTZ, "Electromagnetische Theorie der Farbenzerstreuung," 'Wied. Ann.,' xlviii., 1893. The analysis of VON HELMHOLTZ consists, as usual with him, in a tentative process of fitting known electric laws into a minimum theorem which is an extended form of the Principle of Least Action. For this purpose he uses two sets of variables to represent what we have here called the true current and the displacement current of the free æther; and he varies them independently of each other. There is no distinction drawn between the polarization and the conduction parts of the true current; so that the current of conduction appears in the potential energy function, thus being assumed to imply elastic strain of the medium, in opposition to the views that have been here set forth. It has been shown by REIFF ('Wied. Ann.,' l, 1893, p. 361) that the theory of VON HELMHOLTZ does not lead to FRESNEL'S formula for the influence of moving media on the velocity of propagation, unless the æther is supposed to partially partake of the motion of the material medium; but that when certain terms are omitted from his potential energy function, it is not necessary to assume that the æther is moved with the matter. But it does not appear that any reason is assigned for such modification of the theory, which, as already remarked, seems to be intrinsically at variance with the view here taken.

† KUNDT, 'Phil. Mag.,' 1888 (2). It had been already shown by VOIGT ('Wied. Ann.,' xxiv., 1885), that the ordinary optical formulæ for prismatic deviation apply when the metallic prism is of very minute angle.

‡ *Cf.* the numbers quoted from DRUDE in Professor J. J. THOMSON'S "Recent Researches . . . . " § 355.

When the third and fourth terms in K′ are predominant, the dispersion will usually be in the abnormal direction, as it is known to be for the great majority of metals, a few of the more conducting ones being however exceptions ; and the value of κ will usually increase with the frequency, which is also in accordance with fact.

[Aug. 25.—The introduction of effective inertia into the equation of conduction, as above, thus appears to be essential to the theory. For the actual negative value of the real part of K′ in metals cannot arise from dielectric polarity of the material medium ; that, as has often been remarked, would imply instability and consequent destruction of such polar structure. To make the other agency which is at work, namely conduction, effective for that purpose, its equation must involve inertia : and the influence of this inertia appears in fact also in other ways ; thus the work done by the electric force on the ions in an electrolyte must be used up proximately in accelerating their velocities, while the increased average velocity reveals itself as Joulean heat. If there were a large number of dissociated ions along a wave-length, which is indeed the condition that the above continuous analysis be literally applicable, it is easy to see by consideration of molecular magnitudes that the effective inertia of an ion would have to be very much greater than its actual mass, or else the effect would be excessive. But this difficulty is only apparent. When the ions are more thinly scattered through the medium, there will be two sets of waves propagated ; the waves of free æther modified somewhat by the presence of the ions, but not extremely different from what they would be if the ions were held fixed in the medium ; and much slower waves propagated from ion to ion with the intervening æther nearly in an equilibrium condition at each instant. The former class alone would be sensibly excited by optical means : it may be formally represented by the above scheme of equations with the inertia coefficient large, and it is wide enough to include the phenomena both of transparent dielectrics and of opaque metallic media. It is found that, for the wave-lengths of luminous radiation, the real and imaginary parts of the square of the index of refraction are of about the same order of magnitude for all metals. This possibly indicates that the depth to which the light can penetrate in the metal, and therefore also the coefficient of absorption, depends essentially on the ratio of molecular magnitudes to wave-length, which would be about the same for all.]

Refraction distinct from Dispersion.

12. In the former paper, a physical foundation was assigned for MacCullagh's theory of dispersion. That theory being a statical one, must rest on the discreteness of the medium being comparable with the wave-length of the radiation ; but conside-

rations similar to those given in that paper (§ 123) in connection with CAUCHY'S theory, show that the number of molecules in the wave-length is too great to allow this cause to account for the magnitude of the actual dispersion. Thus by far the greater part of ordinary dispersion, as distinct from refraction, is to be assigned to the sympathetic vibrations of the molecules as here discussed.

[Aug. 25.—For waves of long period and therefore great length, p approaches the value zero; thus K′ approaches the limiting value K and a statical theory represents the phenomena, the molecules being at each instant in the equilibrium position corresponding to the strained state of the surrounding æther. In any case, we may conveniently designate this constant part of K′, the square of the index, by the name of the *refraction*, and the variable part, which depends on the period and ultimately vanishes when the period is long, by the name of the *dispersion*. In the very wide class of media for which the specific inductive capacity, that is the square of the index of refraction for very long waves, is nearly equal to the square of the index for ordinary light-waves, the dispersion is thus small compared with the refraction. In all such cases a statical theory of refraction, which, according to the argument of the previous paper, must be MACCULLAGH'S theory, will certainly be correct; and there is ground for making this conclusion general. Thus, in particular, MACCULLAGH'S theory of the double refraction in crystalline media will hold good as the first approximation. But just like ordinary refraction, this crystalline refraction is subject to dispersive variation when the wave-length of the light is altered. This dispersion of the optic axes in crystals is usually small compared with the double refraction itself, which justifies the present mode of treatment of it as a subsidiary effect to be joined to the main part of the double refraction. In order to obtain equations of propagation in which it shall be included, we have only to add to the effective coefficient of inertia of the æther in MACCULLAGH'S equations a subsidiary æolotropic part. This part may be complex instead of real when the medium is not perfectly transparent, thus including the effect of conduction arising from the presence of free ions; the general equations of absorbing doubly refracting media may in fact be formulated without difficulty on the lines of § 11.

For transparent media, this generalization of MACCULLAGH'S theory preserves the wave-surface, corresponding to any given period of the light, exactly FRESNEL'S. The character of the laws of crystalline reflexion also remains unaltered, but the constants that are involved in them are no longer exactly the same constants that occur in the equation of the wave-surface.

The asymmetric refraction of higher order, which evidences itself by rotation of the plane of polarization, is obviously of a structural kind, and so is correctly represented by MACCULLAGH'S terms. It is itself a highly dispersive phenomenon, on account of the higher differential coefficients on which it depends; thus the dispersion due to variation of its constants with the wave-length may usually be neglected in comparison.]

Influence of Motion of the Medium on Light-Propagation.

13. To deduce Fresnel's law for moving media directly from these principles we have to remember that a movement of the material dielectric through the æther with velocity v parallel to the axis of x produces an additional displacement current at any point fixed in the æther, which (§ 31) is equal to $v d\mathfrak{D}'/dx$. Thus in Ampère's circuital relation $d\mathfrak{D}'/dt$ must be replaced by $(d/dt + v d/dx)\mathfrak{D}'$. Again, from the mode in which the other circuital relation appears in the dynamical theory of the medium, $d\mathfrak{B}/dt$ must mean the total acceleration of velocity of the æther, due in part to change of time and in part to movement of the material dielectric; thus this d/dt also, when it operates on $\mathfrak{D}'$, must be replaced by $d/dt + v d/dx$. In the present connection we may neglect dispersional phenomena and so take $\mathfrak{D} + \mathfrak{D}' = \mathrm{K}\mathfrak{D}$. Thus finally

$$\mathrm{c}^2\nabla^2\mathfrak{B} = \frac{d^2\mathfrak{B}}{dt^2} + (\mathrm{K} - 1)\left(\frac{d}{dt} + v\frac{d}{dx}\right)^2 \mathfrak{B},$$

which leads directly to Fresnel's formula as in § 124 of the previous paper.

14. A clear appreciation of what is involved in the unexplained null result of Michelson and Morley may be obtained by transferring this equation of propagation to coordinates fixed in the moving material system. Let us assume new coordinates x', t' given by

$$x' = x - vt, \quad t' = t + \alpha x,$$

while y and z are unaltered. Then

$$\frac{d}{dx} = \frac{d}{dx'} + \alpha\frac{d}{dt'}, \quad \frac{d}{dt} = \frac{d}{dt'} - v\frac{d}{dx'}.$$

Thus the equation of propagation becomes, on dropping the accents,

$$\left(\frac{d}{dt} - v\frac{d}{dx}\right)^2\mathfrak{B} + (\mathrm{K} - 1)(1 + \alpha v)^2\frac{d^2}{dt^2}\mathfrak{B} = \mathrm{c}^2\left\{\frac{d^2}{dy^2} + \frac{d^2}{dz^2} + \left(\frac{d}{dx} + \alpha\frac{d}{dt}\right)^2\mathfrak{B}\right\}.$$

This will be again of the type of an equation of simple wave-propagation if the coefficient of $d^2\mathfrak{B}/dx\,dt$ vanishes, that is if $v = -\mathrm{c}^2\alpha$; it then becomes

$$\left(1 - \frac{v^2}{\mathrm{c}^2}\right)\left\{1 + \left(1 - \mathrm{K}^{-1}\right)\frac{v^2}{\mathrm{c}^2}\right\}\mathrm{K}\frac{d^2\mathfrak{B}}{dt^2} = \mathrm{c}^2\left\{\frac{d^2}{dy^2} + \frac{d^2}{dz^2} + \left(1 - \frac{v^2}{\mathrm{c}^2}\right)\frac{d^2}{dx^2}\right\}\mathfrak{B}.$$

This means that, referred to a standard of time that varies from point to point, corresponding times at different points being those for which $t - vx/\mathrm{c}^2$ is the same, the propagation relative to the moving material homogeneous isotropic medium is precisely the same as if that medium were at rest with respect to the æther,* provided it be taken slightly æolotropic, in the sense of being less elastic in the direction of the velocity v than in other directions, the difference depending on v^2/c^2, and so being excessively minute. This different reckoning of time at different points does not affect the paths of the rays, which are determined by FERMAT's principle of minimum time of transit; thus the paths of the rays relative to the moving material medium are precisely the same as if that medium were fixed, and at the same time made very slightly æolotropic. But so long as this æolotropy is not compensated by some other second-order correction, the time of passage of a ray from one point to another and back again will depend on the direction as well as the length of the line joining these points, which is in opposition to the experimental result.

Following up a conjecture already thrown out by G. F. FITZGERALD and H. A. LORENTZ,† we can find this compensating second-order correction in a change of the dimensions of a material body produced by motion through the æther. For this purpose we must assume, in conformity with the considerations already given in the previous paper (§ 115), that material systems are built up solely out of singular points in the æther which we have called electrons and that atoms are simply very stable collocations of revolving electrons. Now the equation already given is the one which governs phenomena, when the axes of coordinates as well as the material system are moving through the æther with velocity v parallel to the axis of x. If we write $x' = x\,(1 - v^2/\mathrm{c}^2)^{-\frac{1}{2}}$, it assumes the form

$$\left(1 - \frac{v^2}{\mathrm{c}^2}\right)\left\{1 + \left(1 - \mathrm{K}^{-1}\right)\frac{v^2}{c^2}\right\} \mathrm{K}\,\frac{d^2\mathfrak{B}}{dt^2} = \mathrm{c}^2\left(\frac{d^2}{dx'^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right)\mathfrak{B};$$

and this is now of the same type as if the axes of coordinates were not moving. That is, if we know an actual configuration, steady or varying, of matter at rest in the æther, we derive from it a configuration of matter moving through the æther with velocity v by compressing it in the direction of this velocity in the ratio of $(1 - v^2/\mathrm{c}^2)^{\frac{1}{2}}$ to unity, and at the same time adopting a standard of time which varies from point to point so that each point has its own origin from which time is reckoned. Now the movable framework of MICHELSON's experiments forms a steady configuration of matter when it is at rest, the element of time only entering in connection with molecular motions; hence when it is moving, the condition of continued steadiness requires that it should change its steady dimensions in such way that dx becomes dx'. Thus when linear measurements are estimated with respect to this moving

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[*] FRESNEL's law for the effect of motion of the medium is involved in this statement, as has been shown by LORENTZ, *loc. cit.*, § 339 *infra*.

[†] *Cf.* especially LORENTZ, *loc. cit.*, § 92, who suggests a possible explanation which comes to the same as the one here advanced. It would necessitate an annual inequality of excessively minute amount, in the length of the sidereal day, owing to the Earth's orbital motion.

framework the equation which governs the relative phenomena is of precisely isotropic type; and the negative result of MICHELSON is explained. This transformation is independent of K altogether; it will therefore hold good equally if the atoms are supposed to have intrinsic inertia in addition to the electric inertia which arises directly from the relations of the electrons to the surrounding æther.

If this argument be allowed, it follows (§ 80, previous paper) that movements of perfectly transparent matter, if without rotation, cannot affect the course of a train of light-waves which passes across that matter, in conformity with the result previously deduced from the law of entropy. It is to be noted, however, that if the radiation exerted a resultant mechanical forcive on the moving matter so as to do work on it in its motion, the argument from entropy would break down. This illustrates the limitations of that kind of argument; but in the present case we shall find (§ 28) that the argument is justified, the resultant forcive in question being null when the matter is perfectly transparent.

Method of Separate Electrons.

15. In the Appendix to the previous paper, the dynamics of the mutual actions of electrons was briefly sketched, under the restriction that the velocities of the individual electrons did not approach the velocity of radiation. If these velocities do not exceed say one per cent. of the velocity of light, the strain in the æther will at each moment be practically in the steady equilibrium state which it would have exactly were the velocity of radiation infinite, and the whole circumstances of the field will depend solely on the coordinates which specify the positions of the electrons. In an approximate theory of this kind the phenomena of radiation cannot, of course, themselves be included.

To obtain an exact theory, free from this restriction, it will be necessary to express the distribution of energy in the æther in terms of coordinates of a kind which will allow us to include among them the position coordinates of the various electrons; we must, therefore, transform the kinetic energy into an expression involving change of the rotational strain (f, g, h) in the æther instead of its translational velocity $(\dot{\xi}, \dot{\eta}, \dot{\zeta})$. This procedure will be of fundamental importance, because the relation of matter to æther must resolve itself into the relation of electrons to æther, and the various forcives between æther and matter must be derivable in that way.

If, for facility of interpretation, we adhere to the ordinary electromagnetic system of units, we have

$$T = \frac{1}{8\pi}\int(\dot\xi^2 + \dot\eta^2 + \dot\zeta^2)\,d\tau$$

$$= \frac{1}{8\pi}\int\left\{\left(\frac{dH}{dy} - \frac{dG}{dz}\right)\dot\xi + \left(\frac{dF}{dz} - \frac{dH}{dx}\right)\dot\eta + \left(\frac{dG}{dx} - \frac{dF}{dy}\right)\dot\zeta\right\}d\tau$$

$$= \frac{1}{8\pi}\int\left\{F\left(\frac{d\dot\zeta}{dy} - \frac{d\dot\eta}{dz}\right) + G\left(\frac{d\dot\xi}{dz} - \frac{d\dot\zeta}{dx}\right) + H\left(\frac{d\dot\eta}{dx} - \frac{d\dot\xi}{dy}\right)\right\}d\tau,$$

on integration by parts, a surface integral over the infinite sphere being neglected, as usual, because the energy is all considered to be in a finite region. Now, in free æther, the curl of the velocity of the æther $(\dot\xi, \dot\eta, \dot\zeta)$, divided by 4π, is equal to the displacement current; in ponderable media it is, on the other hand, equal to the total current composed of this displacement current and the drift of electrons, that total current being circuital in accordance with this relation. If the total current be (u, v, w), we have, therefore,

$$T = \tfrac{1}{2}\int(Fu + Gv + Hw)\,d\tau.$$

In these formulæ $(\dot\xi, \dot\eta, \dot\zeta)$ is the curl of the vector (F, G, H), so that

$$\nabla^2 F - \frac{d}{dx}\left(\frac{dF}{dx} + \frac{dG}{dy} + \frac{dH}{dz}\right) = -\,4\pi u,$$

with two similar equations. The unique solution of these equations satisfying the condition $dF/dx + dG/dy + dH/dz$ null is

$$(F, G, H) = \int(u, v, w)r^{-1}\,d\tau.$$

Thus (F, G, H) is the vector potential of the total current, including, when necessary, Amperean molecular currents; and

$$T = \Sigma\Sigma\,(uu' + vv' + ww')r^{-1}\,\delta\tau\,\delta\tau'.$$

If an electron e moving with velocity v, say $(\dot x, \dot y, \dot z)$, constitute a part of the total current, we have, setting out at length all the part of the energy depending on e,

$$T = \tfrac{1}{2}Le^2(\dot x^2 + \dot y^2 + \dot z^2) + (e\dot xF + e\dot yG + e\dot zH) + \ldots$$

Hence the motion provides a kinetic reaction which must be balanced by stress in the æther, just in the same way as the reversed mass-acceleration of a moving body balances the applied forcive by D'Alembert's principle; and the component of this kinetic reaction on the electron which has reference to translation along x, is in this case, by LAGRANGE's dynamical equation,

$$P_1' = -\left(\frac{\delta}{dt}\frac{d\mathrm{T}}{d\dot{x}} - \frac{d\mathrm{T}}{dx}\right)$$

$$= -\mathrm{L}e^2\ddot{x} - e\left(\frac{\delta \mathrm{F}}{dt} + \dot{x}\frac{d\mathrm{F}}{dx} + \dot{y}\frac{d\mathrm{G}}{dx} + \dot{z}\frac{d\mathrm{H}}{dx}\right),$$

in which δ/dt implies total rate of variation, due both to the change of time and to change of position of the electron in that time, so that

$$\frac{\delta}{dt} = \frac{d}{dt} + \dot{x}\frac{d}{dx} + \dot{y}\frac{d}{dy} + \dot{z}\frac{d}{dz}.$$

Hence finally,

$$P_1' = -\mathrm{L}e^2\ddot{x} + e\left(c\dot{y} - b\dot{z} - \frac{d\mathrm{F}}{dt}\right),$$

where

$$(a, b, c) = \mathrm{curl}\,(\mathrm{F}, \mathrm{G}, \mathrm{H}).$$

It is simplest to split this up into two parts, and to say that the motional forcive (P_1, Q_1, R_1) typified by

$$P_1 = e\left(c\dot{y} - b\dot{z} - \frac{d\mathrm{F}}{dt}\right),$$

will go on increasing the acceleration of the electron, of effective mass $\mathrm{L}e^2$, until the motion has brought the system of electrons into such a configuration that this forcive is compensated by traction due to the elastic strain of the surrounding æther, or in case of currents of conduction by Ohmic resistance to diffusion.

16. The potential energy of the system resides wholly in the æther and is

$$\mathrm{W} = 2\pi \mathrm{C}^2 \int (f^2 + g^2 + h^2)\, d\tau.$$

The variation of W will give the statical forcive which would work in altering (f, g, h): but to this forcive must be added a constraining forcive required to maintain the condition of constraint between f, g, and h, which is represented by

$$\frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} = \rho,$$

where ρ is the density of free electrons. This constraint will be included if we conduct the variation of the equivalent expression

$$\mathrm{W}' = 2\pi \mathrm{C}^2 \int (f^2 + g^2 + h^2)\, d\tau - \int \Psi \left(\frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} - \rho\right) d\tau,$$

and afterwards determine suitably this function of position Ψ. Now

$$\delta W' = \int\left\{\left(4\pi c^2 f + \frac{d\Psi}{dx}\right)\delta f + \ldots + \ldots\right\} d\tau + \int \Psi\,\delta\rho\,d\tau - \int \Psi\,(lf + mg + nh)\,dS,$$

after the usual integration by parts. Hence we have the forcive on free æther

$$(P, Q, R) = \left(4\pi c^2 f + \frac{d\Psi}{dx},\quad 4\pi c^2 g + \frac{d\Psi}{dy},\quad 4\pi c^2 h + \frac{d\Psi}{dz}\right).$$

And we also observe, from the form of $\delta W'$, that Ψ represents the potential of the distribution of electrons in the sense that a change of density in the volume $\delta\tau$ increases the statical energy by $\Psi\,\delta\rho\,\delta\tau$; so that the movement of an electron e is assisted by a force

$$-e\left(\frac{d}{dx},\ \frac{d}{dy},\ \frac{d}{dz}\right)\Psi.$$

To understand how this latter forcive works, let us consider an electron, in the electric field represented by a rotational strain (f, g, h) in free æther. The energy from which the traction is derived is

$$W = 2\pi c^2 \int (f^2 + g^2 + h^2)\,d\tau,$$

where in the immediate neighbourhood of the electron

$$(f, g, h) = \left(-\frac{e}{4\pi}\frac{dr^{-1}}{dx} + f',\ -\frac{e}{4\pi}\frac{dr^{-1}}{dy} + g',\ -\frac{e}{4\pi}\frac{dr^{-1}}{dz} + h'\right),$$

the first terms representing the part of the field due to the electron itself. Thus expressing only the terms which represent the interaction between the electron and the extraneous part of the field (f', g', h'),

$$\begin{aligned} W &= -c^2 e \int \left(f'\frac{d}{dx} + g'\frac{d}{dy} + h'\frac{d}{dz}\right) r^{-1}\,d\tau + \ldots \\ &= -c^2 e \int (lf' + mg' + nh')\,r^{-1}\,dS + c^2 e \int \left(\frac{df'}{dx} + \frac{dg'}{dy} + \frac{dh'}{dz}\right) r^{-1}\,d\tau + \ldots \\ &= c^2 e \Sigma \frac{e'}{r} + \ldots, \end{aligned}$$

provided the nucleus of the electron sensibly maintains its spherical form. The resultant traction of the medium over the surface of the electron is obtained by varying W, and is therefore a forcive

$$-e\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right)\Psi, \quad \text{where } \Psi = c^2\Sigma\frac{e'}{r},$$

which helps towards compensating the kinetic reaction aforesaid.

This result has been obtained by separating away the part of the stress which increases indefinitely as the electron under consideration is approached, and which therefore represents the field of that electron itself in the neighbouring æther. It is noteworthy that the resultants of the stresses between the electrons at finite distances apart are represented simply by their electrostatic attractions, whether the field of æther is in equilibrium or is disturbed in any manner whatever; a result which depends on the smallness of the nuclei, and the consequent intensity of the permanent strain near their surfaces.

The forcive thus derived, partly kinetic and partly static, is named the electric force, because in the collocation of polarized molecules and free electrons forming an unelectrified body, it tends to produce electric separation by driving the positive electrons one way and the negative ones the opposite way, while it has no tendency to move the element of volume as a whole. But if there is an excess of one kind of electrons over the other in an element of volume of the body, so that the element has a charge q, it also provides a force, equal to q multiplied by its intensity, acting on the element of volume of the charged body. This mechanical forcive may in certain cases be represented as the unbalanced part of a stress in the manner of MAXWELL'S stress in the medium; but in our present order of ideas it is the force itself that is the reality, and the stress is only a mathematical mode of representing it, which has no physical significance as it does not represent the actual stress either in the æther or in the material medium.

In one case this analysis must go deeper, namely in deducing the traction on an element of surface of the vacuous core of an electron. One mode of procedure would be to imagine the æther to be continuous throughout the core, but unstrained inside it, thus avoiding internal boundaries,—a method which has been already applied (previous paper, §51) in more intricate cases. We must then imagine the electric charge as freely distributed over the surface of the core in order to maintain this state of equilibrium, and the traction will simply be the usual forcive on this electric charge, namely, a normal pull equal to $\sigma^2/8\pi$ per unit area, where σ is its surface-density, as above. The same result would be more directly derived by varying the energy in the æther with respect to a displacement of the surface of the core.

17. It remains to find the electrokinetic part of the forcive tending to increase the electric displacement (f, g, h) in an element $\delta\tau$ of free æther. The kinetic energy associated with the element is

$$T_2 = (\dot{f}F + \dot{g}G + \dot{h}H)\,\delta\tau;$$

therefore

$$P_2 = -\frac{\delta}{dt}\frac{dT_2}{d\dot{f}} = -\frac{dF}{dt}.$$

No difference is here introduced by motion of the æther such as represents magnetic induction: in fact we have seen that the kinetic reaction on an electron e moving along with the æther is $-e d\mathrm{F}/dt$ simply; and to a doublet moving in this way the electric displacement in the element of æther itself may be assimilated. This forcive $(\mathrm{P}_2, \mathrm{Q}_2, \mathrm{R}_2)$ strains the æther, and is compensated jointly by the stress which it thus calls forth and the kinetic reaction of the motion involved in change of strain.

18. We have now to formulate the intensity (u, v, w) of the total circuital current in an extended body, as made up of conduction current, polarization current, æthereal displacement current, convection currents of free electrons and of polarized molecules, and the current which produces redistributions of electric charges in a conductor. If the vector $\mathfrak{D}'$ denote the aggregate result of the polarization in the body, made up of orientation of the electrons with the æthereal displacement bound to them, and $\mathfrak{D}$ the free displacement in the æther which excites it, the POISSON-MOSSOTTI theory of polarization will give a relation $\mathfrak{D} + \mathfrak{D}' = \mathrm{K}\mathfrak{D}$, where the constant K denotes specific inductive capacity; thus of the total displacement $\mathfrak{D} + \mathfrak{D}'$, the fraction $1 - \mathrm{K}^{-1}$ is bound to the polarization of the molecules and only the fraction K^{-1} is free elastic displacement.

When an element of volume of a conductor moves across a magnetic field with velocity (p, q, r), the force (P, Q, R) acting on its contained electrons, supposed at rest in it, is of the type

$$\mathrm{P} = cq - br - \frac{d\mathrm{F}}{dt} - \frac{d\Psi}{dx},$$

acting on the positive electrons one way, on the negative ones the reverse way. When the electrons are in motion forming a current, it might at first sight appear that the ones that drift in the same direction as the conductor moves would experience the greater force, and so carry most of the current. But no difficulty of this kind occurs; for it is only the component of the velocity of the conductor at right angles to the direction of the current that is effective, and this acts equally on both sets of electrons. When a material body is in motion, the force tending to produce electric separation of its electrons, that is the electric force exerted in it, is given by this formula whether it carries a current or not. Strictly, the velocity (p, q, r) which occurs in this formula is relative to the æther, so that it will involve the velocity of the æther itself which constitutes the magnetic field: but it follows from optical experiments that the latter is extremely minute in comparison with ordinary velocities of material bodies.

On the other hand, the force tending to produce free rotational displacement in the æther is $(\mathrm{P}', \mathrm{Q}', \mathrm{R}')$, where

$$\mathrm{P}' = -\frac{d\mathrm{F}}{dt} - \frac{d\Psi}{dx}.$$

The former force acts on $\mathfrak{D}'$, this one on $\mathfrak{D}$. Under steady circumstances the corresponding parts of the current must have for x component

$$\frac{\mathrm{K}-1}{4\pi\mathrm{C}^2}\frac{\delta\mathrm{P}}{dt}+\frac{1}{4\pi\mathrm{C}^2}\frac{d\mathrm{P}'}{dt};$$

in which δ/dt represents $d/dt + p\,d/dx + q\,d/dy + r\,d/dz$, as appears from the fact that mere convection of a steady polarization by motion of the matter through the æther itself constitutes a current.

The total current is thus (u, v, w) given by

$$u = \sigma\mathrm{P} + \frac{\mathrm{K}-1}{4\pi\mathrm{C}^2}\frac{\delta\mathrm{P}}{dt}+\frac{1}{4\pi\mathrm{C}^2}\frac{d\mathrm{P}'}{dt} + p\rho + u_0,$$

where

$$4\pi\rho = \frac{d\mathrm{P}'}{dx}+\frac{d\mathrm{Q}'}{dy}+\frac{d\mathrm{R}'}{dz} = -\nabla^2\Psi;$$

this is on the hypothesis that the material polarization is all induced and therefore circuital, and so adds nothing to the convergence of $(\mathrm{P}', \mathrm{Q}', \mathrm{R}')$. If the velocity of the material medium is supposed uniform, we have

$$\frac{d\mathrm{P}}{dx}+\frac{d\mathrm{Q}}{dy}+\frac{d\mathrm{R}}{dz} = -4\pi(pu+qv+rw) - \nabla^2\Psi;$$

hence, when the material medium is at rest, the condition of circuitality of the current (u, v, w) is

$$0 = \left(4\pi\sigma + \mathrm{K}\mathrm{C}^{-2}\frac{d}{dt}\right)\rho - \frac{d\rho}{dt};$$

so that, if initially there is any volume density of electrification, it at once diffuses on to the interfaces in the case of conducting media.

The only part of this scheme of equations that is incomplete is the specification of (u_0, v_0, w_0), that portion of the current which redistributes free electrifications. In an ordinary conductor this current is of the same order of magnitude as the polarization current, which is itself wholly masked by the current of conduction, except in the case of optical phenomena. In a dielectric this current does not exist at all. In an electrolyte we may form a provisional scheme of it on the lines of NERNST's theory of migration of the ions, by putting $(u_0, v_0, w_0) = k\rho\ (\mathrm{P}, \mathrm{Q}, \mathrm{R})$, on the supposition that the medium is uniform, where k is analogous to a coefficient of ionic mobility: but we shall thereby destroy the linearity of the system of equations. The fact that free electrons act on each other simply by their electrostatic attractions, however the æther between them be disturbed, shows that they will tend to drive each other to the surfaces of the conductors, so that the current (u_0, v_0, w_0) will usually be a transient phase at the beginning of the settling of the disturbance, in agreement with the above, if it ever exist at all; and once the free charges are on the surfaces of the conductors they will remain there and be redistributed by ordinary conduction currents. This part of the current may therefore possibly be left out of account; that is, the electric density in a good conductor at rest may be taken to be always null.

19. It is the migration of the positive and negative ions through the unelectrified material medium in opposite directions that constitutes the conduction current: a movement of the medium itself carries as many positive as negative electrons along with it, and so adds nothing to the current: the medium in a sense moves through the current, without carrying it along,—in opposition to the assumption usually implied in the notion of a current element. This is true irrespective of the manner in which the current is distributed between flux of positive electrons one way, and flux of negative electrons the opposite way. In all cases, however, in which there is an electrolyte in the circuit, the law of FARADAY shows, not that the current is equally divided between positive and negative ions, but that the numbers of these ions crossing in opposite directions any section of the steady current are equal,—although this necessitates extensive changes of concentration in the electrolytic solution after the current has become steady, when the ions have different rates of migration, as has been demonstrated by HITTORF. If we were to assume that such changes of concentration of ionic electrons are not important in the metallic portion of the circuit, the interface between two metallic media not being sensibly polarizable, it would follow that the velocities of migration of the positive and negative electrons in such conductors would be sensibly equal; and this equality would not be altered by the presence of a magnetic field. (*Cf.* however § 23.)

The conduction current does not involve elastic displacement; if it flows in a complete circuit so that electrons are not allowed to accumulate and exert a back electric force, it will go on permanently, a limit being set to it only by the *quasi*-frictional resistance to the motions of the ions through the medium in the sense of the kinetic theory of gases, which is expressed by the law of OHM.

The relation to the principle of energy of the force (P, Q, R) which tends to produce electric separation in a conductor is expressed by the formula that, if there is a drift of positive electrons one way and negative the opposite way, so as to form a *true* current (u', v', w'), the time-rate at which the potential energy of the field is thereby exhausted is $u'\mathrm{P} + v'\mathrm{Q} + w'\mathrm{R}$ per unit volume. The electric force as here introduced therefore agrees with its formulation in the ordinary elementary theory of true currents, in connection with OHM'S law.

20. The system of electromotive equations may now be collected together. The dynamical equations in the free æther are of type

$$4\pi C^2 f + \frac{d\Psi}{dx} = -\frac{dF}{dt}, \text{ where } F = \int \frac{u}{r}\, d\tau + \int\left(B\frac{d}{dz} - C\frac{d}{dy}\right)\frac{1}{r}\, d\tau.$$

The force acting on an electron e, moving with velocity $(\dot{x}, \dot{y}, \dot{z})$, is of type

$$eP = e\dot{y}c - e\dot{z}b - e\frac{dF}{dt} - e\frac{d\Psi}{dx}, \text{ where } (a, b, c) = \text{curl}\,(F, G, H);$$

and its coefficient of electric inertia is Le^2, where L depends on the radius of its nucleus.

The total current in an element of matter moving with velocity (p, q, r) through the æther is of type

$$u = \sigma P + \frac{K-1}{4\pi C^2}\frac{\delta P}{dt} + \frac{df}{dt} + p\rho + u_0,$$

where

$$P = qc - rb - \frac{dF}{dt} - \frac{d\Psi}{dx},$$

and δ/dt represents $d/dt + p\, d/dx + q\, d/dy + r\, d/dx$; while

$$\rho = \frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz}$$

where

$$f = \frac{1}{4\pi C^2} P' = -\frac{1}{4\pi C^2}\left(\frac{dF}{dt} + \frac{d\Psi}{dx}\right).$$

21. When the material system is moving through the æther with uniform velocity p parallel to the axes of x, the equations become

$$(P, Q, R) = \left(\frac{dF}{dt} - \frac{d\Psi}{dx},\quad -\frac{dG}{dt} - \frac{d\Psi}{dy} - pc,\quad -\frac{dH}{dt} - \frac{d\Psi}{dz} + pb\right).$$

Thus

$$\frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz} = p\nabla^2 F - \nabla^2\Psi; \text{ also, in all cases } \frac{dF}{dx} + \frac{dG}{dy} + \frac{dH}{dz} = 0.$$

As the total current (u, v, w) is circuital, we must have

$$0 = \left(\sigma + \frac{K-1}{4\pi C^2}\frac{\delta}{dt}\right)(p\nabla^2 F - \nabla^2\Psi) - \frac{1}{4\pi C^2}\frac{d}{dt}\nabla^2\Psi + \left(\frac{\delta\rho}{dt} - \frac{d\rho}{dt}\right) - \frac{\delta\rho}{dt}.$$

It will suffice at present to confine ourselves to a non-conducting medium, so that σ, ρ, and (u_0, v_0, w_0) are null; then this condition of circuitation gives

$$\left\{\frac{d}{dt} + (1 - K^{-1})\, p\frac{d}{dx}\right\}\Psi = (1 - K^{-1})\, p\left(\frac{d}{dt} + p\frac{d}{dx}\right)F,$$

or neglecting cubes of p/c,

$$\Psi = (1 - \mathrm{K}^{-1})p\left(1 + \mathrm{K}^{-1}p\frac{d/dx}{d/dt}\right)\mathrm{F}.$$

We have also, when all the magnetism is induced,

$$\nabla^2(\mathrm{F}, \mathrm{G}, \mathrm{H}) = -4\pi\mu(u, v, w);$$

thus

$$\frac{\mathrm{c}^2}{4\pi\mu}\nabla^2\mathrm{F} = -\frac{\mathrm{K}-1}{4\pi}\left(\frac{d}{dt} + p\frac{d}{dx}\right)\left(-\frac{d\mathrm{F}}{dt} - \frac{d\Psi}{dx}\right) - \frac{1}{4\pi}\frac{d}{dt}\left(-\frac{d\mathrm{F}}{dt} - \frac{d\Psi}{dx}\right)$$

$$= \frac{\mathrm{K}}{4\pi}\frac{d^2\mathrm{F}}{dt^2} + \frac{\mathrm{K}-1}{4\pi}\left\{\frac{d^2\Psi}{dx\,dt} + p\frac{d^2\mathrm{F}}{dx\,dt} + p\frac{d^2\Psi}{dx^2}\right\} + \frac{1}{4\pi}\frac{d^2\Psi}{dx\,dt},$$

so that

$$(\mathrm{K}\mu)^{-1}\mathrm{c}^2\nabla^2\mathrm{F} = \frac{d^2\mathrm{F}}{dt^2} + 2(1 - \mathrm{K}^{-1})p\frac{d^2\mathrm{F}}{dx\,dt} + (1 - \mathrm{K}^{-1})p^2\frac{d^2\mathrm{F}}{dx^2}.$$

Again

$$\frac{\mathrm{c}^2}{4\pi\mu}\nabla^2\mathrm{G} = -\frac{\mathrm{K}-1}{4\pi}\left(\frac{d}{dt} + p\frac{d}{dx}\right)\left\{-\frac{d\mathrm{G}}{dt} - \frac{d\Psi}{dy} - p\left(\frac{d\mathrm{G}}{dx} - \frac{d\mathrm{F}}{dy}\right)\right\}$$

$$- \frac{1}{4\pi}\frac{d}{dt}\left(-\frac{d\mathrm{G}}{dt} - \frac{d\Psi}{dy}\right);$$

so that

$$(\mathrm{K}\mu)^{-1}\mathrm{c}^2\nabla^2\mathrm{G} = \frac{d^2\mathrm{G}}{dt^2} + 2(1 - \mathrm{K}^{-1})p\frac{d^2\mathrm{G}}{dx\,dt} + (1 - \mathrm{K}^{-1})p^2\frac{d^2\mathrm{G}}{dx^2},$$

and a similar equation holds for H.

Thus F, G, H all satisfy equations of precisely the same type up to and including terms of the second order in p/c; so that if the vector $\mathfrak{A}$ represent the vector potential, we have

$$(\mathrm{K}\mu)^{-1}\mathrm{c}^2\nabla^2\mathfrak{A} = \frac{d^2\mathfrak{A}}{dt^2} + 2(1 - \mathrm{K}^{-1})p\frac{d^2\mathfrak{A}}{dx\,dt} + (1 - \mathrm{K}^{-1})p^2\frac{d^2\mathfrak{A}}{dx^2},$$

which is the same equation (with p substituted for v) that was found (§ 13) for the magnetic induction $\mathfrak{B}$ by the method of averaged fluxes.

22. Hitherto we have omitted the complication which arises in optical applications owing to dispersion. In the case when the material medium is at rest, it is however easy to include the effects of dispersion and opacity. The total current is made up, as before, according to the specification

$$\mathfrak{C} + \mathfrak{D} + \mathfrak{D}' + (u_0, v_0, w_0),$$

where, as in § 11,

$$\mathfrak{C} = k\left(m\frac{d}{dt} + \sigma'\right)(\mathrm{P}, \mathrm{Q}, \mathrm{R}), \quad \mathrm{P} = -\frac{d\mathrm{F}}{dt} - \frac{d\Psi}{dx},$$

$$\mathfrak{D} + \mathfrak{D}' = \mathrm{K}\mathfrak{D} - \mathrm{Ki}\frac{d^2\mathfrak{D}'}{dt^2}$$

$$\nabla^2\Psi = -4\pi\rho, \quad \frac{du_0}{dx} + \frac{dv_0}{dy} + \frac{dw_0}{dz} = -\frac{d\rho}{dt}$$

The condition of circuitation shows, as in § 18, that a volume-density ρ subsides in a conducting medium in a non-vibrational manner, so that Ψ and ρ may be left out of account. Then $(P, Q, R) = -d/dt\,(F, G, H)$, and we arrive at the same equations as previously in § 11.

Ponderomotive Forces.

23. When a conductor carrying a current is in a magnetic field, the positive and negative electrons* are urged equally by the magnetic force in a common direction perpendicular to the current and to the field, when they are themselves drifting with equal velocities in opposite directions. A difference in their velocities of migration, such as has actually been found to exist in electrolytes, would lead to a greater force on one kind than the other, and so produce a small transverse electric force representing the HALL effect. When the flow in the magnetic field has become steady, the distribution of drifting free positive and negative electrons across the section will not be quite uniform, thus involving also change of resistance.

In any case the aggregate of the transverse forcives acting on the free electrons will constitute a mechanical electrodynamic forcive per unit volume,

$$(X, Y, Z) = (v'c - w'b,\ w'a - u'c,\ u'b - v'a),$$

acting on the conductor, where however (u', v', w') is the *true* current, namely $(u - \dot{f},\ v - \dot{g},\ w - \dot{h})$. There will in addition be a mechanical forcive $\rho\,(P, Q, R)$, if the element of volume contain an excess of one kind of electrons, including as a limiting case, a normal traction $\frac{1}{2}\sigma N$ over the surface of each charged conductor; and there will be the forcives acting on the magnetic and electric polarization of the element, derived respectively from potential functions $\frac{1}{2}\kappa(\alpha^2 + \beta^2 + \gamma^2)$ and $(K - 1)/8\pi\,.\,(P^2 + Q^2 + R^2)$, leaving out of account the part which merely produces molecular stress; while, if part of the magnetic or electric polarity is permanent, there will also be couples as in §§ 33, 35.

24. We have thus attained to a complete scheme of equations of the electric field, simply on the assumption that a material medium contains electrons, as many positive as negative in the element of volume unless it is electrified; that these electrons are in part combined into systems which are, or belong to, the molecules, some of these molecules being neutral, and some having an excess of positive or negative electrons, being therefore ions. The only other assumption is that the nuclei of the electrons occupy a negligibly small part of the whole space. As regards the manner in which the electromotive and the ponderomotive forcives in the electric field are accounted for, a similarity with WEBER'S molecular theory may be remarked.† In that theory electric molecules act on one another directly at a distance according to a law of force

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* The argument in these sections is equally applicable, whether the current in metallic conductors is supposed to be carried by material ions, or by electrons considered as immaterial.

† W. WEBER, "Electrodynamische Maasbestimmungen;" MAXWELL, "Treatise," §§ 846–860.

which involves their relative velocity; on the present theory actions are transmitted from one moving electron to another solely by the intervention of the æther. The Weberian theory has been subjected to destructive criticism by von Helmholtz, on the ground that it implies the possibility of a perpetual motion; the mode of genesis of the present theory obviates such a criticism. The features of the Weberian theory above mentioned were characterized by Maxwell as "eminently successful" ("Treatise," § 856), although this commendation is afterwards limited by the assertion that these features are necessarily connected by the principle of energy. It is now, however, recognized that this principle cannot by itself furnish more than one relation between the various quantities that enter into the problem of electrodynamic induction, so that the fact that any theory, not otherwise discredited, accounts for the two types of forcive in the electric field, ought to weigh strongly in its favour. The range of a theory of moving electrons of the present type, with its underlying æther, is of course much wider than that covered by Weber.

When the mechanical forcive is exerted only on the discrete electrons contained in the element of volume, it clearly cannot involve directly in its constitution an equilibrating internal stress, such as we found must be included if we take the current element as a connected physical entity; it is for this reason that Weber's theory, though in other respects different from the present one, agrees with it in giving the Ampère-Maxwell ponderomotive force, involving the *true* current however, not the total current as in Maxwell's formula.

Unipolar Induction.

25. The phenomenon of unipolar induction, in which a current is induced when a magnet revolves round its axis of symmetry through its own field of force, is deprived of all difficulty or ambiguity when it is considered under the present point of view. The electrons in the magnet, as they are moved across the magnetic field of the æther, are each subject to a forcive which is proportional to the component magnetic intensity in the meridian plane, and which produces electric separation by drifting the positive ions towards the axis and in the direction of the length of the magnet one way, and negative ions the opposite way. This constitutes an electromotive force along the revolving magnet.

It follows for instance that a magnet symmetrical around its principal axis will, on rotation round that axis in its own field, acquire an electrification of excessively minute amount when the circuit is incomplete, but still sufficient to compensate the electric force induced by the motion. We can utilize Maxwell's equations of electric force, modified so as to refer to a system of axes moving through the æther ("Treatise," § 600, or as in 'Phil. Mag.,' Jan., 1884, p. 12), to infer at once that for a *solid* magnet of any form, in motion of any type, the induced electric force is derived from a potential $-(\mathrm{F}p + \mathrm{G}q + \mathrm{H}r)$, where (p, q, r) is the velocity through the æther of the element of the magnet at the point considered; so that it can at each instant be compensated by the static force due to a minute induced electrification. The maximum difference of potential to be thus compensated, between the axial and circumferential parts of the rotating magnet, is of the order 10^{-4} volt in the case of the Earth. If the force were not thus derivable from a potential, a magnet in motion would induce currents in itself even when there is no extraneous field, and the energy of the absolute motion through the æther, of the Earth and all other magnets, would gradually be converted into heat owing to this cause; as things are, only the energy of relative motion of magnets is subject to dissipation in this way.

[Aug. 3, 1895.—26. The case of a solid conductor spinning steadily round an axis of symmetry, in a magnetic field which is also symmetrical round that axis, has an important bearing on theory on account of the simplicity of the conditions. If it were legitimate to specify the electrodynamic energy of the system in terms of current elements and their mutual configurations, then in this case the energy belonging to a current element associated with an element of volume of the conductor would remain invariable, on account of the steady configuration of the motion; therefore there would be no electric force induced in such an element. Thus, for a conductor spinning in this manner there could then be no differences of electric potential caused by the motion. And, moreover, if a so-called unipolar circuit is completed by means of a fixed conducting wire, attached to the spinning conductor by sliding contacts at its equator and one of its poles, there could be no electric force induced in this wire either; and the electromotive force of the current which actually flows round this circuit would have to be sought for wholly in the sliding contacts, which, considering the definiteness of this electromotive force and the great variety of types of contact that are possible, seems to be an untenable alternative.

On the other hand, the formulæ of the present paper, which considers an electrification to be made up of discrete elements each surrounded by free æther, make the induced electromotive force along any open line of material particles consist of two parts, (i) a part due to motion of this line of particles with respect to the quiescent æther, and equal to the time-rate at which it cuts across the tubes of magnetic induction, themselves supposed to be stationary in this computation, and (ii) a part equal to the line integral of $-d/dt$ (F, G, H), to be computed by the integral formula for (F, G, H) given above, which represents the effect of change in the inducing system. When the electromotive force is taken round a complete circuit, all theories are of course in agreement. In a case of steady motion, such as is now under consideration, the second of these parts is null, and the electromotive force induced even in an open circuit is given by FARADAY'S original rule, in terms of the number of tubes of magnetic induction which cut across the current.

In the present order of ideas, a distinction has to be observed between (i) the electric stress in the æther, which is the tangential shear derived from its potential energy and represents the whole forcive acting on free æther, and (ii) the electric force which acts on electrons and moves them through the æther, thus polarizing dielectric media and producing true electric currents in conductors. In the hands of FARADAY and MAXWELL, the current of conduction was completed or rendered circuital by an effect across dielectrics, which was equivalent to a current, and there was no difference contemplated in this effect depending on whether the dielectric was a material substance or free æther, both being considered to be merely polarizable ; on the present more complete view this dielectric action has to be divided into the true polarization current in the material dielectric, which is excited by the electric force orientating its polar molecules, and the rotational strain in the æther which is regulated by the laws of elasticity of that medium. Thus, the question whether there is electric force induced in the free æther itself is, on our present view, nugatory, there being no electrons on which it could operate.

The calculations given in 'Phil. Mag.,' Jan. 1884, of the differences of potential, and the consequent electrification, induced in a sphere or other conductor of revolution, rotating in a symmetrical magnetic field, will thus, on the present view, be absolutely correct when the conductor is moving in a vacuum. When it is rotating in a gaseous dielectric, like air, there will be aerial flow produced owing to viscosity, after the manner of the action of a fan, and there will therefore be electric force induced in the moving portions of the air ; but the capacity of air for dielectric polarization is so small that the polarity thereby induced will not sensibly affect the state of electrification of the system. If, on the other hand, the conductor were rotating in a liquid dielectric, the polarity induced in the moving parts of the liquid would depend in part on its motion, which would thus very materially influence the distribution of electrification in a manner which it would not be difficult to calculate if it were necessary to do so. The unipolar current obtained on completing the conducting circuit will, of course, in any case, unless the conductivity is almost evanescent, be practically independent of the nature of the surrounding dielectric.

The analytical theory of MAXWELL's "Treatise" being based on current elements, that theory should, when correctly developed, give in the rotating conductor the null electrification of the beginning of this section, instead of, as here, an electrification based on FARADAY's rule. And this is easily verified if to MAXWELL's original equations of electric force are added, in accordance with VON HELMHOLTZ's correction, terms derived from a potential function which is the scalar product of the vector potential and the velocity of the material medium ; in computing the radial force, which is in cases of symmetrical rotation the total force, these terms exactly cancel the FARADAY part.

The discrepancy between these two theories, is put to a test in a classical experiment already made by VON HELMHOLTZ.* He found that when a conductor was spun in a magnetic field symmetrical around the axis of rotation, there was a difference of potential induced between the axial and circumferential parts as evidenced by resulting electrification, which agreed with FARADAY'S rule within three per cent., a quantity well inside the limits of uncertainty of measurement. He drew the conclusion that an electrodynamic potential theory of the NEUMANN type is thus proved inadmissible unless it recognizes polarization currents in the dielectric. The considerations stated at the beginning of this section seem to show that it could not even thereby be helped out; and that the true inference from the experiment must be a wider one, namely the abandonment of a theory of currents ultimately continuous in favour of one which regards them as made up of discrete electrons separated from each other by free æther.

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* 'Berlin Monatsber.,' 1875; 'Ges. Abhandl.,' I., p. 783. MAXWELL had very early considered the question whether in his total kinetic energy as specified in terms of the current elements and the velocities of the conductors, there are any terms which involve products of these quantities, and he had drawn a negative conclusion from experiment; 'Treatise,' II., chap. vi., secs. 568–577. The

27. The electrification induced in a conductor by rotation in a symmetrical magnetic field has just been examined; we pass on naturally to the conjugate problem of the magnetic field induced by a rotating electrified conductor, where similar considerations must crop up. The difficulties involved in the interpretation of the results of ROWLAND'S experiment, on the hypothesis that an electric charge in a conductor is a continuous distribution of electricity, and an electric current a continuous flux, have been already considered in the previous paper,* and proved surmountable only on the extremely precarious assumption that the rotating gilded glass discs of the experiments were divided into mutually insulated segments by the scratches which were intended to prevent FOUCAULT currents. The translation of an isolated electric charge carries on its own surrounding electric field and so alters the electric intensity at each point in the æther; and can therefore certainly induce a magnetic field with consequent reaction on the moving charge. But the steady rotation round its axis of a charged conductor of revolution in no respect affects the field of electric strain in the surrounding æther; that remains steady, and therefore no magnetic, that is kinetic, energy can be locally generated anywhere in it. And there is also the related difficulty previously enforced, that if a charged conductor connotes merely a field of self-locked electric strain in the surrounding dielectric, the elasticity breaking down when the surface of the conductor is reached, the rotation of the conductor round its axis of symmetry could exert no grip on this field of strain, which would therefore not be affected at all. These difficulties will not vanish unless the electric charge on the conductor is made up of discrete portions, separated by dielectric spaces however narrow. The circumstances will then be no longer symmetrical with respect to the axis of rotation so far as these spaces are concerned; the magnetic field induced, at a place at finite distance from the conductor, will depend not only on the change of electric intensity at that place, which is null as before, but also on the surface conditions which obtain along the boundaries of the dielectric region.

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electrification in VON HELMHOLTZ'S crucial experiment might, however, be formally expressed on that system as due to energy terms of this mixed type, but, of course, extremely minute.

[*] 'Phil. Trans.,' A, 1894, p. 764.

The phenomena are thus accounted for on the hypothesis that the electric charge on a conductor consists of a distribution of discrete electrons over its surface. It is true that the facts of electric vibrations on conductors show that these electrons must be extremely mobile and sensitive to electric force: this is because of the very strong charges involved in them, but it does not imply that when the conductor is rotated, the electrons will slip backward over its surface and remain where they were, owing to the electric inertia. They will not take up the motion of the conductor just at once; but there is no electric force which would tend to prevent that result, and the same steady viscous agencies that produce electric resistance when a steady current of electrons is flowing in a fixed conductor, will ultimately make them move with the rotating conductor, after a time relatively long but probably absolutely very short.

It does not, in fact, appear that there are any of the hitherto outstanding difficulties of pure electrodynamic theory that are not removed by the hypothesis of moving electrons, to which, from the consideration of several distinct classes of phenomena, and apart altogether from electrochemical theory, we have been compelled to resort. This hypothesis, in its wider aspect involving the nature of matter itself, seems also to have a philosophical necessity; for the location of causes of disturbance of the uniform all-pervading medium in permanent discrete singularities or nuclei of strain or motion, that belong to it and can move about through it, is the only way of avoiding the introduction into theory of either direct distance actions, or else those assumptions of independent media superposed in the same space and discharging different functions, which violate the maxims of modern physics. Without a precise conception of the causes which produce disturbances in it and form one side of the play of action and reaction, a theory of the æther can be merely descriptive; while any assumed causes that are not of the nature of singularities arising from the constitution of the medium itself, must introduce a foreign element and so deprive the theory of its interconnection and self-contained character.]

Mechanical Pressure of Radiation.

28. An application of the expression for the ponderomotive forcive will be to the examination of MAXWELL's mechanical pressure of radiation (" Treatise," §§ 792, 793). Let us consider a train of plane waves moving along the axis of x, with their magnetic induction c along z, and their current v along y. The circuital relations give

$$4\pi\mu v = -\frac{dc}{dx}, \qquad \frac{dQ}{dx} = -\frac{dc}{dt};$$

while the equation connecting electric force with current is

$$v = \sigma Q + \frac{K}{4\pi C^2}\frac{dQ}{dt}.$$

Hence

$$\frac{d^2c}{dx^2} = K\mu C^{-2}\frac{d^2c}{dt^2} + 4\pi\sigma\mu\frac{dc}{dt};$$

leading to

$$c = c_0 e^{-px}\cos(nt - qx),$$

where

$$(p + \iota q)^2 = K\mu C^{-2}n^2 - 4\pi\sigma\mu n\iota.$$

Of the total current v, the part $v' = v - \frac{1}{4\pi C^2}\frac{dQ}{dt}$ is the true current, derived from motion of electrons and not including the æthereal rotation, C being the velocity of radiation in free æther. The mechanical forcive per unit volume of the material medium is thus $X = v'c$; hence

$$\int X\,dx = -\frac{c^2}{8\pi\mu} - \int\frac{c}{4\pi C^2}\frac{dQ}{dt}dx.$$

Let us first take the case of a transparent medium, for which

$$\frac{dQ}{dt} = \frac{4\pi C^2}{K}v = -\frac{C^2}{\mu K}\frac{dc}{dx};$$

so that

$$\int X dx = -\frac{c^2}{8\pi\mu}\left(1 - \frac{1}{K}\right) = -\frac{c^2}{8\pi\mu}\left(1 - m^{-2}\right),$$

where m is the index of refraction.

The value of this indefinite integral, taken over an exact number of half wave lengths in a homogeneous medium, gives a null result for the total forcive; but if an interface between two media is included in the volume of integration, there will also be terms in it which may be represented by averaged tractions $\frac{1}{2}\frac{c_0^2}{8\pi\mu}(1 - m^{-2})$ exerted by this interface on its two sides, or what is the same thing, equal pressures acting on the interface. In air this pressure practically vanishes. This is the result which replaces MAXWELL's formula $c_0^2/8\pi\mu$ for the mechanical pressure produced by radiation falling on the surface separating two media.

29 When the train of waves falls on an absorbing medium, the circumstances are different. From the value of c above given, it follows that

$$\frac{dQ}{dt} = n^2 c_0 \frac{e^{-px}}{(p^2 + q^2)^{\frac{1}{2}}}\cos(nt - qx - \epsilon'), \text{ when } \tan\epsilon' = \frac{q}{p};$$

so that

$$c\frac{dQ}{dt} = \tfrac{1}{2}n^2{c_0}^2\frac{e^{-2px}}{(p^2+q^2)^{\frac{1}{2}}}\{\cos(2nt - 2qx - \epsilon') + \cos\epsilon'\},$$

and

$$\int\frac{c}{4\pi\mathrm{C}^2}\frac{dQ}{dt}dx = \frac{n^2{c_0}^2}{8\pi\mathrm{C}^2}e^{-2px}\left\{\frac{\cos(2nt-2qx-\epsilon')}{2(p^2+q^2)} - \frac{\cos\epsilon'}{2p(p^2+q^2)^{\frac{1}{2}}}\right\}.$$

Hence

$$\int \mathrm{X}\,dx = \frac{{c_0}^2}{8\pi\mu}\cos^2 nt + \frac{n^2{c_0}^2}{8\pi\mathrm{C}^2}e^{-2px}\{\ldots\ldots\},$$

in which the lower limit is at the interface, and the upper one at a place where the radiation has been practically extinguished. Thus here there is an average pressure $\frac{1}{2}\frac{{c_0}^2}{8\pi\mu}$ acting on the interface towards the opaque material medium, which is half of MAXWELL's result.

Interfacial Conditions.

30. It is important to emphasize the dynamical distinction between electric force and electric displacement in a dielectric. The electric force is that vector which occurs in the rotational stress in the æther, and which, divided by the proper elastic coefficient $\mathrm{C}^2/8\pi$, gives the rotational strain or electric displacement in it. This is not circuital when there are moving electrons present, but by adding a fictitious electric displacement representing the drift of the electrons, a circuital total displacement is obtained. This fictitious displacement or true current represents, however, the rotation in a hydrodynamical flow of the æther which the motion of the electrons sets up, which belongs to the kinetic energy and in ordinary electrodynamical applications constitutes practically the whole of it. At an interface separating media of different dielectric qualities, the electric stress in the æther must be continuous, and as this is tangential, equal to the tangential component of the electric force but at right angles to its direction, it follows that the tangential components of the electric force must be continuous.

Again, in a magnetic medium, the magnetic induction represents the smoothed-out velocity of flow due partly to motions communicated by distant disturbances, and partly to the circulatory motions of the magnetic vortices that are caused by the very rapid orbital rotations of the electrons in the molecule. The magnetic induction is therefore always circuital, so that its normal component is continuous at an interface. But as regards the tangential component, we must divide the whole induction into two parts, one of them representing the effect of the vortices in the immediate neighbourhood of the point under consideration, and the other including all the rest of the flow. The latter part is the magnetic force, in the ordinary phraseology, and it is clearly only this part whose tangential component must be continuous, when we cross an interface into a region in which the distribution of the magnetic molecular vortices is different.

It is a check on the averaged form of the dynamical equations for a material medium that these six interfacial conditions regarding force and flux should be consistent with the incompressibility of the æther ; and in fact the relations given above show that only four of the six are independent. It is on this account, for example, that the general requirements of the problem of physical optics are satisfied.

Molecular Current Systems replaced by Equivalent Continuous Currents.

31. To secure a perfectly homogeneous specification of the electrodynamic field in a dynamical theory, there is no alternative but to reduce magnetism to molecular electric currents. As, however, our analytical equations for an extended medium involve (u, v, w), the current according to a volume specification, while the molecular currents are minute whirls not involving continuous flow in any direction, it is necessary to determine the volume specification of currents that shall be their equivalent when the element of volume is so great as to contain a large number of whirls of which the average effect only is required. Such a specification is clearly possible; and when we bear in mind that the volumes occupied by the cores of the whirling electrons form only an excessively small part of the total space, it is also clear that the type of magnetic force which represents the velocity of the incompressible æther must be a circuital one, that is, the magnetic induction of MAXWELL. There is no difficulty in verifying by direct analysis that the magnetic force at a point due to a system of molecular currents is the same as the magnetic induction due to the equivalent magnet ; in fact it is shown (previous paper, § 120) that the said force is equal to the curl of a vector potential

$$\int (\mathrm{B}d/dz - \mathrm{C}d/dy,\ \mathrm{C}d/dx - \mathrm{A}d/dz,\ \mathrm{A}d/dy - \mathrm{B}d/dx)\, r^{-1}\, d\tau,$$

the same vector potential from which the magnetic induction of the equivalent magnet is derived.

Now let us consider the projections of the molecular circuits parallel to the plane of xy : let these projections swell out in area until they come into contact filling up the whole plane, the currents round them being reduced in the same ratio as their areas are increased. Along each edge common to two circuits there will be a differential current flowing, and by making the enlarged circuits rectangular of the form $\delta x\, \delta y$, it becomes clear that the aggregate of these differential currents make up a volume distribution of currents $(-d\mathrm{A}/dy,\ d\mathrm{B}/dx,\ 0)$ together with a current flowing round the boundary. Hence, adding up for the projections on all three coordinate planes, we obtain the distribution represented by $-$ curl (A, B, C) together with a current sheet on the external bounding surface equal to $(\mathrm{B}n - \mathrm{C}m,\ \mathrm{C}l - \mathrm{A}n,\ \mathrm{A}m - \mathrm{B}l)$ per unit area. The validity of this substitution is verified by the analytical transformation

$$F = \int \frac{u}{r}\, d\tau + \int \left(B \frac{d}{dz} - C \frac{d}{dy} \right) \frac{1}{r}\, d\tau = \int \frac{u}{r}\, d\tau + \int (Bn - Cm) \frac{1}{r}\, d\tau - \int \left(\frac{dB}{dz} - \frac{dC}{dy} \right) \frac{1}{r}\, d\tau.$$

Thus if the magnetism is wholly induced, so that $(A, B, C) = \kappa (\alpha, \beta, \gamma)$, the equivalent volume distribution of currents is $4\pi\kappa (u, v, w)$, provided κ is constant throughout the field; hence the induced magnetism may be ignored if in estimating the induction we multiply the current system by $1 + 4\pi\kappa$ or μ.

More generally, if a part (A_0, B_0, C_0) of the magnetism is permanent, we have in a region of constant μ

$$\nabla^2 F = -4\pi\mu u + 4\pi \left(\frac{dC_0}{dy} - \frac{dB_0}{dz} \right);$$

of which the last term vanishes when there is no permanent magnetism, or when it is of lamellar type. At the interface between two different media (F, G, H) must be continuous. These relations are sufficient to determine (F, G, H) completely in terms of (u, v, w).

Again, a steady electric polarization (f', g', h') in a dielectric, moving across the æther with velocity (p, q, r), is equivalent in its kinetic effects (previous paper, § 125) to a molecular current system or magnetization $(rg' - qh',\ ph' - rf',\ qf' - pg')$ together with a current sheet over the bounding surface, *only provided* the velocity is uniform. In any case, however, it is equivalent to the current system

$$\left(p \frac{d}{dx} + q \frac{d}{dy} + r \frac{d}{dz} \right) (f', g', h').$$

Mechanical Forcives acting on Magnetically and Electrically Polarized Media.

32. We have still to calculate the steady forcive on a molecular current. The force on a revolving electron e has been shown to be $e(P, Q, R)$ where

$$P = c\dot{y} - b\dot{z} - \frac{dF}{dt} - \frac{d\Psi}{dx},$$

$(\dot{x}, \dot{y}, \dot{z})$ being its velocity, and Ψ the potential of the static part of the electric field due to such free charges as may exist in the material media. The first two terms, involving the velocity of the electron, will give rise to magnetic forcive, the remaining part will make up into the electric forcive, in a polarized medium.

The averaged value of the first two terms, over the orbit of a single electron, gives $\iota \int (c\, dy - b\, dz)$, where ι the equivalent current is equal to e multiplied by its velocity v and divided by the length of the orbit; this is the same as $\iota \int (l\, da/dx + m\, db/dx + n\, dc/dx)\, dS$ over a surface bounded by the orbit, by Stokes' theorem in conjunction with the circuital character of (a, b, c). Summing up for all the orbits of electrons, this would give for the translational magnetic forcive per unit volume (X′, Y′, Z′), where

$$X' = A\frac{da}{dx} + B\frac{db}{dx} + C\frac{dc}{dx},$$

and (A, B, C) is the magnetization of the medium. Now the actual magnetic forcive must certainly, from the most cursory observation of its intensity for iron, involve the magnetic force (α, β, γ) and not the induction (a, b, c); whence then this discrepancy? The forcive on an electron involves correctly (a, b, c), for the velocity of the incompressible æther is circuital. But it must be borne in mind that the molecule contains as many negative revolving electrons as positive, and therefore that the forcive on the molecule is a differential one, positive and negative electrons pulling on the average opposite ways; so that in treating of single molecules we cannot take the actual velocity of the æther to be an averaged function of position such as (a, b, c).

33. To find the forcive on a molecule, we must in the first place divide this velocity into two parts, one independent of the immediate surroundings of the point considered and depending only on the general character of the field, the other representing the effect of local configuration. The former is the magnetic force as usually represented by (α, β, γ); it is made up of the force due to the distribution of ideal magnetic matter introduced by POISSON into the theory, together with that due to the distribution of electric flow. The former element in it is independent of local peculiarities by the ordinary theory of the gravitation potential; the latter also has never any term involving the flow at the point considered, for similar reasons. The purely local portion of the velocity of the æther consists itself of two parts, a steady one, and one of varying type with very rapid alternation characteristic of the orbits of the electrons; the part last mentioned averages to nothing, while the former part adds on to the magnetic force to produce the total averaged velocity of flow of the æther, that is the magnetic induction (a, b, c).

Now even if instead of the actual velocity of the æther, rapidly varying from point to point, we substituted this average velocity (a, b, c), we should still obtain both a forcive which depends on the general character of the field, and one (though only the regular part of it) which depends on the interaction of the molecule with its immediate surroundings. The latter forcive, which cannot be completely expressed in terms of the quantities of the present theory, belongs to an intermolecular stress of cohesive type; it is self-equilibrating and contributes nothing to the forcive on the material medium as a whole, though, as we shall see later, it produces deformation of its parts. If we agree to consider it as a separate molecular forcive, we have for the magnetic forcive proper (X′, Y′, Z′) the formula

$$X' = A\frac{d\alpha}{dx} + B\frac{d\beta}{dx} + C\frac{d\gamma}{dx}.$$

There will also be a magnetic couple (L', M', N') per unit volume, where $L' = B\gamma - C\beta$; but there will clearly be no additional molecular couple acting in the element of volume.

34. The steady magnetic force and couple thus obtained may be provisionally considered as derived from an energy function (kinetic) $A\alpha + B\beta + C\gamma$ per unit volume, in which (A, B, C) is unvaried (*cf.* MAXWELL, "Treatise," § 639); the regular part of the molecular forcive may be considered as involving an internal energy function $2\pi(A^2 + B^2 + C^2)$ in which (A, B, C) is varied.

More fundamentally, if we divide (α, β, γ) into a part $(\alpha', \beta', \gamma')$ due to the magnet itself and a part $(\alpha_0, \beta_0, \gamma_0)$ due to the inducing field, the actual energy associated with magnetic force is

$$\tfrac{1}{2}(A\alpha' + B\beta' + C\gamma') + A\alpha_0 + B\beta_0 + C\gamma_0,$$

in which (A, B, C) is now to be varied; and this, when there is no permanent magnetism, leads to the same mechanical forcive as an energy function $\frac{1}{2}\kappa(\alpha^2 + \beta^2 + \gamma^2)$, where κ is the coefficient of magnetization, the part of it not thus compensated being connected with the internal work of orientation of the molecules. This energy function is not the whole kinetic energy of the æther; that would be in the present units the space-integral of $(a^2 + b^2 + c^2)/8\pi$ together with a molecular part, that is the space-integral of $(a\alpha + b\beta + c\gamma)/8\pi + \frac{1}{2}\kappa(\alpha^2 + \beta^2 + \gamma^2)$, of which the latter term is exactly compensated by the above mechanical forces acting on the magnetized body, while the other term which remains over goes to produce electrodynamic effects, and in part to represent intrinsic energy of magnetization, and is in fact the electrodynamic energy formulated in MAXWELL'S scheme.

It is to be remarked that the magnetic vector potential, obtained in the previous paper as the vector potential of the Amperean currents in the form

$$F = \int (B\,d/dz - C\,d/dy)\,r^{-1}d\tau,$$

implies that the orbital velocities of the electrons do not approximate very closely to the velocity of radiation, a condition which in reality is sufficiently satisfied: if that were not so, this expression would require correction for an æthereal displacement part of the molecular current.

35. The magnetic forcive (X', Y', Z') acting on the actual magnetization I is different from the forcive that would act on the three components of the magnetization A, B, C, in case there are currents flowing in the magnet; so that in this connection it is not permissible to treat the magnetization as a vector and resolve it into components. The reason here is that we have actually to do with molecular currents; and if we replace a molecular circuit by its three components we thereby alter the character of its linking with the lines of flow of the current flowing across the medium. But there is a consideration of a less special kind which shows that

this cannot be done: if we were to resolve the magnetization, we should obtain results for the forcive which would be physically different according to the system of coordinate axes that is adopted. If, however, we sum up forcives on separate magnetic poles, instead of calculating forcives on resolved magnetic moments, we arrive again at the correct result, independent of the coordinate system: as will be illustrated immediately in connection with the forcive of electric origin.

36. The ponderomotive forcive of electric origin, acting on a dielectrically polarized medium, is made up of a bodily force (X, Y, Z) and a bodily couple (L, M, N), where

$$X = f'\frac{dP}{dx} + g'\frac{dQ}{dx} + h'\frac{dR}{dx}, \quad L = g'R - h'Q;$$

in these formulæ (f', g', h') is the total polarization of the material medium, consisting of the ordinary induced polarization $(K - 1)/4\pi \,.\, (P, Q, R)$ and, under circumstances of residual charge, also a permanent part. When the magnetic field is varying, so that (P, Q, R) is not derived from a potential, there will be delicate considerations concerning this force, similar to those discussed above in connection with magnetization. The formula here given for X is however correct, because it is what is directly obtained by grouping the actual electrons of the neutral molecule in pairs to form polar doublets; this is a legitimate procedure although the resolution of the averaged electric moment of an element of volume into three components proves not to be such. Thus the forcive on a doublet δD lying along the axis of x is a force $(\delta D \,.\, dP/dx,\ 0,\ 0)$ and a couple $(0,\ -\delta D \,.\, R,\ \delta D \,.\, Q)$; and when the axes of coordinates are changed so that δD becomes $\delta(f', g', h')$ these expressions change into the ones given above, because $P\,\delta f' + Q\,\delta g' + R\,\delta h'$ is an invariant function as regards change of axes.*

37. The stress of molecular type, produced by magnetization, offers some points of interest. It has been remarked by VON HELMHOLTZ† long ago that in a polarized medium there exists a material tension along the lines of polarization and a pressure at right angles to them. Each of these is, however, proportional to the square of the susceptibility‡ of the medium, and they are not necessarily equal; for media

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* [The circumstances of electrified and magnetized media are not parallel, in the sense that K corresponds to μ. In a magnetized medium the circuital vector, namely the induction (a, b, c), is the smoothed out velocity of the æther; and the magnetic force (α, β, γ) is that part of it which is independent of the immediately adjacent vortices or Amperean currents. In a polarized dielectric (f, g, h), $= (P, Q, R)/4\pi$, is the smoothed out *total* elastic rotation or electric displacement in the æther; (f', g', h') is the time-integral of the polarization current of the material medium which is constituted of movements of orientation of the electrons; while it is the sum of these vectors that is now circuital, representing the total electric current arising from æthereal strain and movement of electrons combined.—August 25.]

† 'Berlin Monatsber.,' Feb., 1881; 'Wissen. Abhandl.,' 1, p. 779.

‡ 'Proc. Roy. Soc.,' April, 1892, p. 63.

of very high susceptibility they are thus far more intense than MAXWELL's hypothetical stress which depends on its first power. Their origin is very conveniently exhibited in a chain of iron nails hanging end to end from a pole of a magnet: the nails hold together longitudinally but repel each other transversely. Now consider a longitudinally magnetized bar: this straining together of opposite polarities in neighbouring molecular groups will produce an internal stress in the bar proportional to I^2, which will usually tend to shorten it, but may conceivably in some cases do the reverse. Furthermore the orientation of the molecular groups due to magnetization will directly alter the length to an extent depending on the first power of I. Hence the whole increase of length will be $AI + BI^2$, in which A and B may each be either positive or negative. When A and B have opposite signs there will be a value of I at which the total effect will reverse its sign by passing through a null value. It appears from experiment that for iron A is positive and B negative; for nickel, A is negative and B negative; for cobalt, A is negative and B positive.*

[Aug. 25.—When A and B have opposite signs there will be an intensity of magnetization, $I = -A/2B$, which corresponds to maximum or minimum elongation. Near this intensity a small change in the magnetization will not affect the length of the bar; and therefore conversely a change of length produced by tension will have no influence on the magnetization, any changes in the two being independent of each other. It follows that the magnetization at which the influence of tension vanishes is only half the VILLARI critical magnetization at which the elongation of the bar is null. This is, of course, on the assumption that the phenomenon is a regular and reversible one, and that there is no sensible term in the elongation involving I^3.]

There must be similar strain effects connected with the polarization of a dielectric in an electric field, though of course they would be far more difficult to detect.

It seems probable that the greater part of the phenomena of electrostriction and magnetostriction is of this character, and that only a small part is due to strain of the material by the direct effect of electric and magnetic attractions between finite portions of the medium.

General Considerations.

38. It has been one aim of the present analysis to examine how far it is possible to identify, in the play of kinetic and potential energy between the electrons, the main phenomena of matter. In order to comply with the requirements of negative optical experiments by LODGE and others, it is necessary to assume the inertia of the æther to be at least comparable numerically with the inertia of derivative character which belongs to dense matter; and the velocities of its movements are thus extremely slow. On the other hand, in the descriptive electric theories that are now commonly held, it is usual to consider the æther to be like ordinary matter as regards

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* *Cf.* EWING, "Magnetic Induction . . .," chapter 9.

its inertia, and also as regards capacity for electric and magnetic polarizations. Its density, for astronomical and other reasons, must then be excessively small; and therefore small forces, if unbalanced, will set it into very rapid motion. It has been recently shown by VON HELMHOLTZ* that it is not possible for an electrically polarizable medium of small inertia so to adjust itself by finite motions, that the electric and magnetic forcives on it shall be in internal equilibrium and thereby avoid producing very intense movements in it,—unless there exists finite slip at the surfaces of the moving bodies which set up the surrounding electric field; and it seems difficult to see how this slip could be allowed, or what circumstances would regulate its magnitude.

On the present view, the inertia of matter is different in kind from that of the æther, and is possibly to be found in the electric inertia which is possessed by electrons. The well-known considerations advanced by Lord KELVIN, which find in the magnetic rotation of light evidence of a rotatory motion round the lines of magnetic force, still obtain a place here, but in a modified form; it is not the æther itself which is in rotation round the lines of force, but the electrons of the ponderable medium; rotation of the free æther would not here affect the elastic propagation except convectively. We should thus expect the magnetization of the *material* medium to rotate the plane of polarization without altering to any corresponding extent the mean velocity of radiation, just as in Lord KELVIN's theory: and the present theory is not open to objection on the score that the FARADAY rotation is easy to observe, while convection of the radiation by a magnetic field has hitherto been sought for in vain.

It is to be observed that a primary condition for the permanence of material phenomena according to a scheme such as has here been sketched, is that the æther should be absolutely devoid of friction. If there were the slightest amount of dissipation, the motions by whose stability the system hangs together would gradually diminish, and finally the positive and negative electrons would fall into each other and thus suffer complete extinction; the whole material universe would in fact gradually vanish, and leave no trace behind.

All formulæ with regard to the conservation, under certain circumstances, of linear or angular momentum, or of momenta of more general type as in the case of cyclic fluid motions, are bound up essentially with this absence of friction in the æthereal medium. If friction were present, the relative motions of parts of the system would always be transferring momentum as well as energy out of the system into the æther, and nothing could remain absolutely steady or permanent.

39. In a recent memoir by H. A. LORENTZ,† on electrical and optical phenomena in moving bodies, the author arrives, starting from the ordinary equations of the electric

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* "Folgerungen aus MAXWELL's Theorie über die Bewegungen des reinen Aethers," 'Sitz. Berl. Acad.,' July, 1893, 'Wiss. Abhandl.,' 3, p. 526.

† "Versuch einer Theorie . . . in bewegten Körpern," Leiden, 1895.

field, at fundamental equations of the same type as are given by the present theory. The method adopted by him is, as here, to find the cause of electrical phenomena in the motion of ions. He considers them merely as volume distributions of electricity confined to limited spaces, and with this guiding idea determines the extension of MAXWELL'S fundamental equations that will most conveniently cover the extension of the problem to include their movements. By transforming, as in § 14, to axes of co-ordinates which partake of the uniform motion of the material system, various results relating to the independence between electrodynamic phenomena and such motion are deduced, and FRESNEL'S formula for the effect on the velocity of radiation is shown to be involved. The conception of a molecule, electrically polar by reason of its positive and negative electrons, of its magnetism due to the orbital motions of these electrons, and of its diamagnetism due to changes in the orbits, and the conception of conduction by a *quasi*-electrolytic separation, do not however occur. And in consequence of the absence of recognition of the fact that in an unelectrified element of volume there are present equal numbers of positive and negative ions, the relation between electromotive force and ponderomotive force is not attained. Professor LORENTZ'S conclusions from his system of equations as to the untenability of MAXWELL'S scheme of stress are in agreement with the results of the present theory; but from the indications given, it is not clear that he would admit the form here deduced for the mechanical forcive exerted by radiation. As this memoir of LORENTZ only came to hand after the greater part of the present paper had been developed, the agreement, so far as it goes, may be taken as additional evidence in favour of the validity of the method of investigation that has been employed.

According to the present views, there is nothing paradoxical in the existence of unsaturated atoms or ions in which the two kinds of electrons do not compensate each other; and the properties of these ions will be very different from those of neutral molecules, as it is well known that they are. The fact that a neutral molecule, say HCl, dissociates so that the same element H is always positive, may possibly be ascribed to the catalytic action of the solvent without whose aid the splitting up would not occur. When the solvent is water, or it may be for a gaseous body merely traces of moisture, the decomposition is produced by the formation of an aggregate with the water-molecules which splits easily in only one way; but when the solvent is changed, the characters of the charges of the ions might be inverted, as in fact sometimes occurs.

40. If we consider a molecule to be made up of, or to involve, a steady configuration of revolving electrons, it will follow that every disturbance of this steady motion will involve radiation and consequent loss of energy. The only steady configuration of which we can assert that it will remain permanent under all circumstances is, therefore, that one which possesses least total energy. Thus, by extending the conception of absolute stability, which already plays a part in the theories of material systems which lose sensible energy by viscosity, to systems which lose energy by radiation, we

obtain a possible mode of accounting for the uniqueness of the atomic configuration and the invariability of its spectral lines. Such a view would also account for diffuse lines or bands being sharp at one edge and hazy at the other.*

It is easy to calculate the configuration of minimum energy for the simple but exceptional system of two electrons of opposite sign, considered in § 118 of the previous paper; in that case, however, the distance between their centres comes out to be rather less than the diameter of one of them, so that they will gradually fall into each other owing to loss of their orbital energy. But this need not be the case if the system contained several electrons of the same sign; for example, if it were roughly of the type of a ring of positive electrons revolving round an inner ring of negative ones. In the special case of that investigation, also, an estimate of the diamagnetic coefficient may be obtained. It is easy to see that the result of imposing a magnetic field H at right angles to the circular orbit of an electron e is the same as if the effective inertia were increased from Le^2 to $Le^2 + eH/\omega$. This effect, though very minute, will be cumulative for all the electrons of the atom, whereas the change of orientation, which represents paramagnetism, will be a differential one as between the positive and negative orbits.

By a representation like the present, the origin of the vast number of spectral lines of some molecules may be simply illustrated. Thus in the analogous astronomical system of the Sun, Earth, and Moon, which has only nine coordinates, three for each body, there exist the much larger number of periodic inequalities or oscillations that are discussed in the Lunar Theory. The fact is that the oscillations of the coordinates of the system are not themselves harmonic or even exactly periodic; it is only when they are analyzed by the mathematical processes suitable to vibrations, or by a physical instrument such as a spectroscope which yields the same results, that the lines of the spectrum come into existence. The radiation traverses the free æther as a homogeneous whole which is a function only of the coordinates of the vibrating molecule: it is only under the influence of dispersion or diffraction by material bodies that it is broken up into harmonic constituents, which are not independent elements of it in case they exceed in number the internal coordinates of the molecule.

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* In this connection a suggestive result has recently been obtained by Mr. H. C. POCKLINGTON, 'Roy. Soc. Proc.,' May, 1895. As the energy associated with a simple vortex ring in perfect fluid increases, its velocity diminishes, so that a theory of gases in which the molecules are simple vortex rings fails to represent the facts. In the previous paper (§ 87), a theory of simple vortex atoms in a rotational æther, with the appropriate molecular electric charges, was noticed. It is now found that the effect of the electrification may be to reverse the above peculiarity, and, in particular, when the ring has nearly the configuration of minimum energy for its given vorticity and given charge, the total energy varies very nearly as the square of its velocity, that is as the temperature. But this configuration is precisely the one into which such a ring would settle down under the influence of radiation. Similar statements apply in a general way to the effective vortex made up of a system of revolving electrons, which is considered in the text, and which is not subject to the instability and the other difficulties that would beset an ordinary electrified vortex with hollow core.

As in connection with the previous paper on the present subject, I am under great obligation to Professor G. F. FitzGerald, and also to Professor O. J. Lodge, for the readiness with which they have allowed me to consult and profit by their opinions on some of the questions here treated, and for their kindness in communicating the results of the experimental tests described in § 3 of this paper.

XXI. *India's Contribution to Geodesy.*

By General J. T. Walker, *R.E., C.B., F.R.S., LL.D., late Surveyor-General of India.*

Received April 27,—Read May 16, 1895.

[Plate 18.]

The science of Geodesy has long been indebted to India for valuable data contributing to a knowledge of the Figure of the Earth. Early in the present century the Great Trigonometrical Survey was commenced, as a basis for the general topography of the Peninsula, by Major Lambton, who found that in order to obtain exact determinations of the latitudes and longitudes of the trigonometrical points from his geodetic calculations, it was necessary to obtain more exact values of the numerical elements of the Earth's figure. He therefore commenced measuring with great care and exactitude a meridional chain of triangles, and determined the latitudes of certain stations in this chain, thereby producing a meridional arc which he carried up from Cape Comorin, in latitude 8°, to the parallel of 18°. After his death this arc was extended 11° further to the north by Colonel Everest, who brought it up to the nearest point to the Himalayan ranges at which it seemed probable that the astronomical latitudes would not be sensibly affected by the attraction of the mountain masses. This work was completed by the year 1842, furnishing a meridional arc 21° in length, which is generally known as the Great Arc of India, and has invariably been employed in all subsequent investigations of the figure of the Earth.

From that time onwards chains of principal triangulation have been carried all over India, in gridiron fashion, that is to say, by a system of meridional series at varying distances of 1° to 3° apart, on either side of the primary Great Arc, tied together by longitudinal series on the parallels of Madras, Bombay, and Calcutta, and along the northern frontier. Thus a large number of additional meridional chains of triangles have been furnished, several of which have been converted into meridional arcs by the addition of latitude observations at a large number of the stations. In addition to this, a large number of longitudinal arcs have been acquired by the electro-telegraphic determinations of the astronomical differences of longitude between certain of the trigonometrical stations. This work, so far as available up to the year 1878, was employed by Colonel Clarke, C.B., of the Ordnance Survey of Great Britain and

Ireland, in his latest determination of the figure of the Earth.* But very much more geodetic material has since become available.

A further contribution to geodesy has been made in the Indian pendulum observations which have been taken at several stations of the survey from Cape Comorin up to the table lands of the Himalayas, and also at islands in the ocean and points on the coast lines; they have been connected with pendulum observations in England, Russia, and America.

Full details of the principal operations of the Survey, as completed as a whole for all India, and finally reduced, are given in the "Accounts of the Operations of the Great Trigonometrical Survey," which are published in the Survey Office at Dehra Dun, in Northern India. These volumes give the measurements of the base-lines and of the principal angles, the astronomical latitudes, the electro-telegraphic longitudes, and the pendulum determinations; they also show the steps which have been taken to make the triangulation consistent throughout with the smallest possible disturbance of the facts of observation, and they give the final results which have been obtained in each instance. Of these volumes, sixteen have now been published in large quarto, the first appearing in 1870. They contain both the work which has been done by the Survey as a contribution to geodesy, as well as what has been executed as a basis for the topography of India. But they are thick, bulky volumes, and are not very readily accessible, though they have been widely distributed to public libraries. The results of the geodetic operations may, however, be summarized and capitulated in a comparatively small space; it seems desirable that this should be done, in order to place them within convenient reach of all mathematicians wishing to investigate the figure of the earth. I have therefore extracted from the published volumes all that is of essential importance in relation to geodesy, and added to it several latitude determinations which I have recently received from the Survey and which have not as yet been published, all which I trust will be found of convenience and utility in future geodetic investigations.

I now propose to give first a general sketch of the operations from their origin to their conclusion; then to describe the measurements of the base-lines and the principal angles, the instruments employed, and the method which was taken to bring the very vast amount of measurements, each one necessarily fallible, into harmony with each other, so as to secure the smallest possible disturbance of the original facts of observation. The next thing will be to give a brief description of the instruments and methods employed for the latitude observations, and a synopsis of the results, and to do the same for the electro-telegraphic determinations of differences of longitude; then to examine the extent to which these operations have been influenced by local deflections of the plumb-line on the meridian and on the prime vertical. It will be seen that great deflections of the plumb-line are met with, not only in the vicinity of mountain masses, but in plains where there are no elevations or irregularities of surface, and where the cause of deflection must be variations of density in the masses below the ground level. This leads to the combination of the observations in groups to eliminate the influence of local attraction as far as possible. Finally, the most probable values of the amplitudes, and the lengths in feet of certain meridional and longitudinal arcs resulting from these combinations, are given.

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* "Geodesy," by Colonel A. R. CLARKE, C.B., Clarendon Press Series. Oxford, 1880.

General Sketch of the Operations.

The Great Trigonometrical Survey of India was commenced in the year 1800, when a systematic triangulation was initiated in the Southern Peninsula, by Major LAMBTON, with a view to determining the exact positions in latitude and longitude of all the objects that appeared best calculated to become permanent geographical marks to be employed as the basis of the general survey of the country. Up to that time whatever surveys had been made had depended on astronomical determinations of latitudes and longitudes, which were liable—the latter more particularly—to far greater errors than the differential determinations of the same elements which might be obtained from a good triangulation. The Astronomical Observatory at Madras had then been founded, and Major LAMBTON made it the origin of his longitudes, through which to refer to the meridian of Greenwich.

But the accuracy of geodetic calculations of latitude and longitude from the data of a triangulation depends on the accuracy of the elements of the figure of the earth which are employed in the calculations. At that time those elements were not known with all desirable accuracy, for though geodetic operations had been completed in various parts of the world, they had not yet been undertaken in regions within 33° of the equator. Thus Major LAMBTON executed a central meridional triangulation with the greatest possible precision, and measured the astronomical latitudes of some of the stations, thereby initiating the Indian Great Arc as a contribution to the science of geodesy, and a means of enhancing the accuracy of his own operations. He carried this arc from Cape Comorin to the parallel of 18°, and made a general triangulation of the hill country to the east and west up to the coast lines, founding it on chain-measured base-lines which were placed at convenient distances apart.

He was succeeded by Colonel EVEREST, who carried the Great Arc up to the Himalayan Mountains, and made many notable improvements in the methods of operation. He introduced luminous signals—heliotropes for observations by day, and lamps with parabolic reflectors for observations by night—which were much more satisfactory than the opaque signals hitherto employed, and which could only be observed during the day-time. He obtained better theodolites for measuring the principal angles, and COLBY'S apparatus of compensation bars and microscopes for measuring base-lines in place of the measuring chains which had hitherto been employed. He also made an important modification in the general design of the principal triangulation, which has resulted in greatly increasing its value for geodetic purposes; instead of throwing it over the whole country from coast to coast as had been done in southern India, he adopted the gridiron form of meridional chains tied together by longitudinal chains, the normal directions to be only deviated from on the frontier and coast lines, the directions of which had necessarily to be followed.

Colonel EVEREST retired in 1843, and was succeeded by Colonel WAUGH, under whose superintendence a large amount of the principal triangulation was executed. Additional great theodolites were obtained, the precision of the *modus operandi* was increased by further refinements, and some of the new chains of triangles became superior in accuracy to the Great Arc. Thus they would become available for meridional and longitudinal arcs for geodetic purposes by the addition of the determination of astronomical latitudes and differential-longitudes, and Colonel WAUGH took steps to obtain new instruments from England for the purpose.

Colonel WAUGH retired in 1861, and was succeeded in the superintendence of the Great Trigonometrical Survey by the author of this paper, under whom the principal triangulation was completed in India proper and carried down through Burma to Mergui, and considerable progress was made with the astronomical latitudes and differential-longitudes. He retired in 1884, after which the astronomical work was carried on under the superintendence of Colonel GEORGE STRAHAN.

It should be here stated that in order to place all the principal triangulation on a satisfactorily uniform basis of accuracy, Major LAMBTON'S portion of the Great Arc has been wholly revised, and the gridiron system has been carried down the east and west coasts of the Peninsula to the parallel of Madras, and down the east coast to Cape Comorin, with a branch series across the Straits of Manaar, to form a connection with the triangulation of the Island of Ceylon. The annexed chart (Plate 18) shows the whole of the principal triangulation which has been executed up to date, and also the secondary triangulation outside the limits of the principal; a vast amount of secondary work has also been executed within the limits of the principal as a basis for topographical surveys; but all this is omitted, as it would only confuse the chart.

I now proceed to give sufficient information regarding the principal triangulation and the base-lines on which it rests, and the method by which the whole has been finally reduced and rendered consistent all over India, to indicate the value of the final results and the amount of confidence which may be placed on them for future geodetic investigations.

The Base-lines.

The base-lines are ten in number; half of them are situated on the Great Arc, at Cape Comorin, Bangalore, Bider, Sironj, and Dehra Dun, and the remainder at Attok, Kurrachee, Sonakhoda, Calcutta, and Vizagaparam, which are situated at the further corners of certain of the gridiron figures which have been made in combination with various sections of the Great Arc. They were all measured with COLBY'S apparatus of compensation bars and microscopes, which is described in detail in vol. 1 of the 'Account of the Operations,' and in many other publications.

Though the base-line measurements were invariably carried on under tents, for the protection of the bars and microscopes from the rays of the sun, it was soon found that the lengths of the compound bars did not remain constant throughout the day; they were consequently compared systematically with the standard 10-foot iron bar of the Survey for three days before and after the measurement of each base-line, at the same hours as the actual measurements, and their lengths were assigned accordingly. It was at first supposed that the compensation points had not been exactly fixed by the makers; but eventually it was found at the Cape Comorin base-line—where a line of about one-fourth the usual length was measured over four times, in order to test the apparatus—that whichever of the two components of a bar was nearest the sun acquired heat most rapidly, notwithstanding that they lay side by side in a stout deal case, and were further protected by tents. The variations in length, however, though very sensible under high microscopic power, were not of a material magnitude; and as in the comparisons of the compound bars with the standard which were made at each base-line, the bars were invariably set up parallel to the line of the measurement and compared during the hours in which the measurements were made, the errors arising from the variations in length must have been largely eliminated.

The mathematical probable errors of the base-lines are investigated in chapters ix. and x. of vol. 1, where it is shown that all the ten base-lines may be practically regarded as of equal weight, and that their probable errors may be taken as equal to ± 2·6 millionth parts of the length measured, which corresponds to 108 feet in the length of the polar axis of the earth.

The Principal Triangulation.

The principal angles were all measured with theodolites whose azimuthal circles ranged from 36 inches down to 15 inches in diameter, the larger circles being furnished with five and the smaller with three micrometer microscopes for reading the graduations. More than three-fourths of the angles were measured with the larger theodolites having circles of 24 or 36 inches in diameter. All the instruments are described in Appendix No. 2 of vol. 2 of the 'Account of the Operations.'

The signals between which the angles were measured were invariably luminous, heliotropes by day, and lamps by night.

The system of operation was specially devised with a view to eliminate the graduation errors of the azimuthal circles to the utmost possible extent, by systematic angular shifts in the position of the zero of the circle in the course of the successive measurements of each angle, so that the microscope readings during the telescope pointings to any object might be taken at several equidistant points on the circle. The distances were taken first at 9°, and afterwards at 7° 12′, for circles with five microscopes, and at 10° for circles with three microscopes, producing a greater number of changes of zero for the latter than for the former. Thus in the final mean result, periodic errors of graduation are eliminated, and accidental errors are largely cancelled because of the large number of graduations employed. Half the measurements were made with the face of the vertical circle to the right, and half with it to the left of the observer, thus cancelling the effects of collimation error, and doubling the number of points at which the circle was read, the number of microscopes being always an odd number. Sometimes two, but more generally three measures were taken as a minimum in each position of the instrument, before changing face or altering the zero setting. The minimum number prescribed for the whole of the measures varied with the adopted arcs for zero setting, and was usually 20, 24, or 30 for theodolites having five microscopes, and 36 for those having three; its completion was an invariable condition of observation, and it was always added to whenever unfavourable atmospheric conditions made it difficult to intersect the signals accurately.

The observations completed, the weight of each angle was deduced as usual from the discrepancies between each measure and the mean of all. For all the later triangulation this has been done by a formula which endeavours to take account of both the personal error of observation, as shown by the differences between measures taken on the same graduations, and the graduation error, as shown by the differences between each zero mean and the mean of all the zeros. The theoretical errors of mean square of observation and of graduation are given in the 'Accounts of Operations,' immediately after the details of the observations of the angles in each chain of triangles, and they are combined in separate groups for each observer and instrument, and also for observations at stations on hills and at stations on plains, as the former are usually taken under more favourable atmospheric conditions. The theoretical errors are given only for single observations and for single zeros, for which they are mathematically exact; but the theoretical error of the value of the angle deduced from the mean of all the measures, which must necessarily be deduced from them eventually, is only exact when the graduation errors are all accidental, for periodic errors of graduation, however large, are wholly eliminated by the systematic changes of zero. When they exist the theoretical errors will be too large. This does not matter for the primary reduction of triangles whose angles have all been measured with the same instrument; but it is of importance when masses of triangulation with different instruments have to be put together and finally adjusted. Thus it eventually became necessary to correct the preliminary weights by the more reliable evidence of average weight, which can be obtained from an examination of the geometrical errors of triangles and polygonal figures, of which a large number have been measured with the same instrument.

The following table gives the average values of the theoretical probable errors of the angles measured by the several theodolites, which were derived from the actual observations. The number of angles and their average probable errors are given for each of the separate sections into which the triangulation was divided for final reduction, as will presently be explained.

Theodolite.	North-West Quadrilateral.		North-East Quadrilateral.		South-East Quadrilateral.		South-West Quadrilateral.		Trigon.		Whole.	
	No.	Probable error.	No.	Probable error.	No.	Probable error.	No.	Probable error.	No.	Probable error.	No.	Probable error.
		″		″		″		″		″		″
Troughton and Simms' 36-inch . . .	1104	± ·21	35	± ·29	566	± ·20	..	..	45	± ·17	1750	± ·21
Barrow's 36-inch	164	·32	156	·19	66	·23	..	..	..	..	386	·25
Troughton and Simms' 24-inch, No. 1	57	·19	..	..	211	·27	..	..	946	·16	1214	·18
Troughton and Simms' 24-inch, No. 2	147	·21	395	·20	..	..	..	..	..	..	542	·20
Barrow's 24-inch, No. 1	..	..	465	·35	..	..	..	..	..	..	465	·35
Barrow's 24-inch, No. 2	373	·14	48	·40	99	·33	..	..	489	·21	1309	·16
Waugh's 24-inch, No. 1	638	·24	123	·37	391	·21	..	..	..	..	1152	·24
Waugh's 24-inch, No. 2	251	·35	33	·35	..	..	..	..	..	..	284	·35
Troughton and Simms' 18-inch, No. 1	81	1·95	276	1·55	13	1·47	..	..	..	..	370	1·63
Troughton and Simms' 18-inch, No. 2	..	..	209	·60	..	..	698	± ·61	..	..	907	·61
Cary's 18-inch	..	..	89	·79	..	..	..	..	..	..	89	·79
Syod Mohsin's 18-inch	..	..	78	·93	..	..	..	..	..	..	78	·93
Harris and Barrow's 18-inch . . .	..	..	195	·69	..	..	..	..	..	..	195	·69
Cary's 15-inch . :	..	..	147	·86	..	..	..	..	..	..	147	·86
Dollond's 15-inch	43	·83	..	..	..	..	190	·88	125	·91	358	·88

The values of the probable error which were obtained from an examination of the triangular and polygonal geometrical errors give results which slightly exceed the above values, excepting in the case of TROUGHTON and SIMMS' 18-inch theodolite, No. 1; this instrument has been found to have a large periodic error, which becomes eliminated by the adopted method of changing zero, as is shown in Appendix No. 4, vol. 2; its true theoretical probable error is only about one-third of the quantity shown in the preceding table.*

The several series of triangles are composed sometimes of single triangles, but most frequently of polygonal figures, which are self-verificatory, and add considerable weight to the triangulation. The single triangles mainly appertain to the portions of the triangulation lying wholly in the plains which were first completed, but, as the work proceeded, polygonal figures were employed in plains as well as in hill districts.

During the recess following each field season, the reduction of the observations was taken in hand, the weight of each angle was determined, the corrections to the observed angles, which were necessary to satisfy the geometrical conditions involved, were computed by the method of minimum squares, the lengths and azimuths of the sides, and the latitudes and longitudes and heights of the stations were calculated; the secondary triangulation was treated somewhat similarly, but with less precision, and thus all the work of the previous field-season was rendered available for immediate topographical requirements.

But it will be obvious that as the several chains of triangles became completed, closing on each other and on the base lines, discrepancies would be met with in the values of length, azimuth, latitude, and longitude, which would be presented by the different chains at the sides of junction. In order to make the triangulation consistent throughout, it was necessary to eliminate these discrepancies by the application of small corrections to the whole of the angles, and it became a matter of great importance to decide how this should be done. To do it with strict theoretical nicety would necessitate the simultaneous reduction of the whole of the triangulation which would have entailed the solution of an algebraical problem, presenting upwards of 8000 unknown quantities, connected together by upwards of 3600 geometrical equations of condition, and the operation could not have been commenced until the whole of the triangulation was completed. Strict theoretical nicety was, therefore, set aside. The triangulation, as a whole, was divided into five great sections, to be treated successively, on the understanding that the whole of the angular corrections for each section should be computed simultaneously, but the angles of chains of triangles, which might be common to any two sections, should retain the corrections obtained for them in the section first reduced, and should not be introduced into the reduction of the second section.

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* TROUGHTON and SIMMS' 36-inch theodolite and their two 18-inch theodolites were landed in India in 1830. The larger instrument was excellently divided by hand by Mr. WILLIAM SIMMS. The two smaller instruments are alleged to have been divided upon TROUGHTON'S first dividing engine, but the marked differences in their performances, and the existence of a considerable periodic error in No. 1, from which No. 2 is free, indicate differences in the method of graduating. The original dividing engine was found to be faulty, and was superseded by the celebrated dividing engine which was completed in the year 1842, and has afterwards been employed in graduating all the large circles manufactured by this firm. Their two 24-inch theodolites which appear in the list on p. 751, were received in India in 1848, and were graduated by the new machine. In the same year BARROW'S two 24-inch theodolites were received; they are remarkably well graduated instruments; the angles measured with No. 2 in the North-West Quadrilateral show the smallest probable errors of any of the observed angles; they were, however, measured under exceptionally favourable atmospheric conditions. WAUGH'S 24-inch theodolites were put together in Calcutta in 1846 from the parts of other instruments; most of the other 18-inch and 15-inch theodolites were similarly constructed, and they all date back to some period anterior to 1830.

The five sections thus decided on for separate treatment comprise four quadrilateral figures for Northern and Central India and a trigon for the Southern Peninsula. The northern sections lie on either side of the Great Arc and meet at the station of Kalianpur, in Central India, which Colonel EVEREST adopted as the origin of the geodetic calculations of the survey; they are respectively called the North-East Quadrilateral, the North-West Quadrilateral, the South-East Quadrilateral, and the South-West Quadrilateral; the fifth section is called the Southern Trigon, and is bisected by the Great Arc.

In the simultaneous solutions of the several sections much simplification was rendered possible by the circumstance that, when a large number of independent equations has to be treated, we may select and solve any groups we please, independently, and apply corrections to the angles as deduced without prejudice to obtaining the most probable values of the angles ultimately, provided the whole of the equations are finally solved simultaneously with such absolute terms as exist after the approximations have been made. An important first approximation had already been made by the correction of the observed values of the angles for the geometrical errors of the triangles and polygonal figures to which they respectively appertained; thus, each polygonal triangulation might be converted into a chain of single triangles by employing the triangles on one flank of the polygons only and discarding those on the other flank, and, as the angles had already been made consistent, it did not matter which flank was employed. After the final values of the angles on the adopted flank had been obtained by the simultaneous solution, those for the discarded angles could be readily obtained, figure by figure, and thus the whole triangulation would be finally adjusted.

It is here desirable to observe that the numerous astronomical determinations of azimuth, latitude, and differential longitude which have been made at various stations of the Survey, furnish a number of external and independent facts of observation with which the triangulation might have been brought into accordance. The azimuths more particularly seemed to have a claim for introduction, as they were measured with the intention of forming a check on the triangulation; they are very numerous, and are given in full detail in the accounts of the operations. But the triangulation of this Survey cannot be forced into accordance with any astronomically determined facts of observation without a liability to introduce errors, arising from deflections of the plumb-line in astronomical observations, which far exceed the errors ordinarily generated in the triangulation. Moreover, other errors would arise, more minute, but becoming very sensible in long distances, from the errors in the numerical elements of the earth's figure which were adopted as the basis of the geodetic calculations of latitude, longitude, and azimuth. If the triangulation were to be forced into accordance with the astronomical determinations, it would become seriously distorted in parts and unfitted for employment in further investigations of the figure of the Earth. Thus, in the final reductions, the lengths of the base-lines and the geometrical consistency of the angles were alone taken into consideration, the only astronomically determined elements which were employed being those which were adopted for Kalianpur, the station of origin of the calculations.

As each section would have to accept the results of the reduction of a previous section as final and unalterable, it was obviously necessary to commence the final reductions with the section containing the most accurately measured angles and follow with the next most accurate section, in order that work of the best class might not be injured by being forced into accordance with work of an inferior nature. The reductions were commenced about the year 1868, when the North-West Quadrilateral was completed, with the exception of two comparatively short chains across the desert tracts of Sind and Rajputana, which were postponed until the whole of the rest of the triangulation had been completed. The North-East Quadrilateral was already completed; but it consists of a number of series of triangles which were the first executed after the completion of the Great Arc, with less precision than the work of subsequent years, and several of them with the inferior 18-inch and 15-inch theodolites; and there was reason to fear that portions of the triangulation which were carried on towers over the plains of Bengal had suffered from slight displacement of the markstones by deflections of the towers during the heavy rains of the Indian monsoons, thus altering the lengths of certain sides by a few inches in the interval between closing and resuming operations.* The North-West Quadrilateral was of much superior accuracy to the North-East Quadrilateral; it was therefore determined to commence the final reductions with that section as already completed, leaving the two intermediate chains to be afterwards fitted in. By the time its reduction was completed the triangulation of the South-East Quadrilateral was also completed, and its final reduction was taken in hand next for similar reasons.

Thus the order adopted for the final simultaneous reductions of the several sections was as follows:—

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* The towers consisted at first of a solid masonry pillar, on which the instrument was placed, surrounded by a platform of sun-dried bricks and earth for the observer and his equipment; after deflection was noticed the pillars were built hollow with a vault below, and the markstone was placed at the basement, with a passage leading to it.

First Section.—The North-West Quadrilateral.
Second Section.—The South-East Quadrilateral.
Third Section.—The North-East Quadrilateral.
Fourth Section.—The Southern Trigon.
Fifth Section.—The South-West Quadrilateral.

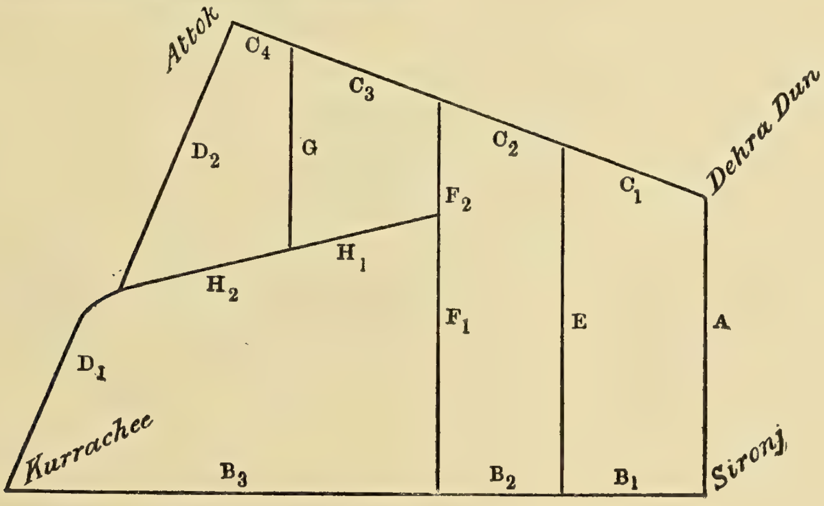
Full details of the measurements of the observed angles and of the successive steps of the reductions and the final results are given in vols. 2, 3, and 4 for the first section, vol. 6 for the second, vols. 7 and 8 for the third, vols. 12 and 13 for the fourth, and vol. 14 for the fifth.

The labour involved in each of the great simultaneous reductions was very considerable; the mass of calculation was enormous, and as, in each case, it was all interdependent, so that a single error might vitiate the results, great precautions had to be taken to eliminate error in each stage of the computations before proceeding to the next stage. A description of the practical procedure in the case of the North-West Quadrilateral is given by Mr. HENNESSEY in Appendix No. 12 to vol. 2. Some modifications and improvements were introduced in the treatment of the subsequent sections by Colonel HERSCHEL and Mr. W. H. COLE. But in every instance the reduction was a very anxious as well as laborious operation, and its successful completion was a matter of much gratulation and satisfaction.

The numerical values of the several discrepancies in length, azimuth, latitude, and longitude, at the sides of junction of the chains of triangles and at the base-lines, which were met with in each of the sections, and adjusted by the processes of simultaneous reduction, will now be exhibited. Afterwards the magnitudes of the corrections to the angles which resulted from the calculations will be set forth.

The North-West Quadrilateral.

The outlines of this figure, as completed up to the time when its final reduction was taken in hand, are given in this sketch.



It is composed of eight chains of triangles, resting on four base-lines, one at each of the corners of the quadrilateral, viz., at Sironj, Dehra Dun, Attok, and Kurrachee. The chains are—

A, the Great Arc, section 24° to 30°.
B, the Kurrachee Longitudinal Series in three sections, B_1, B_2, and B_3.
C, the North-West Himalaya Series in four sections, C_1 to C_4.
D, the Great Indus Series in two sections, D_1 and D_2.
E, the Rahun Meridional Series.
F, the Gurhagarh Meridional Series in two sections, F_1 and F_2.
G, the Jogi-Tila Meridional Series.
H, the Sutlej Series in two sections, H_1 and H_2.

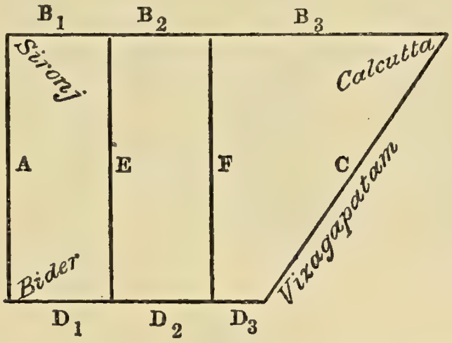
The number of angular errors for simultaneous determination was 1650, which were connected together by 23 equations of condition, some comprising the errors in the direct triangulations between the base lines, and the remainder comprising the errors in the circuits of triangulation. The formation of these equations is to some extent a matter of choice; in the present instance all the linear or side-length equations were first written down, and afterwards the equations to the geodetic errors on the circuits; but eventually it was found preferable, whenever possible, to place the linear and the three geodetic equations appertaining to each circuit together, as this causes less entanglement and labour of calculation in the solution.

In the following table the errors of the logarithms of the side-lengths in feet, and the geodetic errors in seconds of arc, are shown. The column for included triangles shows the number of single triangles forming the chain or circuit and omits the discarded angles of the polygonal figures.

Chains and their sections.	Included triangles.	Errors.			
		Log. side-length.	Latitude.	Longitude.	Azimuth
			″	″	″
A	51	+·000,0044,0			
$B=B_1+B_2+B_3$	96	−·000,0079,6			
$C=C_1+C_2+C_3+C_4$	39	+·000,0071,9			
$D=D_1+D_2$	123	+·000,0163,8			
$C_1-(B_1+E)$	94	+·000,0068,2			
$A+C_1-(B_1+E)$	145	..	+0·391	+0·168	+5·91
$E+C_2-(B_2+F_1+F_2)$	185	−·000,0124,6	−0·390	+0·214	+1·55
$F_2+C_3-(H_1-G)$	87	+·000,0150,9	+0·036	−0·287	−4·23
$G+C_4-(H_2+D_2)$	139	−·000,0005,3	−0·005	−0·290	−3·00
$F_1+H_1+H_2-(B_3+D_1)$	250	..	+0·387	+0·293	−3·25

The South-East Quadrilateral.

This quadrilateral is composed of six chains of triangles, and its corners rest on the base-lines at Sironj, Bider, Vizagapatam, and Calcutta. It is wholly executed with the best instruments, two small meridional chains near Calcutta, which were executed with inferior instruments, having been excluded from the final simultaneous reduction. The chains employed are—



A, the Great Arc, section 18° to 24°.

B, the Calcutta Longitudinal Series in three sections, B_1, B_2, and B_3.

C, the Coast Series.

D, the Bider Longitudinal Series in three sections, D_1, D_2, and D_3.

E, the Jabalpur Meridional Series.

F, the Bilaspur Meridional Series.

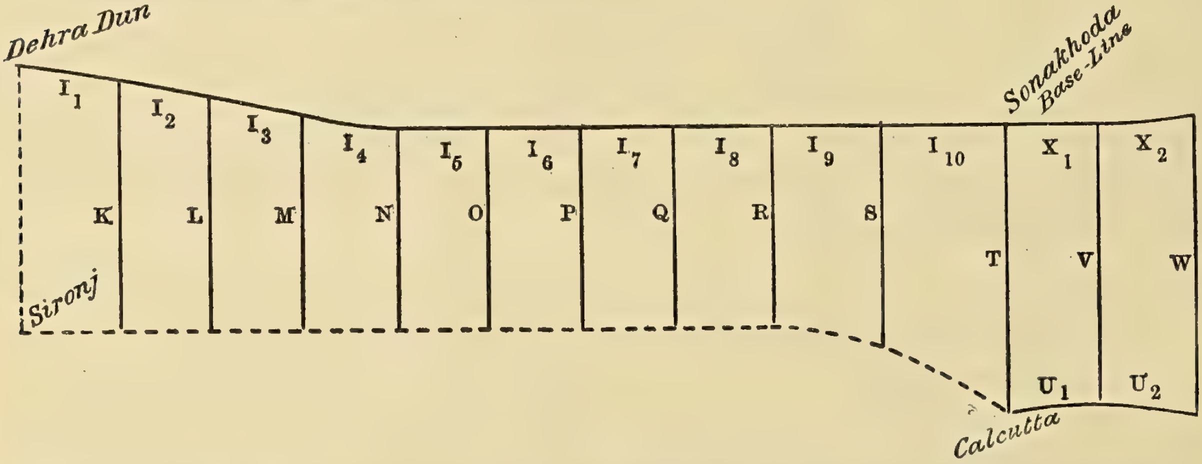
The number of angular errors involved was 831, which were included in fifteen equations of condition, six linear and nine geodetic, presenting the following errors:—

Chains and their sections.	Included triangles.	Errors.			
		Log. side-length.	Latitude.	Longitude.	Azimuth.
			″	″	″
A	40	−·000,0016,4			
$D_1-(B_1+E)$	65	−·000,0054,9			
C	65	−·000,0006,9			
$B=B_1+B_2+B_3$	54	+·000,0042,5			
$D=D_1+D_2+D_3$	39	−·000,0021,3			
$A+D_1-(B_1+E)$	100	..	+0·050	−0·214	+0·21
$E+D_2-(B_2+F)$	109	+·000,0031,9	−0·216	+0·203	−4·97
$F+D_3-(B_3+C)$	154	..	−0·142	+0·239	−3·89

North-East Quadrilateral.

This Quadrilateral includes the whole of the series of triangles east of the Great Arc and north of the parallel of Calcutta. The reduction of this figure rests on the final results already obtained, for the section of the Great Arc to the west and for the Calcutta Longitudinal Series to the south, from the reductions of the North-West and the South-East Quadrilaterals; it also rests on the Sona-Khoda base-line which is situated at the northern extremity of the Calcutta Meridional Series. It very closely touches the base-lines at Calcutta, Sironj, and Dehra Dun, and rests on the polygonal figures of which they form a part.

The outlines of the figure are as follow:—



Thus the angles which came under treatment were those of the following series:—

I, the North-East Longitudinal Series, in ten sections, I_1 to I_{10}.
K, the Rangir Meridional Series.
L, the Amua Meridional Series.
M, the Karára Meridional Series.
N, the Gurwani Meridional Series.
O, the Gora Meridional Series.
P, the Hurilaong Meridional Series.
Q, the Chendwar Meridional Series.
R, the North Parasnath Meridional Series.
S, the North Maluncha Meridional Series.
T, the Calcutta Meridional Series.
U, the East Calcutta Longitudinal Series, in two sections, U_1 and U_2.
V, the Brahmaputra Meridional Series.
W, the Eastern Frontier Series, section 23° to 26°.
X, the Assam Longitudinal Series, in two sections, X_1 and X_2.

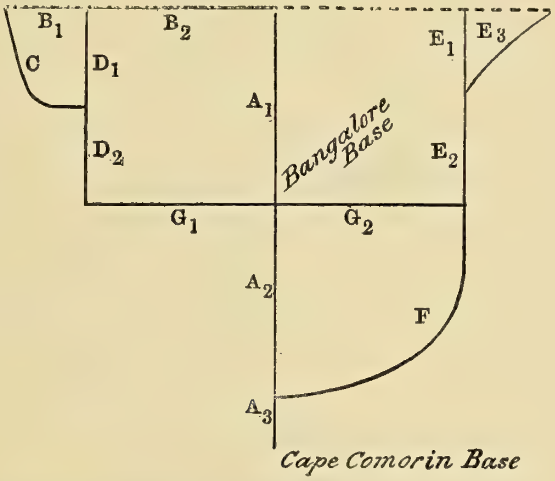
The number of angular errors for determination was 1719, and the number of equations of condition was 49

The errors to be eliminated were as follow :—

Chains and their sections.	Included triangles.	Errors.			
		Log. side-length.	Latitude.	Longitude.	Azimuth.
$K+I_1$	44	+·000,0287,1	+0″·939	+0″·934	+11″·60
$L+I_2-K$	82	−·000,0007,3	−0·352	−1·090	−14·32
$M+I_3-L$	81	+·000,0482,3	+0·925	−0·183	− 6·80
$N+I_4-M$	79	−·000,0641,0	−1·209	−0·060	+10·18
$O+I_5-N$	73	+·000,0169,4	+0·383	+0·237	− 0·79
$P+I_6-O$	73	+·000,0082,1	+0·211	+0·104	+ 4·84
$Q+I_7-P$	69	−·000,0132,9	−0·357	−0·030	− 5·74
$R+I_8-Q$	55	+·000,0196,7	+0·303	−0·027	+ 4·44
$S+I_9-R$	52	−·000,0284,6	−0·314	+0·036	− 2·28
$T+I_{10}-S$	83	..	−0·214	−0·113	+ 0·65
$I_{10}+S$	36	+·000,0190,9			
T	47	+·000,0036,7			
U_1+V-X_1	81	−·000,0193,7			
$U_1+V-(T+X_1)$ . . .	128	..	+0·129	−0·143	+ 2·41
$U_2+W-(V+X_2)$ . . .	113	+·000,0102,0	+0·173	−0·627	−13·14

The Southern Trigon.

The triangulation rests on the Bider Longitudinal Series of the South-East Quadrilateral and on two of the base-lines of the Great Arc, one at Bangalore, the other at Cape Comorin. Its outlines are shown in the following sketch :—



It is composed of seven chains of triangles—

A, the Great Arc from 8° to 18°, in three sections, A_1, A_2, and A_3.
B, the Bombay Longitudinal Series, in two sections, B_1 and B_2.
C, the South Konkan Coast Series.
D, the Mangalore Meridional Series, in two sections, D_1 and D_2.

E, the Madras Meridional and Coast Series, in three sections, E_1, E_2, and E_3.
F, the South-East Coast Series.
G, the Madras Longitudinal Series, in two sections, G_1 and G_2.

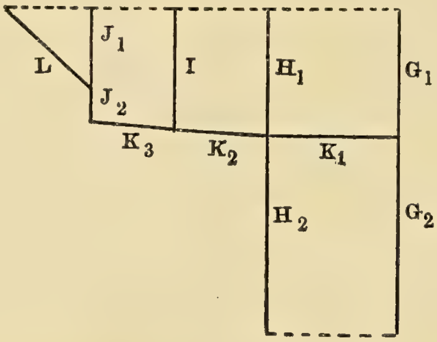
The number of angular errors involved was 909, and they were embodied in 22 equations of condition.

The errors to be eliminated were as follows :—

Chains and their sections.	Included triangles.	Errors.			
		Log. side-length.	Latitude.	Longitude.	Azimuth.
B_1+C-D_1	46	+·000,0136,6	+0″·022	+0″·078	−0″·25
$B_2+D_1+D_2+G_1-A_1$ . .	99	−·000,0022,7	−0·001	+0·288	−3·68
A_1	35	+·000,0020,0			
$A_1+G_2-(E_1+E_2)$ . . .	84	+·000,0039,9	0·000	−0·174	+4·30
G_2+F+A_3	88	+·000,0021,5			
$A_2-(G_2+F)$	107	. . .	−0·273	+0·424	−9·04
A_2+A_3	33	+·000,0001,1			
E_1-E_3	32	−·000,0011,7	−0·042	−0·001	−0·32

The South-West Quadrilateral.

This triangulation rests on the North-West Quadrilateral and on the Southern Trigon. The number of angular errors for determination was 516, and the number of equations of condition 24. Its outlines are shown in the following sketch :—



It consists of six chains of triangles, viz.—

G, the Khanpisura Meridional Series, in two sections, G_1 and G_2.
H, the Singi Meridional Series, in two sections, H_1 and H_2.
I, the Abu Meridional Series.
J, the Kattywar Meridional Series, in two sections, J_1 and J_2.
K, the Guzerat Longitudinal Series, in three sections, K_1, K_2, and K_3.
L, the Cutch Coast Series.

The errors to be eliminated were as follows :—

Chains and their sections.	Included triangles.	Errors.			
		Log. side-length.	Latitude.	Longitude.	Azimuth.
G_1+G_2	30	+·000,0189,8	−0″·637	−0″·571	−7″·81
$H_1-(G_1+K_1)$	36	−·000,0212,5	+0·068	+0·407	−7·11
H_1+H_2	35	+·000,0027,4	−0·435	+0·065	−6·72
$I-(H_1+K_2)$	40	−·000,0185,9	+0·151	−0·013	+2·96
$J_1-(I+K_3+J_2)$	49	−·000,0062,2	+0·139	−0·072	+3·05
J_1+L	48	+·000,0257,1	+0·060	−0·256	−3·91

From the five preceding tables we obtain the following table of the average error generated in ten consecutive triangles in each of the five great figures, the three sections which were mainly executed with the best instruments being shown first, and then the two others, into which work executed with the smaller and less accurate instruments largely enters.

Section.	Average error generated in ten triangles.			
	Log. side-length.	Latitude.	Longitude.	Azimuth.
North-West Quadrilateral	± ·000,0008,7	± 0″·015	± 0″·016	± 0″·22
South-East ,,	·000,0004,7	0·011	0·018	0·25
Southern Trigon	·000,0006,1	0·009	0·026	0·48
Average for first class	·000,0006,5	0·012	0·020	0·32
North-East Quadrilateral	·000,0031,7	0·059	0·038	0·83
South-West ,,	·000,0039,3	0·063	0·058	1·33
Average for second class	·000,0035,5	0·061	0·048	1·08

The magnitudes of the angular corrections range from 0 to 4″; the following table shows the number of corrections of each magnitude which were applied in each section.

Section.	Total number of angles.	Corrections, and number of angles receiving them.									
		0 to ·1″.	·1″ to ·2″.	·2″ to ·3″.	·3″ to ·4″.	·4″ to ·5″.	·5″ to 1″·0.	1″·0 to 1″·5.	1″·5 to 2″·0.	2″·0 to 3″·0.	3″·0 to 4″·0.
North-Western Quadrilateral	1650	1511	116	20	2	1					
South-Eastern ,,	831	706	109	13	2	1					
Southern Trigon	909	777	103	12	4	2	11	2			
North-Eastern Quadrilateral.	1719	780	390	229	106	58	128	17	10	1	
South-Western ,,	516	103	115	109	78	31	49	11	9	9	2

It will be seen that the relative magnitude of the closing errors, and of the corrections to the angles, fully justify the decision which was made regarding the order in which the several sections of the triangulation should be taken up for final reduction.

The preceding details sufficiently indicate the general character of the triangulation; but some further information regarding it is still necessary.

The geodetic calculations were made on the assumption that the Earth's figure is spheroidal and of the following dimensions :—

Major axis 20,922,932 feet
Minor axis 20,853,375 ,,
Ellipticity $\frac{1}{300 \cdot 80}$.

These values were originally computed by Colonel EVEREST from such data as were available in 1830. He computed a second set of constants after the completion of the Great Arc in 1844, but they have never been employed in this Survey, and are now known to be less accurate than the first set. They are given in the following table, with the constants determined by other investigators :—

	Major axis in feet a.	Minor axis in feet b.	$a - b$.	$\frac{a-b}{a}$
EVEREST'S second set . . .	20,920,902	20,853,642	67,260	$\frac{1}{311 \cdot 04}$
LAPLACE	20,919,768	20,852,822	66,946	$\frac{1}{312 \cdot 20}$
AIRY	20,923,713	20,853,810	69,903	$\frac{1}{299 \cdot 33}$
BESSEL	20,923,600	20,853,656	69,944	$\frac{1}{299 \cdot 15}$
CLARKE'S first set	20,926,348	20,855,233	71,115	$\frac{1}{294 \cdot 26}$
CLARKE'S last set	20,926,202	20,854,895	71,307	$\frac{1}{293 \cdot 47}$

The unit of length of the Survey is the 10-foot standard iron bar A whose relations to other well-known standards are as follows at the temperature of 62° F.

Indian 10-foot bar A = 3·33331886 in English standard yards.
,, ,, = 1351·14821 in lines of the toise.
,, ,, = 3047·95942 in millimeters.

The adopted origin of the Survey is the trigonometrical station of Kalianpur, near the Sironj base-line, in Central India. The latitude of this station was determined by EVEREST as 24° 7′ 11″·26 from astronomical observations taken in 1825 and 1839–41, and that value is the basis of all the geodetic latitudes of the Survey. Subsequently additional observations were taken at Kalianpur, and the whole were finally reduced with the latest and best values of star places, as described in vol. 11, 'Astronomical Latitudes;' the value 24° 7′ 11″·10 was then finally obtained. Thus a correction of − 0″·16 has to be applied to all the geodetic latitudes to reduce them to the final value of the latitude of the origin.

The azimuth was determined astronomically at Kalianpur, and eventually at a large number of stations around; a comparison of the subsequent determinations with the initial azimuth, through the triangulation, led to its receiving a correction of − 1″·1 for local attraction; see section 4, chap. xi., vol. 2. Thus corrected it became the fundamental azimuth of the final geodetic calculations.

As the elements of the Earth's figure adopted for the geodetic calculations so long ago as 1830 are not so accurate as those lately determined from fuller data, it is necessary here to give as much of the formulæ employed as is required to facilitate the conversion of the given latitudes and longitudes to the values which would have been obtained had other elements been employed. The formulæ were devised by PUISSANT for the solution of the problem,—Given the latitude and longitude of a station A, the azimuth at A of a station B, and the distance of B from A, to find the latitude and longitude of B, and the azimuth of A at B.

Let a and b be the major and the minor semi-axes, e the eccentricity $= \sqrt{(a^2-b^2)/a^2}$, ρ the radius of curvature to the meridian in latitude λ, ν the normal to the meridian in the same latitude, λ and L the given latitude and longitude of A, and A the given azimuth of B at A, $\lambda + \Delta\lambda$ and $L + \Delta L$ the required latitude and longitude of B, and B the required azimuth of A at B; also let c be the distance between A and B at the sea level. Then, all azimuths being measured from the south, we have

$$\Delta\lambda'' = \left\{ \begin{array}{l} - \dfrac{c}{\rho} \cos A \operatorname{cosec} 1'' \\ - \dfrac{1}{2} \dfrac{c^2}{\rho \, . \, \nu} \sin^2 A \tan \lambda \operatorname{cosec} 1'' \\ - \dfrac{3}{4} \dfrac{c^2}{\rho \, . \, \nu} \dfrac{e^2}{1 - e^2} \cos^2 A \sin 2\lambda \operatorname{cosec} 1'' \\ + \dfrac{1}{6} \dfrac{c^3}{\rho \, . \, \nu^2} \sin^2 A \cos A \, (1 + 3 \tan^2 \lambda) \operatorname{cosec} 1'' \end{array} \right.$$

$$\Delta L'' = \left\{ \begin{array}{l} - \dfrac{c \sin A}{\nu \cos \lambda} \operatorname{cosec} 1'' \\ + \dfrac{1}{2} \cdot \dfrac{c^2}{\nu^2} \dfrac{\sin 2A \tan \lambda}{\cos \lambda} \operatorname{cosec} 1'' \\ - \dfrac{1}{6} \dfrac{c^3}{\nu^3} \dfrac{(1 + 3 \tan^2 \lambda) \sin 2A \cos A}{\cos \lambda} \operatorname{cosec} 1'' \\ + \dfrac{1}{3} \dfrac{c^3}{\nu^3} \dfrac{\sin^3 A \tan^2 \lambda}{\cos \lambda} \operatorname{cosec} 1'' \end{array} \right.$$

$$\begin{array}{r} \Delta A'', \text{ or} \\ B - (\lambda + A) = \end{array} \left\{ \begin{array}{l} - \dfrac{c}{\nu} \sin A \tan \lambda \operatorname{cosec} 1'' \\ + \dfrac{1}{4} \dfrac{c^2}{\nu^2} \left\{ 1 + 2 \tan^2 \lambda + \dfrac{e^2 \cos^2 \lambda}{1 - e^2} \right\} \sin 2A \operatorname{cosec} 1'' \\ - \dfrac{c^3}{\nu^3} \left(\dfrac{5}{6} + \tan^2 \lambda \right) \dfrac{\tan \lambda}{2} \sin 2A \cos A \operatorname{cosec} 1'' \\ + \dfrac{1}{6} \dfrac{c^3}{\nu^3} \sin^3 A \tan \lambda \, (1 + 2 \tan^2 \lambda) \operatorname{cosec} 1''. \end{array} \right.$$

Each quantity is the sum of four terms which are respectively symbolised with the aid of the four prefixes δ_1, δ_2, δ_3, and δ_4 ; thus we have

$$\begin{aligned} \Delta\lambda &= \delta_1\lambda + \delta_2\lambda + \delta_3\lambda + \delta_4\lambda \\ \Delta L &= \delta_1 L + \delta_2 L + \delta_3 L + \delta_4 L \\ \Delta A &= \delta_1 A + \delta_2 A + \delta_3 A + \delta_4 A \end{aligned}$$

Differentiating with respect to ρ, ν and e we have

$$d \, . \, \Delta\lambda = - \Delta\lambda \frac{d\rho}{\rho} - \delta_2\lambda \frac{d\nu}{\nu} - \delta_3\lambda \left(\frac{d\nu}{\nu} - \frac{2de}{(1 - e^2) \, e} \right) - 2\delta_4\lambda \frac{d\nu}{\nu} .$$

$$d \, . \, \Delta L = - \Delta L \frac{d\nu}{\nu} - \delta_2 L \frac{d\nu}{\nu} - (\delta_3 L + \delta_4 \mathrm{L}) \, 2 \frac{d\nu}{\nu} .$$

Also

$$\frac{d\rho}{\rho} = -\frac{1}{a}\{da - 2db - 3(da - db)\sin^2\lambda\},$$

$$\frac{d\nu}{\nu} = +\frac{1}{a}\{da + (da - db)\sin^2\lambda\},$$

$$\frac{de}{e} = \frac{1}{de^2}\{da - db\}.$$

The most probable values of the Earth's axes at present known are those in Colonel CLARKE's 'Geodesy' from his latest investigations. According to them, the errors of the elements of the Indian Survey are $da = -3270$ feet, and $db = -1520$ feet. Inserting these values in the above expressions we have, with sufficient approximation,

$$d.\Delta\lambda = -\{\cdot000{,}011 - \cdot000{,}251\sin^2\lambda\}\,\Delta\lambda + \cdot00795\sin 2\lambda,$$

$$d.\Delta L = +\{\cdot000{,}156 + \cdot000{,}084\sin^2\lambda\}\,\Delta L.$$

The corresponding corrections to arcs of 1° in latitude and 1° in longitude, for successive parallels 5° apart, are as follow :—

Latitude correction.	Parallel.	Longitude correction.
+ 0″·031	5°	− 0″·565
+ 0·008	10	− 0·572
− 0·026	15	− 0·583
− 0·070	20	− 0·598
− 0·128	25	− 0·616
− 0·194	30	− 0·637
− 0·267	35	− 0·662

Corresponding corrections to the primarily computed latitudes and arcs of longitude are given hereafter in the investigations of local attraction.

The Meridional Arcs.

Lambton's first meridional arc was a short one, only 1°·35′ in length, which was situated about 35 miles to the west of Madras. His subsequent arcs were all portions of the Great Arc on the central meridian of India. His operations were at first embarrassed by the result that the lengths of the meridional degrees appeared to decrease instead of to increase with the latitude, but he very soon found that this was due to deflections of the plumb-line at the astronomical stations which were caused by local attractions; this anomaly disappeared when the length of the arc was increased and new latitude stations were introduced

He employed a zenith sector, by Ramsden, which was considered a beautiful instrument for that time. It consisted of an arc of 20°, graduated to a radius of 60 inches, and a telescope of a focal length of 62 inches; the arc was fixed at the bottom of the frame of the instrument, while the telescope was suspended at its object end on the summit of the frame, from a horizontal axis which was placed exactly over the centre of the graduated arc. Thus, in pointing to zenith stars the eye end of the telescope moved over the face of the arc, and it was furnished with a microscope for reading the graduations. The instrument could be rotated horizontally, and stars were observed with the telescope in the two positions east and west on alternate nights. The adjustment for verticality was effected with a plummet suspended over the outer end of the horizontal axis carrying the telescope; an arrangement was provided for setting the thread of the plummet exactly opposite the dot which indicated the centre of the axis; the thread coincided with the central line of the graduated arc when the instrument was truly vertical. A full description and a drawing of the instrument are given in vol. 11.

Now, in all graduated instruments, the errors of graduation have usually most importance. There are no means of controlling such errors for partial arcs, but, for complete circles, they may be very effectively controlled by employing a number of equidistant microscopes to read the circle, and shifting their positions systematically so as eventually to obtain circle readings at a large number of equidistant points, according to the method already described as having been introduced by Everest for the measurements of the horizontal angles of the survey. Thus, in conformity with his treatment of theodolites, Everest abandoned the zenith sector and introduced two astronomical circles in its place. They were constructed by Messrs. Troughton and Sims, in 1835, in the form of alt-azimuth instruments, with large vertical and small horizontal circles. Each instrument was fitted with a pair of graduated vertical circles, 36 inches in diameter, which were bolted together with the telescope between them; the supporting pillars carried a pair of fixed microscopes, 180° apart on one side, and, on the other side, a pair of flying microscopes, also 180° apart, which could be set on various points of the circle through a range of 70°; change of face was made systematically by turning the instrument 180° in altitude and azimuth, and systematic reversals of the pivots in the Y's were also made in order that the circles might be brought in turn opposite the fixed and the flying microscopes. For full details and drawings, see vol. 11.

With these two astronomical circles, EVEREST and WAUGH determined the arcs of amplitude of the two northern sections of the Great Arc, in the years 1839–41; simultaneous observations were taken to eighteen pairs of stars, situated half to the north and half to the south of the zeniths of the two stations, and the same stars were observed at both stations in order to eliminate errors of star's place.

Astronomical observations were then suspended for more than 20 years, during which the strength of the department was wholly devoted to the triangulation. In 1863 two astronomical parties were formed for the purpose of making the recent triangulation subservient to geodesy by the measurement not only of additional meridional arcs, but of longitudinal arcs also, which the recent completion of electric telegraph lines all over the country had rendered possible.

The two astronomical circles being very heavy and difficult to carry about, two new zenith sectors were designed by Colonel STRANGE—a retired officer of this Survey—and were constructed under his superintendence for observing latitudes. Full descriptions and drawings of them are given in vol. 11, but they must be very briefly described here. Each instrument has a pair of sectors, of amplitude 55° with a radius of 18 inches, lying opposite each other in a single casting of brass; they were graduated by TROUGHTON and SIMM'S celebrated dividing engine, on strips of silver hammered into grooves in the brass. The sector plate is permanently attached to the telescope, at right angles, so that the two move together. The sectors are read by four microscopes which are attached to a fixed horizontal frame, in pairs, 20° apart, one pair for each sector. The extreme range of star observation is 17½° on either side of the zenith. Every star is observed in the two positions, "telescope east" and "telescope west" in rapid succession, the first observation being taken about 20 seconds before, and the last as much after, transit over the meridian.

One or other of the two astronomical circles and the two new zenith sectors was employed in all the latitude observations up to the field season of 1890–91. A zenith telescope was then introduced for the first time; it has no vertical arcs, but determines the latitudes by measuring the differences of the zenith distances of north and south stars of nearly equal zenith distance with a micrometer in the eye-piece, in accordance with what is known as the TALCOTT method of observation.

The latitude observations after the year 1862 were first taken at additional stations on the Great Arc at distances of about 1° apart; afterwards at groups of stations composed of one principal and a few secondary stations situated around and within a few miles of the principal, to ascertain whether there were any local variations of density under the surface of the ground to produce deflections of the plumb-line as had been met with on the plains of Moscow. Additional stations were introduced on and in the neighbourhood of the Himalayan Mountains to ascertain the influence of their attraction. Thus forty-eight latitude stations were added to the primary eleven on the Great Arc. Latitude observations have also been made on the following chains of triangles—the Rahun and the Gurhagarh series and their southern extension through the Khanpisura series to Mangalore, which form an important meridional combination to the west of the Great Arc; the Amua, Jabalpur, and the Madras meridional series, forming a similar combination to the east; the Jodhpore series and its extension southwards to Bombay; the Calcutta-Kurrachee longitudinal series, and the Bombay and the Bider longitudinal series.

Latitudes have been observed at 161 stations in all, for a considerable majority of which full details are given in vol. 11; for the remainder the details have not yet been published, but I am able to give a synopsis of the results which has been communicated to me in manuscript.

For the computation of the latitudes in vol. 11 a very careful investigation of the places of all the stars employed was made from twenty-one star catalogues, of which the five Greenwich Catalogues for 1840, 1845, 1860, 1864, and 1872, and Mädler's for 1850 were mainly depended upon. Whenever a star was found in these six catalogues its place was taken from those which gave results within 1″ of the mean; if at least three gave accordant results the mean was accepted; if the star only appeared in two of them, then other catalogues were employed. A catalogue was prepared for the computation which shows the value given by each authority, as well as that which was finally adopted.

There are 111 latitude stations in vol. 11; at 94 of these the observations were taken with one or other of the two astronomical circles by TROUGHTON and SIMMS, or the two zenith sectors by STRANGE; at five they were taken with one of these instruments and also with RAMSDEN's zenith sector, at five more with RAMSDEN's zenith sector only; in five cases they were taken with the 18-inch vertical circle of one of the 36-inch or the 24-inch theodolites; and in two cases, in the Himalayan Mountains, with the 10-inch vertical sectors of a 14-inch theodolite.

As it is a well known fact that some instruments have a tendency to exhibit more or less constant differences between latitudes deduced from north stars and those deduced from south stars, it has been generally a rule to observe as many north as south stars at each station. In the final reductions, even when this rule has not been followed, a value of the latitude is always deduced from the north stars and another from the south stars, and the arithmetical mean of the two is adopted for the final value.

Astronomical Circle No. 2, and STRANGE's Zenith Sector No. 2, have given results throughout which do not show constant differences between north and south stars. The two sister instruments have, however, shown marked constant differences, which must be here noticed, though it is believed that the errors of each are cancelled in the means.

Astronomical Circle No. 1 gives the following values of North—South :—

	″
In 1839–40, at Kaliánpúr	+ 1·00
,, 1840–41, at Damergida	− 1·69
,, 1855, at Kurrachee	+ 1·04
,, 1864, mean of five stations	+ 1·65
,, 1865, mean of six stations	+ 1·12
,, 1866, January and February, mean of four stations	+ 0·72
,, 1866, October, to 1867, October, four stations	+ 5·95

The instrument must have met with some mishap between February and October, 1866, but the nature of the mishap has baffled all search. As a check the latitude was re-observed at two of the stations of the last group with Astronomical Circle No. 2, and results were obtained very closely coinciding with the means of north and south stars originally deduced, showing that the latitudes obtained with the instrument are quite reliable. It was, however, set aside, and has not been used since 1867.

STRANGE'S Zenith Sector No. 1 has invariably given greater latitudes by south than by north stars; the difference increases systematically with the zenith distance, rising from 1″·9 for a mean zenith distance of 4° 16′ to 5″·4 for 15°. The latitudes by south stars being greatest, the zenith distances are too great; this suggests that the sector may have contracted after having been graduated; the graduation is known to have been performed when the sector was a new casting, which may have undergone some change before finally settling down. The sister sector is believed not to have been graduated until some time after the casting. Whatever the cause, it has been recently shown that the mean of north and south stars is free from error, for the instrument has been employed at three stations both as a zenith sector and as a zenith telescope, and the latitudes obtained from the sectors differed by less than 0″·1 from the values with the telescope alone.

The following tables give the results of the latitude observations at each station, together with the longitude of the station and its height above the sea-level, the instrument employed, and the year of observation. The first table has been wholly abstracted from vol. 11; the second gives results which have not yet been published. A majority of the stations are principal stations of the great triangulation; but in certain localities the latitude has been observed at several points around a principal station whose positions were fixed by secondary triangulation; such points are practically as valuable for geodetic purposes as the principal stations, but they are entered as secondaries in the tables.

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Longitude of station.	Height in feet above sea-level.	Instrument employed.	Year.	Name of station.	Astronomical latitude.	Number of stars. N.	Number of stars. S.	Seconds of observed latitude. By North stars.	Seconds of observed latitude. By South stars.
° ′ 77 37	1557	Z. S. No. 2 . . .	1872	Akampalle. *Secy.*	° ′ ″ ″ 17 10 50·39 ± ·04	20	15	″ ″ 50·26 ± ·06	″ ″ 50·51 ± ·06
77 44	3140	A. C. No. 1 . . .	1864	Amsot	30 22 16·02 ·07	40	54	16·81 ·10	15·30 ·07
75 2	1532	A. C. No. 2 . . .	1869	Aramlia	24 25 2·66 ·05	35	35	2·46 ·07	2·83 ·07
77 47	55	Z. S. No. 2 . . .	1871	Arasákulam	8 13 41·96 ·04	27	14	42·04 ·05	41·87 ·06
77 39	1128	,, . . .	1872	Badgaon	20 44 15·54 ·04	18	19	15·48 ·05	15·60 ·06
77 3	1447	,, . . .	1871	Bandúr. *Secy.*	14 57 44·41 ·04	16	22	44·39 ·07	44·42 ·04
77 42	3016	,, . . .	1870	Bangalore Base-Line } N.E. end	13 4 53·17 ·04	22	22	53·28 ·05	53·15 ·06
77 37	3126	,, . . .	1869–70	Bangalore Base-Line } S.W. end	13 0 36·12 ·03	58	28	36·10 ·04	36·14 ·05
78 3	7433	Theod. 18-inch vert. cir.	1851	Banog	30 28 4·18 ·14	9	9	3·77 ·20	4·60 ·21
76 11	1870	A. C. No. 1 . . .	1866	Bánksho	26 50 2·37 ·07	32	32	2·84 ·09	1·91 ·11
78 5	346	Z. S. No. 2 . . .	1870	Black Station. *Secy.*	9 31 4·22 ·05	21	17	4·15 ·06	4·28 ·07
77 30	2005	Z. S. R.	1806	Bommasundra. *Secy.*	13 59 42·63 ·22	4	8	42·95 ·23	42·31 ·37
88 24	25	A. C. No. 2 . . .	1864–65	Calcutta. *Secy.*	22 32 55·58 ·08	43	43	55·13 ·13	56·04 ·09
85 29	2817	,, . . .	1865–66	Chendwar	23 57 16·82 ·06	17	17	16·24 ·07	17·39 ·09
77 14	1516	Z. S. No. 2 . . .	1871	Chikalgurki. *Secy.*	14 59 5·16 ·04	16	22	5·39 ·05	4·92 ·05
77 43	1944	Z. S. R.	1815	Damargida { 1st visit	18 3 14·62 ·14	4	9	16·20 ·09	13·04 ·26
,,	,,	A. C. No. 1 . . .	1840–41	Damargida { 2nd visit	18 3 15·22 ·06	16	16	14·41 ·08	16·03 ·09
				Combination	18 3 15·13 ·06				
77 41	767	A. C. No. 1 . . .	1864	Datairi	28 43 58·67 ·07	46	46	59·33 ·11	58·01 ·09
78 6	2289	Theod. 18-inch v. c.	1852	Dehra	30 19 19·56 ·18	8	8	19·13 ·25	20·03 ·23
75 35	1727	A. C. No. 2 . . .	1869	Deo Dongri	23 26 43·17 ·05	36	36	42·96 ·08	43·35 ·06
77 44	1593	Z. S. No. 2 . . .	1872	Devanúr. *Secy.*	17 10 56·88 ·04	20	16	56·78 ·07	56·98 ·05
77 44	1332	,, . . .	1871	Devaragat. *Secy*	16 6 31·98 ·05	18	21	31·98 ·06	31·98 ·07
80 21	439	,, . . .	1885	Dewarsán	26 15 58·32 ·08	17	16	58·31 ·13	58·33 ·10
75 15	1553	A. C. No. 2 . . .	1870	Dhaigaon	19 30 30·82 ·05	36	36	30·66 ·06	30·96 ·07
77 44	1135	Z. S. No. 2 . . .	1872	Dhánura. *Secy.*	20 44 3·35 ·04	18	17	3·12 ·06	3·57 ·06
77 40	3003	Z. S. R.	1805–6	Doddagunta. *Secy.* { 1st visit .	12 59 50·89 ·18	9	7	51·54 ·13	50·23 ·33
,,	,,	Z. S. No. 2 . . .	1870	Doddagunta. *Secy.* { 2nd visit .	12 59 52·14 ·04	23	23	52·24 ·05	52·04 ·06
				Combination	12 59 52·08 ·04				

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Longitude of station.	Height in feet above sea-level.	Instrument employed.	Year.	Name of station.	Astronomical latitude.	Number of stars.		Seconds of observed latitude.	
						N.	S.	By North stars.	By South stars.
° ′ 77 35	1140	Z. S. No. 2	1872	Dotra. *Secy.*	° ′ ″ ″ 20 41 22·25 ± ·04	10	13	″ ″ 22·08 ± ·05	″ ″ 22·43 ± ·06
80 42	429	”	1885	Etora	26 54 22·63 ·09	14	15	22·81 ·15	22·44 ·10
75 4	1204	A. C. No. 2	1868	Garinda	27 55 30·05 ·04	35	35	29·25 ·07	30·83 ·05
77 48	1225	Z. S. No. 2	1872	Gattináráyantippa. *Secy.*	16 7 48·95 ·04	18	20	48·86 ·07	49·04 ·04
74 43	7752	Theod. 10-inch sectors	1860	Gogipatri. *Secy.*	33 51 46·93 ·68	4	4	57·33 ·98	36·43 ·93
76 7	1360	A. C. No. 1	1865	Gurária	24 25 31·98 ·07	30	30	32·41 ·11	31·57 ·09
82 20	2083	A. C. No. 2	1866	Gurwáni	24 1 28·93 ·09	15	15	28·69 ·15	29·16 ·11
77 43	1335	Z. S. No. 2	1872	Halda. *Secy.*	19 9 24·41 ·05	15	19	24·17 ·08	24·65 ·05
75 36	1816	A. C. No. 2	1869	Harnása { 1st visit	22 47 26·56 ·10	6	6	26·55 ·14	26·57 ·15
”	”	”	”	2nd visit	22 47 26·84 ·04	36	36	26·44 ·06	27·21 ·06
				Combination	22 47 26·80 ·04				
75 13	2775	Z. S. No. 1	1872	Honnavalli	14 16 30·76 ·04	15	16	30·75 ·06	30·76 ·05
77 8	1579	Z. S. No. 2	1871	Honnúr	14 55 22·20 ·04	11	11	22·36 ·05	22·03 ·05
84 24	1378	A. C. No. 2	1866	Huriláong	24 2 16·74 ·09	25	25	16·43 ·08	17·07 ·15
76 9	874	”	1867	Isanpur	30 38 16·03 ·06	33	34	16·19 ·09	15·83 ·07
80 31	536	Z. S. No. 2	1884	Jarúra	27 59 50·22 ·09	13	13	51·18 ·13	49·25 ·13
74 21	1967	A. C. No. 2	1868	Jetgarh	26 18 8·02 ·04	35	35	7·94 ·06	8·09 ·06
77 42	828	”	1839–40	Kaliána	29 30 47·98 ·08	18	18	47·80 ·11	48·15 ·12
77 42	1765	Z. S. R.	1824–25	Kaliánpur { 1st visit	24 7 10·76 ·13	6	11	11·08 ·19	10·44 ·18
”	”	A. C. No. 1	1839–40	2nd visit	24 7 10·92 ·08	18	18	11·42 ·11	10·42 ·11
”	”	A. C. No. 2	1840–41	3rd visit	24 7 11·18 ·07	16	16	11·68 ·09	10·69 ·10
”	”	A. C. No. 1	1865	4th visit	24 7 11·44 ·07	40	40	12·03 ·09	10·84 ·11
”	”	”	1865	5th visit	24 7 10·90 ·07	31	31	11·18 ·09	10·63 ·10
				Combination	24 7 11·10 ·03				

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Longitude of station.	Height in feet above sea-level.	Instrument employed.	Year.	Name of station.	Astronomical latitude.	Number of stars.		Seconds of observed latitude.	
						N.	S.	By North stars.	By South stars.
° ′ 80 28	416	Z. S. No. 2	1885	Kánákhera	° ′ ″ ″ 25 51 25·97 ± ·08	16	16	″ ″ 25·88 ± ·14	″ ″ 26·06 ± ·09
76 10	1652	A. C. No. 1	1866	Kánkra	25 37 58·75 ·06	31	31	59·11 ·09	58·40 ·07
67 4	35	”	1855	Karáchi (Kurrachee)	24 49 50·14 ·07	17	17	50·66 ·07	49·62 ·12
81 18	1966	A. C. No. 2	1866–67	Karára	24 4 42·20 ·08	25	25	41·85 ·09	42·57 ·12
80 2	1625	Theod. 18-inch v. c.	1867	Karaundi	23 10 45·07 ·29	11	11	46·23 ·50	43·90 ·28
77 43	878	Z. S. No. 2	1870	Kátpálaiyam. *Secy.*	10 56 36·66 ·04	26	24	36·60 ·05	36·71 ·05
75 21	1951	A. C. No. 2	1870	Kem	18 10 45·68 ·05	34	34	45·45 ·07	45·87 ·08
77 43	1487	A. C. No. 1	1864–65	Kesri	25 46 41·57 ·06	48	48	42·15 ·09	41·00 ·09
74 50	1393	A. C. No. 2	1868	Khámor	25 45 11·00 ·04	37	37	10·69 ·05	11·28 ·06
74 49	2751	”	1870	Khánpisura	18 45 22·63 ·06	28	31	22·53 ·12	22·73 ·12
75 3	731	A. C. No. 1	1867	Khimúána	30 22 11·74 ·10	25	25	14·80 ·12	8·67 ·16
77 41	1906	Z. S. No. 2	1872	Kodangal	17 7 53·74 ·04	21	17	53·45 ·07	54·02 ·05
75 1	2527	Z. S. No. 1	1872	Koramúr	14 8 1·71 ·05	15	16	1·72 ·06	1·69 ·07
77 44	175	Z. S. No. 2	1871	Kudankulam	8 10 23·41 ·03	27	27	23·52 ·05	23·30 ·04
75 17	2147	Z. S. No. 1	1871	Kundgol 1st visit	15 15 14·34 ·05	23	24	14·35 ·07	14·33 ·07
”	”	”	1872	2nd visit	15 15 14·58 ·05	16	16	14·54 ·07	14·62 ·07
				Combination	15 15 14·46 ·04				
78 3	347	Z. S. No. 2	1870	Kutipárai	9 28 47·09 ·03	28	29	47·19 ·04	46·99 ·03
77 45	1875	A. C. No. 1	1865	Ládi	23 8 39·10 ·05	63	63	40·29 ·07	37·91 ·08
77 45	1815	Z. S. No. 2	1872	Linganapalle. *Secy.*	17 7 13·40 ·04	21	16	13·29 ·06	13·50 ·05
80 12	1923	A. C. No. 2	1866	Lora	23 29 46·30 ·05	25	25	45·93 ·07	46·70 ·08
74 29	2613	Z. S. No. 1	1872	Majala	16 46 55·45 ·05	22	22	55·35 ·07	55·54 ·08
87 8	970	A. C. No. 2	1865	Malúneha	23 54 29·64 ·08	16	16	31·41 ·11	27·86 ·11
77 46	1294	Z. S. No. 2	1872	Mandála. *Secy.*	19 2 42·84 ·06	16	19	42·73 ·10	42·94 ·07
74 53	186	Z. S. No. 1	1872	Mangalore	12 52 17·76 ·04	22	22	17·82 ·06	17·69 ·05
74 50	2582	”	1872	Mávinhúnda	16 25 4·47 ·05	19	19	4·45 ·07	4·49 ·08
73 27	7000	Theod. 18-inch v. c.	1858	Murree. *Secy.*	33 54 37·35 ·28	11	11	39·49 ·33	35·23 ·45
78 7	6937	A. C. No. 1	1866	Mussooree 1st visit	30 27 4·45 ·18	6	6	7·23 ·34	1·62 ·12
”	”	”	1867	2nd visit	30 27 3·35 ·25	15	18	6·44 ·42	0·26 ·26
”	”	A. C. No. 2	1867	3rd visit	30 27 4·27 ·06	6	6	4·58 ·08	3·90 ·10
				Combination	30 27 4·24 ·06				

DETERMINATIONS of Latitude, published in Volume 11—(continued).

Longitude of station.	Height in feet above sea-level.	Instrument employed.	Year.	Name of station.	Astronomical latitude.	Number of stars.		Seconds of observed latitude.	
						N.	S.	By North stars.	By South stars.
77° 39′	1169	Z. S. R.	1811	Namthabad. *Secy.*	15° 5′ 51″·75 ± ″·11	8	11	51″·93 ± ″·15	51″·58 ± ″·15
75 6	2445	Z. S. No. 1	1872	Navalúr	15 25 28·48 ·04	16	16	28·46 ·07	28·49 ·05
77 14	1565	Z. S. No. 2	1871	Nimbágal. *Secy.*	14 51 56·14 ·06	18	20	56·12 ·10	56·16 ·07
80 32	486	,,	1885	Nimkár. *Secy.*	27 21 8·16 ·09	14	14	8·33 ·14	7·99 ·11
77 41	710	A. C. No. 1	1864	Noh	27 50 53·13 ·07	47	47	54·20 ·09	52·04 ·11
77 43	929	,,	1863–64	Nojli	29 53 14·12 ·08	46	46	15·00 ·09	13·23 ·12
77 40	970	Z. S. R.	1806	Pachapálaiyam. *Secy.* 1st visit	10 59 40·70 ·16	13	7	41·39 ·21	40·00 ·24
,,	,,	Z. S. No. 2	1870	Pachapálaiyam. *Secy.* 2nd visit	10 59 40·91 ·03	25	23	40·84 ·03	40·98 ·05
				Combination	10 59 40·90 ·03				
77 44	1641	A. C. No. 1	1865	Pahárgarh	24 56 6·47 ·05	45	45	6·78 ·07	6·17 ·08
78 8	217	Z. S. No. 2	1870–71	Pandalagudi. *Secy.*	9 23 30·55 ·04	21	17	30·84 ·05	30·26 ·06
77 19	3022	Z. S. R.	1805	Pávagada	14 6 18·80 ·24	3	3	19·60 ·10	18·00 ·47
80 47	481	Z. S. No. 2	1885	Pavia	25 27 21·18 ·06	18	17	21·19 ·09	21·17 ·08
77 47	1090	,,	1872	Peddapád. *Secy.*	16 17 14·13 ·03	13	15	13·90 ·05	14·35 ·04
77 39	1869	,,	1872	Pialmudi. *Secy.*	17 4 1·06 ·05	8	2	1·00 ·07	1·11 ·08
74 32	8323	Theod. 10-inch sectors	1860	Poshkar	34 2 3·78 ·46	5	7	14·16 ·75	1 53·40 ·52
81 0	993	Z. S. No. 2	1885	Potenda	24 37 24·71 ·08	15	16	24·73 ·12	24·69 ·10
77 40	48	Z. S. R.	1809	Punnæ 1st visit	8 9 30·18 ·13	14	4	30·08 ·14	30·28 ·23
,,	,,	Z. S. No. 2	1871	Punnæ 2nd visit	8 9 29·66 ·04	27	15	29·69 ·04	29·63 ·08
				Combination	8 9 29·70 ·04				

DETERMINATIONS of Latitude, published in Volume 11—(continued).

Longitude of station.	Height in feet above sea-level.	Instrument employed.	Year.	Name of station.	Astronomical latitude.	Number of stars.		Seconds of observed latitude.	
						N.	S.	By North stars.	By South stars.
° ′ 77 45	167	Z. S. No. 2 . . .	1871	Rádhápuram	8 17 1·75 ± ·05	25	13	″ ″ 1·77 ± ·05	″ ″ 1·73 ± ·09
76 9	785	A. C. No. 1 . . .	1866	Rákhi	29 17 20·76 ·07	32	32	21·25 ·09	20·26 ·11
67 3	50	Theod. 18-inch v. c.	1853	Rámbágh	24 51 20·58 ·32	6	6	23·57 ·54	17·63 ·36
75 3	951	A. C. No. 2 . . .	1868	Rám Thal	28 29 38·81 ·05	35	35	38·21 ·08	39·39 ·07
80 31	541	Z. S. No. 2 . . .	1884–85	Rámuápur	28 22 0·10 ·09	14	14	0·59 ·12	21 59·61 ·14
79 28	1184	A. C. No. 2 . . .	1867	Rangír	24 0 19·28 ·06	25	25	19·08 ·08	19·59 ·09
77 38	1046	Z. S. No. 2 . . .	1872	Rángrai	20 48 7·16 ·05	17	16	6·89 ·09	7·43 ·06
74 19	1542	A. C. No. 2 . . .	1868	Rewat	26 53 54·74 ·05	35	35	54·54 ·07	54·89 ·07
75 5	779	A. C. No. 1 . . .	1867	Sangatpur { 1st visit	31 17 35·46 ·08	21	21	37·94 ·12	33·00 ·12
,,	,,	A. C. No. 2 . . .	1868	Sangatpur { 2nd visit	31 17 35·37 ·08	21	21	35·53 ·11	35·20 ·11
				Combination. . . .	31 17 35·42 ·06				
75 6	697	A. C. No. 1 . . .	1867	Sawaipur	29 39 13·13 ·15	25	25	16·62 ·18	9·62 ·25
75 8	830	,, . . .	1866–67	Sháhpur	32 1 34·23 ·12	18	22	37·25 ·20	31·21 ·13
77 59	333	Z. S. No. 2 . . .	1870	Shúlakarai. *Secy.*	9 32 15·53 ·04	21	19	15·75 ·06	15·31 ·05
77 41	1094	Z. S. R.	1823–24	Takalkhera. *Secy.*	21 5 50·17 ·18	15	9	50·82 ·12	49·52 ·35
77 40	1233	Z. S. No. 2 . . .	1872	Tâlegaon. *Secy.*	19 1 21·65 ·04	16	18	21·32 ·06	21·98 ·05
77 41	176	,, . . .	1871	Tanakarakulam	8 13 57·50 ·05	22	14	57·65 ·06	57·35 ·08
76 15	2050	A. C. No. 1 . . .	1866	Tásing	27 52 59·49 ·08	30	30	59·84 ·10	59·15 ·13
75 27	851	A. C. No. 2 . . .	1869–70	Thíkri	22 1 3·92 ·04	37	37	3·63 ·07	4·17 ·05
79 45	125	Z. S. R.	1808	Tiruvendipuram. *Secy.* . . .	11 44 43·40 ·22	4	4	43·70 ·30	43·10 ·31
77 37	1133	Z. S. No. 2 . . .	1872	Tonsalgutta. *Secy.*	16 18 2·36 ·03	17	17	2·16 ·06	2·55 ·03
77 37	1450	,, . . .	1871	Tuagat. *Secy.*	16 9 46·73 ·05	21	12	46·78 ·06	46·67 ·08
77 40	810	A. C. No. 1 . . .	1864	Usira	26 57 0·50 ·07	46	46	1·25 ·09	56 59·77 ·10
75 14	1125	A. C. No. 2 . . .	1870	Valvádi	20 44 21·27 ·05	36	36	20·91 ·07	21·60 ·08
77 49	90	Z. S. No. 2 . . .	1871	Vijayápati	8 12 10·67 ·04	27	15	10·79 ·06	10·54 ·05
77 37	1439	,, . . .	1872	Voi. *Secy.*	19 7 14·69 ·04	16	19	14·41 ·06	14·06 ·06
77 1	1698	,, . . .	1871	Yerragúnta. *Secy.*	14 48 27·31 ·04	18	21	27·47 ·06	27·15 ·05
77 53	617	,, . . .	1870	Yettimalai	11 3 52·10 ·04	28	28	52·21 ·05	51·98 ·07

DETERMINATIONS of Latitude, hitherto unpublished.

Longitude of station.	Height above sea.	Instrument employed.	Year.	Name of station.	Astronomical latitude.	Pairs of stars.
77° 2′	2274	Zenith Telescope	1892–93	Achola	18° 14′ 44″·91 ± ″·08	39
78 3	550	Z. S. No. 1	1893–94	Agra	27 9 34·52 ·06	43
74 55	770	,,	,,	Amritsar	31 38 2·53 ·08	19
79 39	1463	Z. S. No. 2	1888–89	Ankora	19 24 26·68 ·05	29
79 49	1490	,,	,,	Bhimsain	20 57 28·60 ·05	34
72 42	774	Z. S. No. 1	1893–94	Bithnok	27 53 25·04 ·05	45
78 34	1600	Zenith Telescope	,,	Bolarum	17 30 7·50 ·06	47
79 50	1370	Z. S. No. 2	1888–89	Bolikonda	17 42 29·12 ·04	35
72 51	75	Zenith Telescope	1892–93	Bombay (Colaba Observatory)	18 53 39·16 ·06	39
79 44	983	Z. S. No. 2	1888–89	Burgpaili	18 54 3·51 ·05	31
72 38	1065	Z. S. No. 1	1892–93	Chamu	26 39 53·49 ·05	24
72 35	953	,,	,,	Chaniána	24 6 25·45 ·04	25
80 0	300	,,	1890–91	Dánapa	15 55 59·70 ·05	45
79 57	195	Zenith Telescope	,,	Darutippa	15 0 33·52 ·04	61
78 1	1958	,,	1892–93	Dehra Dun Base Line, East end	30 16 37·29 ·08	19
70 59	490	Z. S. No. 1	1893–94	Debra Din Panah .	30 33 59·62 ·09	25
72 14	443	,,	1892–93	Deesa	24 15 21·18 ·05	24
74 12	2939	Zenith Telescope	,,	Dhauleshvar . . .	18 25 42·91 ·05	57
80 8	245	Z. S. No. 2	1888–89	Dhúlipalla	16 25 53·48 ·06	28
79 35	967	,,	,,	Díwai	19 49 26·90 ·05	30
80 4	292	Zenith Telescope	1890–91	Gudali	14 1 10·65 ·06	45
72 48	5200	Z. S. No. 1	1892–93	Guru Sikkar . . .	24 37 47·80 ·04	21
72 34	772	,,	,,	Jambo	27 16 31·99 ·04	25
75 46	2610	Zenith Telescope	,,	Kanheri	18 29 21·94 ·05	61
74 49	2751	,,	,,	Khanpisura* . . .	18 45 22·57 ·05	57
72 42	603	Z. S. No. 1	1893–94	Khirsar	28 29 43·77 ·06	48
79 48	458	Zenith Telescope	1890–91	Kistáma	14 27 12·30 ·05	45
72 2	470	Z. S. No. 1	1893–94	Ladimsir	29 21 39·84 ·06	62
80 11	1400	Z. S. No. 2	1886–87	Lingmára	21 42 55·41 ·05	33
73 35	4121	Zenith Telescope	1892–93	Mandvi	18 37 48·09 ·05	53
80 17	54	,,	1890–91	Madras Observatory	13 4 9·88 ·07	39
80 14	250	,,	,,	Madras, St. Thomas' Mount	13 0 20·66 ·08	39
71 29	420	Z. S. No. 1	1893–94	Mooltan	30 10 56·23 ·07	47
79 46	1144	Z. S. No. 2	1888–89	Niálamuri	17 1 25·96 ·04	35
76 19	2289	Zenith Telescope	1892–93	Nitali	18 17 2·80 ·05	61
80 5	250	,,	1890–91	Ongole	15 29 52·90 ·05	47
81 15	684	,,	1893–94	Parampúdi . . .	17 12 32·62 ·06	43
78 38	2090	,,	,,	Pirmulo	17 52 58·48 ·07	48
78 7	2900	,,	1892–93	Rajpúr	30 23 9·33 ·11	10
79 47	1070	Z. S. No. 2	1886–87	Rajuli	20 12 51·29 ·04	40
79 34	1779	,,	1888–89	Rámgír	18 35 26·96 ·04	30
72 37	846	Z. S. No. 1	1892–93	Samdári	25 48 59·61 ·04	27
82 44	2142	Zenith Telescope	1893–94	Sanjib	17 31 12·32 ·07	41
80 5	1100	Z. S. No. 2	1886–87	Sarandi Pat . . .	22 12 50·71 ·05	25
80 59	714	Zenith Telescope	1893–94	Singáwáram . . .	17 45 8·69 ·07	44
80 22	1237	Z. S. No. 2	1886–87	Sítápár	21 24 43·87 ·05	32
72 48	250	Z. S. No. 1	1892–93	Sonáda	23 7 15·64 ·09	30
72 17	470	,,	1893–94	Telu	28 56 12·42 ·05	44
72 25	856	,,	1892–93	Thob	26 3 2·94 ·05	24
79 25	1664	Zenith Telescope	1893–94	Vánákonda . . .	17 36 0·30 ·06	46
83 22	500	,,	,,	Waltair	17 43 20·38 ·05	52

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* The latitude of Khanpisura obtained in 1870 being 18° 45′ 22″·63 (vol. 9), the value to be adopted is 18° 45′ 22″·60.

The results given in the two last tables have now to be arranged for the meridional and the longitudinal arcs to which they respectively appertain; also the geodetic latitudes must be set forth by the side of the astronomical, that the differences between the two sets of results may be exhibited.

Occasional combinations of results must be made in order to place the data in the most convenient form for future investigations; thus for all localities in which groups of stations have been observed, with a view to cancelling the effects of local attraction as much as possible, it is obviously necessary to employ the mean of all the astronomical and the mean of all the geodetic values, instead of the individual values; the hypothetical points thus introduced are as serviceable for geodetic investigations as the actual points of observation. And this process must be extended to wider regions, where there was no intentional grouping of stations; thus where several astronomical latitudes are forthcoming within a meridional belt of moderate breadth—say not much exceeding 1°, so as not to be sensibly influenced by the normal flattening between the equator and the poles—the mean of these will obviously be less affected by local attraction than the single values, and will therefore represent a truer value of the astronomical latitude; either this must be done or some of the astronomical data must be arbitrarily rejected. For example: on the Great Arc there are five stations between the parallels of 13° and 14° 6′; three of them are LAMBTON'S stations, viz., Doddagunta to the south showing northerly attraction, and Bommasundra and Pavagada to the north, showing southerly attraction; LAMBTON expressly stated his belief that the plumb-line was deflected in opposite directions by intermediate masses between the north and south stations; but in their investigations of the figure of the earth, both BESSEL and CLARKE have rejected the northern stations and employed the southern one only. Since then two southern stations have been added, at the ends of the Bangalore base-line near Doddagunta, which also show northerly attraction; and it now seems obviously necessary to accept the mean of all the stations or to reject them all, rather than to accept some and reject the others. Thus the principle of grouping has, whenever practicable, been carried through the following tabular presentation of astronomical and geodetic latitudes.

Separate tables have been prepared for the Great Arc, for the meridians of Madras, Mangalore, and Bombay; for the Calcutta and the Bombay and Bider longitudinal triangulations, and for the stations influenced by Himalayan attraction. As the latest astronomical value of the latitude of the station of origin, Kalianpur, has been found to be $0''\cdot16$ less than the value employed for the geodetic calculations, a correction of $-0''\cdot16$ has been applied to all the geodetic latitudes in the published volumes and the addenda to obtain the values given in these tables.

In the tables the geodetic values are those which were obtained with the elements of the Earth's figure, which EVEREST computed in 1830, and have ever since been employed in all the geodetic calculations of the Indian Survey. The quantities $A - G$ are the differences between them and the astronomical values. The quantities $A - G_c$ are those which would have been obtained if CLARKE's latest elements had been employed instead; they are shown in the table for Himalayan stations and also in subsequent tables.

The last column of these tables gives rough statements as to whether there is any apparent meridional attraction at any station, and if so, whether its direction is northerly or southerly, and its amount small or great. In most instances this information is supplied from the notes of the observers, otherwise it is estimated from the topographical details in the sheets of the Indian Atlas.

Comparisons of Astronomical with Geodetic Latitudes.

Great Arc Meridional Series.

Name of station.	Longitude.	Height in feet above sea.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A − G.	
Punnæ	77° 40′	48	8° 9′ 29″·70	8° 9′ 27″·63	+2″·07	All in plains within a few miles of the Indian Ocean. No attraction
Kudankulam	77 44	175	8 10 23·41	8 10 21·39	+2·02	
Vijayápati	77 49	90	8 12 10·67	8 12 8·18	+2·49	
Arasákulam	77 47	55	8 13 41·96	8 13 39·36	+2·60	
Tanakarakulam	77 41	176	8 13 57·50	8 13 55·23	+2·27	
Rádhápuram	77 45	167	8 17 1·75	8 16 59·28	+2·47	
Group 1	77 44	..	8 12 47·50	8 12 45·18	+2·32	
Pandalagudi	78 8	217	9 23 30·55	9 23 27·53	+3·02	No attraction
Kutiparai	78 3	347	9 28 47·09	9 28 44·71	+2·38	
Black Station	78 5	346	9 31 4·22	9 31 1·14	+3·08	
Shulakarai	77 59	333	9 32 15·53	9 32 13·12	+2·41	
Group 2	78 4	..	9 28 54·35	9 28 51·63	+2·72	
Kátpalaigam	77 43	878	10 56 36·66	10 56 35·81	+0·85	No attraction
Pachapalaiyam	77 40	970	10 59 40·90	10 59 39·72	+1·18	
Yettimalai	77 53	617	11 3 52·10	11 3 49·84	+2·26	
Group 3	77 45	..	11 0 3·22	11 0 1·79	+1·43	

GREAT Arc Meridional Series—(continued).

Name of station.	Longitude.	Height in feet above sea.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A − G.	
Doddagúnta	77° 40′	3003	12° 59′ 52″·08	12° 59′ 55″·60	−3″·52	Very uncertain; possibly slightly south
Bangalore base, S.W. end .	77 42	3016	13 0 36·12	13 0 40·75	−4·63	
Bangalore base, N.E. end .	77 37	3126	13 4 53·17	13 4 55·89	−2·72	Very slight and uncertain
Bommasundra	77 30	2005	13 59 42·63	13 59 36·18	+6·45	Probably northern
Pávagada	77 19	3022	14 6 18·80	14 6 15·23	+3·57	Slightly southern
Group 4	77 34	..	13 26 16·56	13 26 16·73	−0·17	
Yerragunta	77 1	1698	14 48 27·31	14 48 23·10	+4·21	Slightly southern
Nimbágal	77 14	1565	14 51 56·14	14 51 52·27	+3·87	None
Honnúr	77 8	1579	14 55 22·20	14 55 18·80	+3·40	Slightly southern
Bandúr	77 3	1447	14 57 44·41	14 57 42·16	+2·25	Slightly northern
Chikalgurki	77 14	1516	14 59 5·16	14 59 4·37	+0·79	Slightly northern
Namthabad	77 39	1169	15 5 51·75	15 5 52·24	−0·49	Slightly northern
Group 5	77 13	..	14 56 24·50	44 56 22·16	+2·34	
Devaragat	77 44	1332	16 6 31·98	16 6 37·11	−5·13	Slightly northern On plateau between rivers Tungabudra and Kistna
Gattináráyantippa . . .	77 48	1225	16 7 48·95	16 7 54·65	−5·70	Uncertain
Tuagat	77 37	1450	16 9 46·73	16 9 51·50	−4·77	None
Peddapád	77 47	1090	16 17 14·13	16 17 20·22	−6·09	None
Tonsalgutta	77 37	1133	16 18 2·36	16 18 6·75	−4·39	None
Group 6	77 43	..	16 11 52·83	16 11 58·05	−5·22	

GREAT Arc Meridional Series—(continued).

Name of station.	Longitude.	Height in feet above sea.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A—G.	
Pialmudi	77° 39′	1869	17° 4′ 1″·06	17° 4′ 5″·89	—4″·83	Somewhat northern
Linganapálle	77 45	1815	17 7 13·40	17 7 16·50	—3·10	Uncertain
Kodangal	77 41	1906	17 7 53·74	17 7 57·19	—3·45	Uncertain
Akampalle	77 37	1557	17 10 50·39	17 10 53·80	—3·41	Uncertain
Devanúr	77 44	1593	17 10 56·88	17 11 0·27	—3·39	None, or possibly southern
Group 7	77 41	..	17 8 11·09	17 8 14·73	—3·64	
Damargida	77 43	1944	18 3 15·13	18 3 17·19	—2·06	None
Achola	77 2	2274	18 14 44·91	18 14 47·96	—3·05	None
Group 8	77 23	..	18 9 0·02	18 9 2·58	—2·56	
Talegaon	77 40	1233	19 1 21·65	19 1 26·48	—4·83	Uncertain
Mandála	77 46	1294	19 2 42·84	19 2 48·08	—5·24	None
Voí	77 37	1439	19 7 14·69	19 7 19·73	—5·04	Somewhat northern
Halda	77 43	1335	19 9 24·41	19 9 29·22	—4·81	Uncertain
Group 9	77 42	..	19 5 10·90	19 5 15·88	—4·98	
Dotra	77 35	1140	20 41 22·25	20 41 28·75	—6·50	Uncertain
Dhanúra	77 44	1135	20 44 3·35	20 44 10·68	—7·33	Uncertain
Badgaon	77 39	1128	20 44 15·54	20 44 22·90	—7·36	Uncertain
Rángrai	77 38	1046	20 48 7·16	20 48 14·52	—7·36	None
Takalkhera	77 41	1094	21 5 50·17	21 5 56·60	—6·43	Decidedly (5″) northern
Group 10	77 39	..	20 48 43·69	20 48 50·69	—7·00	

GREAT Arc Meridional Series—(continued).

Name of station.	Longitude.	Height in feet above sea.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A − G.	
Ládi	77° 45'	1875	23° 8' 39"·10	23° 8' 43"·97	−4"·87	Uncertain
Kaliánpur	77 42	1765	24 7 11·10	24 7 11·10	..	None
Pahárgarh	77 44	1641	24 56 6·47	24 56 6·76	−0·29	Uncertain
Kesri	77 43	1487	25 46 41·57	25 46 35·65	+5·92	None
Group 11	77 44	..	25 21 24·02	25 21 21·21	+2·81	
Usira	77 40	810	26 57 0·50	26 57 6·06	−5·56	Slightly northern
Agra	78 3	550	27 9 34·52	27 9 39·77	−5·25	None
Group 12	77 52	..	27 3 17·51	27 3 22·92	−5·41	
Noh	77 41	710	27 50 53·13	27 50 52·92	+0·21	Slightly southern
Dataíri	77 41	767	28 43 58·67	28 44 4·33	−5·66	None
Group 13	77 41	..	28 17 25·90	28 17 28·63	−2·73	
Kaliána	77 42	828	29 30 47·98	29 30 54·54	−6·56	Possibly northern to Himalayas.

MADRAS Meridional Triangulation.

Stations.	Longitude.	Height in feet.	Latitude. Astronomical.	Latitude. Geodetic.	Latitude. A − G.	Eye estimate of direction and magnitude of meridional attraction.
Tíruvendipuram	79° 45′	125	11° 44′ 43″·40	11° 44′ 37″·53	+5″·87	No attraction
Madras, St. Thomas' Mount	80 14	250	13 0 20·66	13 0 14·63	+6·03	No attraction
Madras Observatory . .	80 17	54	13 4 9·88	13 4 4·00	+5·88	
Group 1	80 16	. .	13 2 15·27	13 2 9·31	+5·96	
Gudali	80 4	292	14 1 10·65	14 1 9·29	+1·36	Slight southerly
Kistama	79 48	458	14 27 12·30	14 27 14·40	−2·10	None
Group 2	79 56	. .	14 14 11·48	14 14 11·85	−0·37	
Darutippa	79 57	195	15 0 33·52	15 0 36·31	−2·79	None
Ongole	80 5	250	15 29 52·90	15 29 56·69	−3·79	Southern
Group 3	80 1	. .	15 15 13·21	15 15 16·50	−3·29	
Dánapa	80 0	300 ?	15 55 59·70	15 55 59·98	−0·28	None
Dhúlipala	80 8	245	16 25 53·48	16 25 56·59	−3·11	A little northern
Group 4	80 4	. .	16 10 56·59	16 10 58·29	−1·70	

Madras Meridional Triangulation—(continued).

Stations.	Longitude.	Height in feet.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A − G.	
Niálamari	79° 46′	1144	17° 1′ 25″·96	17° 1′ 33″·47	−7″·51	Slight northern
Bolikonda	79 50	1370	17 42 29·12	17 42 35·66	−6·54	Insignificant
Group 5	79 48	..	17 21 57·54	17 22 4·57	−7·03	
Ramgir	79 34	1779	18 35 26·96	18 35 25·96	+1·00	Some southern
Burgpaili	79 44	983	18 54 3·51	18 54 7·04	−3·53	Slight southern
Group 6	79 39	..	18 44 45·23	18 44 46·50	−1·27	
Ankora	79 39	1463	19 24 26·68	19 24 34·59	−7·91	Slight southern
Diwai	79 35	967	19 49 26·90	19 49 32·41	−5·51	None
Group 7	79 37	..	19 36 56·79	19 37 3·50	−6·71	
Rájuli	79 47	1070	20 12 51·29	20 12 55·29	−4·00	None
Bhimsain	79 49	1490	20 57 28·60	20 57 35·80	−7·20	1″ northerly
Group 8	79 48	..	20 35 9·95	20 35 15·55	−5·60	
Sitapár	80 22	1237	21 24 43·87	21 24 50·38	−6·51	Small northerly
Lingmara	80 11	1400	21 42 55·41	21 43 2·91	−7·50	Large northerly
Sarandi Pat	80 5	1100 ?	22 12 50·71	22 12 55·45	−4·74	Large northerly
Group 9	80 13	..	21 46 50·00	21 46 56·25	−6·25	

MADRAS Meridional Triangulation—(continued).

Stations.	Longitude.	Height in feet.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A − G.	
Karaundi	80° 2′	1625	23° 10′ 45″·07	23° 10′ 39″·86	+5″·21	Slight southerly
Lora	80 12	1923	23 29 46·30	23 29 41·37	+4·93	None
Group 10	80 7	..	23 20 15·69	23 20 10·62	+5·07	
Karára	81 18	1966	24 4 42·20	24 4 41·85	+0·35	Northerly
Potenda.	81 0	993	24 37 24·71	24 37 22·88	+1·83	If anything slight southerly
Group 11	81 9	..	24 21 3·46	24 21 2·37	+1·09	
Pavia	80 47	481	25 27 21·18	25 27 17·23	+3·95	Slight southerly
Kanakhera	80 28	416	25 51 25·97	25 51 20·79	+5·18	None
Dewarsán	80 21	439	26 15 58·32	26 15 52·73	+5·59	Slight northerly
Group 12	80 32	..	25 51 35·16	25 51 30·25	+4·91	
Etora	80 42	429	26 54 22·63	26 54 17·69	+4·94	None
Nimkar	80 32	486	27 21 8·16	27 21 7·93	+0·23	None
Group 13	80 37	..	27 7 45·40	27 7 42·81	+2·59	
Jarúra	80 31	536	27 59 50·22	27 59 55·78	−5·56	Possibly slight Himalayan attraction

MANGALORE Meridional Triangulation, combining the Rahun and the Gurhagarh Meridional Series.

Stations.	Longitude.	Height in feet.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A—G.	
Mangalore	74° 53′	186	12° 52′ 17″·76	12° 52′ 14″·60	+3″·16	Slightly northern
Koramur	75 1	2527	14 8 1·71	14 8 6·43	−4·72	Northern
Honnavalli	75 13	2775	14 16 30·76	14 16 32·30	−1·54	Southern
Group 1	75 7	..	14 12 16·24	14 12 19·37	−3·13	
Kundgol	75 17	2147	15 15 14·46	15 15 15·12	−0·66	Slightly southern
Navalúr	75 6	2445	15 25 28·48	15 25 31·01	−2·53	Slightly southern
Group 2	75 11	..	15 20 21·47	15 20 23·07	−1·60	
Mavinhúnda	74 50	2582	16 25 4·47	16 25 4·03	+0·44	None
Majála	74 29	2613	16 46 55·45	16 46 56·66	−1·21	Slightly southern
Group 3	74 40	..	16 35 59·96	16 36 0·35	−0·39	
Kem	75 21	1951	18 10 45·68	18 10 48·74	−3·06	None
Khanpisúra	74 49	2751	18 45 22·60	18 45 30·49	−7·89	Slightly northern
Group 4	75 5	..	18 28 4·14	18 28 9·62	−5·48	

MANGALORE Meridional Triangulation, combining the Rahun and the Gurhagarh Meridional Series—(continued).

Stations.	Longitude.	Height in feet.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A − G.	
Dhaigaon	75° 15′	1553	19° 30′ 30″·82	19° 30′ 34″·88	−4″·06	Slightly northern
Valvádi	75 14	1125	20 44 21·27	20 44 27·57	−6·30	Very slightly northern
Group 5	75 15	..	20 7 26·05	20 7 31·23	−5·18	
Thikri	75 27	851	22 1 3·92	22 1 2·61	+1·31	Slightly southern
Harnása	75 36	1816	22 47 26·80	22 47 29·75	−2·95	Slightly southern
Deo Dongri	75 35	1727	23 26 43·17	23 26 47·63	−4·46	Slightly southern
Group 6	75 33	..	22 45 4·63	22 45 6·66	−2·03	
Gurária	76 7	1360	24 25 31·98	24 25 32·30	−0·32	Inappreciable
Aramlia	75 2	1532	24 25 2·66	24 25 7·11	−4·45	Slightly northern
Group 7	75 35	..	24 25 17·32	24 25 19·71	−2·39	
Kánkra	76 10	1652	25 37 58·75	25 37 59·37	−0·62	Inappreciable
Khamor	74 50	1393	25 45 11·00	25 45 14·85	−3·85	None
Group 8	75 30	..	25 41 34·87	25 41 37·11	−2·24	
Jetgarh	74 21	1967	26 18 8·02	26 18 6·23	+1·79	Southern
Banksho	76 11	1870	26 50 2·37	26 50 7·73	−5·36	Decided northern
Rewat	74 19	1542	26 53 54·74	26 53 53·82	+0·92	None
Group 9	74 57	..	26 40 41·71	26 40 42·59	−0·88	

MANGALORE Meridional Triangulation, combining the Rahun and the Gurhagarh Meridional Series—(continued).

Station.	Longitude.	Height in feet.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A − G.	
Tásing	76° 15′	2050	27° 52′ 59″·49	27° 52′ 59″·31	+0″·18	None
Garinda	75 4	1204	27 55 30·05	27 55 30·39	−0·34	None
Ram Thal	75 3	951	28 29 38·81	28 29 39·11	−0·30	None
Group 10	75 27	. .	28 6 2·78	28 6 2·93	−0·15	
Rákhi	76 9	785	29 17 20·76	29 17 21·12	−0·36	None
Sawaipur	75 6	697	29 39 13·13	29 39 13·80	−0·67	None
Group 11	75 38	. .	29 28 16·94	29 28 17·46	−0·52	
Khimuána	75 3	731	30 22 11·74	30 22 14·66	−2·92	None
Isanpúr	76 9	874	30 38 16·03	30 38 19·85	−3·82	None
Group 12	75 36	. .	30 30 13·89	30 30 17·26	−3·37	
Sangatpúr	75 5	779	31 17 35·42	31 17 34·27	+1·15	None
Amritsar	74 55	770	31 38 2·53	31 37 58·56	+3·97	Possibly distant Himalayan
Shahpur.	75 8	830	32 1 34·23	32 1 33·61	+0·62	
Group 13	75 3	. .	31 39 4·06	31 39 2·15	+1·91	

COMPARISONS of Astronomical with Geodetic Latitudes.

Bombay Meridional Triangulation.

Name of Station.	Longitude.	Height above sea level.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A − G.	
Bombay (Colaba Observatory)	72° 51′	75	18° 53′ 39″·16	18° 53′ 49″·32	−10″·16	None
Sonáda	72 48	250	23 7 15·64	23 7 19·73	− 4·09	None
Chaniána	72 35	953	24 6 25·45	24 6 36·48	−11·03	Northern
Deesa	72 14	443	24 15 21·18	24 15 29·19	− 8·01	Northern
Gúrú Sikkar	72 48	5200	24 37 47·80	24 37 50·80	− 3·00	Uncertain, but the station is situated almost over the centre of gravity of Mount Abu
Group 1	72 32	..	24 19 51·48	24 19 58·82	− 7·34	
Samdari	72 37	846	25 48 59·61	25 48 59·39	+ 0·22	Very slight if any
Thob	72 25	856	26 3 2·94	26 3 5·69	− 2·75	None
Chamu	72 38	1065	26 39 53·49	26 39 52·58	+ 0·91	None
Group 2	72 33	..	26 10 38·68	26 10 39·22	− 0·54	
Jambo	72 34	772	27 16 31·99	27 16 28·72	+ 3·27	None
Bithnok	72 42	774	27 53 25·04	27 53 21·87	+ 3·17	None
Group 3	72 38	..	27 34 58·52	27 34 55·30	+ 3·22	
Khirsar	72 42	603	28 29 43·77	28 29 40·75	+ 3·02	None
Telu	72 17	470	28 56 12·42	28 56 11·18	+ 1·24	None
Ladimsir	72 2	470	29 21 39·84	29 21 41·42	− 1·58	None
Group 4	72 20	..	28 55 52·01	28 55 51·12	+ 0·89	
Mooltan	71 29	420	30 10 56·23	30 10 58·54	− 2·31	None
Dera Din Panáh	70 59	490	30 33 59·62	30 34 1·71	− 2·09	Slight northerly
Group 5	71 14	..	30 22 27·93	30 22 30·13	− 2·20	

Comparisons of Astronomical with Geodetic Latitudes—(continued).

Calcutta—Kurrachee Longitudinal Series, omitting Stations which have already been included in the Meridional Arcs.

Name of station.	Longitude.	Height in feet above sea.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A – G.	
Calcutta	88° 24′	25	22° 32′ 55″·58	22° 32′ 54″·51	+1″·07	None
Malúncha	87 8	970	23 54 29·64	23 54 28·86	+0·78	Uncertain
Chendwar	85 29	2817	23 57 16·82	23 57 13·59	+3·23	Southerly
Rangír	79 28	1184	24 0 19·28	24 0 20·21	–0·93	None
Gurwáni	82 20	2083	24 1 28·93	24 1 25·55	+3·38	Southerly
Kurrachee	67 4	35	24 49 50·14	24 49 50·09	+0·05	None
Rambagh	67 3	50	24 51 20·58	24 51 21·28	–0·70	None
Group	79 34	..	24 1 5·85	24 1 4·87	+0·98	

The station of Hurilaong is omitted from the above list; it shows 10″·91 southerly attraction; it was originally chosen as a latitude station merely because it happened to be the station of origin of the Hurilaong meridional series; but it is obviously unsuited for a latitude station, as the country to the North is described as "an extended plain, while the country to the South is exceedingly hilly."

Bombay Longitudinal Series, omitting Stations which have been already included in the Meridional Arcs.

Name of station.	Longitude.	Height in feet above sea.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A – G.	
Mándvi	73° 35′	4121	18° 37′ 48″·09	18° 37′ 50″·95	–2″·86	Uncertain, possibly southern
Dhauleshvar	74 12	2939	18 25 42·91	18 25 41·48	+1·43	Southerly
Kanheri	75 46	2610	18 29 21·94	18 29 30·59	–8·65	Possibly some northern
Nitáli	76 19	2289	18 17 2·80	18 17 7·00	–4·20	None
Group	74 58	..	18 27 28·94	18 27 32·51	–3·57	

COMPARISONS of Astronomical with Geodetic Latitudes—(concluded).

BIDER Longitudinal Series, omitting Stations which have already been included in the Meridional Arcs.

Name of station.	Longitude.	Height in feet above sea.	Latitude.			Eye estimate of direction and magnitude of meridional attraction.
			Astronomical.	Geodetic.	A − G	
Bolarum	78° 34′	1600	17° 30′ 7″·50	17° 30′ 13″·25	−5″·75	Very slight northern
Pirmulo	78 38	2090	17 52 58·48	17 53 2·65	−4·17	Possibly some northern
Vánákonda	79 25	1664	17 36 0·30	17 36 6·71	−6·41	Very slightly southern
Singáwáram	80 59	714	17 45 8·69	17 45 10·22	−1·53	Slight southern
Parampúdi	81 15	684	17 12 32·62	17 12 38·12	−5·50	None
Sanjib	82 44	2142	17 31 12·32	17 31 18·52	−6·20	Uncertain
Yaltair	83 22	500	17 43 20·38	17 43 29·15	−8·77	Somewhat northern
Group	80 42	. .	17 35 54·33	17 35 59·80	−5·47	

Himalayan Stations and Stations obviously influenced by Himalayan Attraction.

Name of station.	Longitude.	Height in feet above sea.	Latitude.		A − G.	A − G_c.
			Astronomical.	Geodetic.		
Rámuápur	80° 31′	541	28° 22′ 0″·10	28° 22′ 10″·88	−10″·78	−10″·15
Nojli	77 43	929	29 53 14·12	29 53 27·60	−13·48	−12·86
Dehra Base, East end . . .	78 1	1958	30 16 37·29	30 17 7·19	−29·90	−28·90
Dehra Observatory	78 6	2289	30 19 19·56	30 19 56·91	−37·35	−36·35
Amsot	77 44	3140	30 22 16·02	30 22 44·70	−28·68	−27·66
Rajpur	78 7	2900*	30 23 9·33	30 23 56·51	−47·18	−46·16
Mussooree	78 7	6937	30 27 4·24	30 27 40·39	−36·15	−35·12
Banog	78 3	7433	30 28 4·18	30 28 36·75	−32·57	−31·54
Murree	73 27	7000*	33 54 37·35	33 54 57·19	−19·84	−18·00
Gogipatri	74 43	7752	33 51 46·93	33 51 43·73	+ 3·20	+ 5·03
Poshkar	74 32	8323	34 2 3·78	34 1 48·85	+14·93	+16·79

* Approximate values.

The Longitudinal Arcs.

These arcs lie between certain stations of the triangulation which have been put into connection with the wires of the great electric telegraph lines, thus enabling comparisons to be made between the geodetic and the astronomical differences of longitude.

Two transit instruments with collimators, two astronomical clocks, and two chronographs, with such other electric appliances as were necessary, were obtained from England for the operations. Colonel Strange—a retired officer of the Survey who was then in charge of the Mathematical Instrument Department of the India Office—was entrusted with the provision of the apparatus; and he employed Messrs. Cooke and Sons of York for the construction of the transit instruments and their collimators, Messrs. Frodsham of London for the clocks, and Messrs. Eichen and Hardy of Paris for the chronographs and electric appliances. Full descriptions and drawings of the instruments are given in vol. 9 of the Accounts of the Operations, &c.; a brief description is all that is needed here.

The transit instruments were constructed, one of gun-metal, the other of aluminium-bronze; their telescopes have a focal length of nearly 6 feet and are furnished with 5-inch object glasses. For portability they are constructed in three pieces, viz., the axis, the object end and the eye end, which travel in separate cases. The axis has a central cube having $9\frac{1}{2}$-inch sides with conical arms tapering outwards from $9\frac{1}{4}$ to 3 inches in diameter, and terminating in enlarged cylindrical shoulders $3\frac{3}{4}$ inches in diameter and 2 inches wide, into each of which a steel pivot is shrunk which projects 1·9 inches beyond the shoulders; the total length of the axis is 37·3 inches, and the thickness of metal is about 0·37 inch throughout, the cube and cones being cast in one piece. The steel pivots are 1·9 inches in diameter, one having a perforation 0·9 inch in diameter, through which light is passed for the illumination of the wires. The object and eye end tubes of the telescope are attached to the cube of the axis by 12 powerful steel bolts which pass through a projecting flange at the base of each tube and screw into the metal of the cube. The two faces of the cube which are at right angles to the faces carrying the telescope are perforated in their centres by $3\frac{1}{4}$-inch openings through which the collimators are set on each other. The eye piece is fitted with two diaphragms, one containing a single vertical and two horizontal wires, the other a set of 25 vertical wires arranged in groups of 5 each for the observation of star-transits, the former fixed, but the latter movable by a micrometer screw. Two kinds of illumination are provided, which give the observer a choice of either dark wires in a bright field or bright wires in a dark field.

The pivots of the axis rest on Y's in gun-metal beds, one of which has footscrews for the adjustment to horizontality, while the other has a provision for the adjustment in azimuth. These pivot beds rest on iron foundation plates, which are placed on the heads of the masonry supporting piers ; the piers are sunk to a considerable depth in the ground and are carefully insulated from the tread of the observer.

The levelling is performed with a Bohnenberger eyepiece, over a trough of mercury in the nadir ; no spirit-levels are employed.

The two collimators are telescopes with focal lengths of 24 inches and 2½-inch object glasses ; they are set up one to the north the other to the south of the transit instrument, usually at a distance of about 15 feet, on insulated masonry pillars.

The chronographs drive a revolving drum by clockwork which is furnished with a very exact regulator designed by M. FOUCAULT. The drum is about 12 inches wide and 36 inches in circumference ; it carries a paper on which the seconds of the clock time and the moments of the observed transits are impressed electrically by a pair of recording styles ; it revolves once in two minutes and is wholly traversed by the styles in three hours ; a second of time corresponds to three-tenths of an inch linear space on the drum.

The astronomical clocks are furnished with mercurial pendulums, and carry a wheel with 60 teeth, which revolves once in a minute and breaks the electric current at every second, the tooth opposite 0^s on the dial being cut off to mark the commencement of each minute.

The observations were always taken in a portable observatory tent made of canvas and provided with shutters and curtains admitting of a meridional aperture from the horizon to the zenith ; as it was only sufficiently large to hold the transit telescope, the chronograph and clock were provided for in a room in a neighbouring building.

The stations were almost invariably situated in the neighbourhood of a Government Telegraphic Office, and were invariably connected with the nearest stations of the Great Triangulation, so that their geodetic latitudes and longitudes may be accepted with confidence.

The instruments were received in India in 1872, and were at first employed at Bangalore and Madras, which were nearly on the same parallel, and were connected by a railroad, so that the arc between the two places was a very suitable one for testing the capabilities of the instrumental equipment. The initial operations were far from satisfactory; one of the transit instruments, No. 2, showed discordances in the determination of the "constant for collimation," which were eventually traced to a defect in construction. This was duly rectified by the mathematical instrument maker at Madras, and for the three next field seasons the instruments were apparently in excellent working condition.

During the observations at each pair of stations, the same stars were observed at both stations, at their transits across the two meridians, thus eliminating error of star's place. The chronograph at each station received its time signals from the clocks at both stations in alternate batches, and the retardation of the electric current in its passage along the wire was thus measured very exactly.

Great care was taken throughout the operations to make them self-verificatory. The stations were selected in such a manner as to form a network of great triangles over the entire face of the country, and observations were taken on every side of each triangle; thus in every case a triangular circuit was completed, the closing error of which was a valuable indication of the general accuracy of the operations.

In the field season of 1875–76 six arcs were measured; in 1876–77, three arcs, and the differences in longitude between Bombay, Aden and Suez, to complete a connection of India with England through the determination of Greenwich to Suez for the Transit of Venus Expedition in 1874. The Afghan War and other circumstances led to the suspension of operations until 1880–81, when they were resumed and eight arcs were measured. The operations of these three field seasons complete seven circuits, with errors ranging from $\cdot007^s$ to $\cdot083^s$, with the average value $\cdot035^s$, which were as small as could be reasonably expected. (See page 260 of vol. 9.)

In 1881–82 seven arcs were measured, completing three circuits, of which the closing errors were $\cdot046^s$, $\cdot291^s$, and $\cdot556^s$, two of them so large as to show that something had gone wrong. The collimation determinations showed that Transit Instrument No. 2 had again become unsatisfactory, as it was in 1874–75. It was rectified in the Mathematical Instrument Department at Calcutta, and operations were resumed in the two following field seasons, during which three of the arcs of 1881–82 were re-measured, and six new circuits were completed with small closing errors ranging from $\cdot027^s$ to $\cdot081^s$.

Thus it seemed that the faulty transit instrument had been put into a satisfactory working condition; but as some uneasiness was still felt regarding the instruments, they were returned to the makers in England for examination and rectification; afterwards they were carefully tested at the Greenwich Observatory, and were pronounced to be in perfect condition.

Operations were resumed in 1885–86; the instruments worked very satisfactorily, but again large circuit errors were obtained, and similar results were met with up to the close of the operations in 1891–92. The published annual reports of the Survey show what anxious attention was paid to these mysterious circuit errors, and what a variety of possible causes of error were examined from time to time with a view to ascertaining the actual cause. Happily it was eventually discovered, as will now be shown.

The transit instruments had invariably been set up between two collimating telescopes for the determination of their collimation constants by GAUSS's well-known method. They were also invariably reversed on their Y's at every station, so that half the observations were taken with the illuminated pivot to the east, and half with it to the west; at first there were not more than one or two reversals during the whole of the observations at a station, but latterly the number was much increased, and a reversal might be made in the course of a single night's work Now, if the collimation error is really constant and invariable, reversal of pivot removes all necessity for determining its magnitude, as the error it produces in a single observation of a star is eliminated in the mean of a pair of observations to the same star in the two instrumental positions. But values of the error had always been determined from the very first by observations to the two collimators ; a value was found for each position of the transit instrument, and applied to all stars taken in that position. Very frequently the values found in the two positions were closely identical, but occasionally they differed materially, and when this took place it was attributed partly to errors of observation, but mainly to actual changes in the collimation angle on reversal. The process of reversal was not an easy matter ; the weight of the instrument was considerable, 137 lbs. ; there was no apparatus for raising it vertically and dropping it down again after reversal; thus, the following procedure had to be adopted for reversing:—the telescope was laid horizontally, two long poles were attached to the cube of the transit axis, the instrument was then raised by four men who walked round the supporting pillars with it on their shoulders and then lowered it into position, the observer standing at one end and his assistant at the opposite end to guide the pivots exactly into the Y's.

When large discrepancies were met with they were for a long time assumed to be due in some way to this process of reversal. But the largest of all occurred at Deesa, in 1885, only a few weeks after the return of the instruments from the hands of the makers in England ; there the instrument was reversed every night for six nights, and the collimation error was determined in each position ; those in the same position agreed with each other very closely, within an extreme range of ·8 of a division of the micrometer, while those in opposite position differed by as much as 8·3 divisions, a quantity far exceeding the possible errors of observation. It was clear, therefore, that the collimation angle had been constant, and that there was some error in its determination. This led CAPTAIN BURRARD to infer that the observations should be reduced with the mean collimation constant for the two instrumental positions instead of the single constant for each position, and when this was done the circuit errors of large magnitude all disappeared, as will be seen from the following statement.*

The number of circuit errors exceeding ·50^{s} was reduced from . .	4	to	0
,, ,, between ·50 and ·30^{s} was reduced from	2	,,	0
,, ,, ,, ·30 ,, ·20 ,, ,,	5	,,	0
,, ,, ,, ·20 ,, ·15 ,, ,,	3	,,	0
,, ,, ,, ·15 ,, ·09 ,, ,,	5	,,	0
,, ,, ,, ·09 ,, ·05 was increased from	3	,,	7
,, ,, less than ·05 was increased from . . .	4	,,	19
	26		26

Clearly, therefore, the fault lay in the determination of the collimation constant.

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* See Annual Report for 1889–90, p. 71.

The object glasses of the transit telescopes were 5 inches in diameter, those of the collimators were $2\frac{1}{2}$ inches in diameter, and the apertures in the cube of the transit axis through which the collimators were pointed at each other were $3\frac{1}{4}$ inches in diameter; this left an annulus of $\frac{3}{8}$ inch between the rims of the object glasses of the collimators and the rim of the aperture when the collimators were placed exactly in line with the centre of the transit instrument. They may, however, be placed an inch or more to either side without sensibly affecting the operation of laying their wires on each other, notwithstanding that the displacement causes considerable portions of their object glasses to be cut out of operation by the intermediate cube of the transit instrument. More or less displacement had frequently occurred in practice, as the necessity for exact central adjustment had never impressed itself on the observers, and the alignment was always considered sufficiently good if clear vision of the collimator wires was obtained when they were being set on each other. Displacement would act in two ways; it would bring different parts of the object glass of a transit instrument into play before and after reversal of pivots, and it would cut off the same parts of the object glasses of the collimators throughout the operations; in both cases error would be caused if the object glasses were not truly homogeneous throughout and with truly spherical surfaces. All the object glasses were therefore tested. The transit instruments were set on a suitable distant object, which was observed with the full field of the glass, and then with the upper, lower, eastern, and western half-field in succession; and the experiments showed that the object glass of transit instrument No. 1 was perfect, and that of No. 2 nearly perfect. Then the collimators were set on each other, and similar observations were taken with the full field and the half-fields; it was at once found that their object glasses were very imperfect, showing differences between the east and the west halves, which fully accounted for the discordances which had been met with in the determination of the collimation constant. The experiments are fully detailed at pp. 379 to 387 of vol. 15.

Thus, this very perplexing mystery was at length traced to its source, and with the happy result that nine out of ten years' operations gave precise and satisfactory results by the employment of the mean collimation constant, instead of the positional constants which had been previously employed. The work of 1881–82 had to be rejected, because the collimation observations, given in volume 10, show that the tube of the telescope of No. 2 had been shaky, and consequently inefficient, throughout the field season, necessitating the rejection of the whole of the operations during that field season.

Thus, finally, fifty-five longitudinal arcs were obtained. They present thirty-one circuits, of which the errors are known, and thus give so many equations of condition for solution to determine the most probable values of the errors of the individual arcs. The equations were solved in the usual manner by the method of minimum squares, giving equal weights to all the arcs.

The following tables give, *first*, a list of the longitude stations, with their latitudes, longitudes, and heights; *secondly*, a list of the arcs, arranged from west to east, with their distinguishing numbers and observed values; *thirdly*, a list of the circuits, with their errors; and, *fourthly*, a list of the most probable corrections to the individual arcs, as determined by the process of calculation just described. These are followed by Table V., which gives a comparison of the Astronomical with the Geodetic values of the Arcs.

TABLE I.—Geodetic Elements of the Longitude Stations.

	Latitude.	Longitude.	Height, in feet.
	° ′ ″	° ′ ″	
Agra	27 9 39·93	78 3 29·07	550
Akyab	20 8 14·29	92 56 16·57	20
Amritsar	31 37 58·72	74 54 50·63	770
Bangalore	13 0 41·29	77 37 27·37	3110
Bellary	15 8 33·06	76 58 6·76	1500
Bolarum	17 30 13·41	78 33 38·30	1600
Bombay (Colaba) .	18 53 49·49	72 51 16·28	40
Calcutta	22 32 54·99	88 23 56·28	20
Chittagong . . .	22 20 19·53	91 52 43·03	90
Deesa	24 15 29·35	72 13 32·03	440
Dehra Dun . . .	30 19 29·13	78 5 49·30	2200
Fyzabad	26 46 40·66	82 10 35·33	330
Jabalpur	23 10 10·10	79 59 29·79	1350
Jalpaiguri	26 31 17·39	88 46 39·95	280
Kalianpur	24 7 10·91	77 41 44·75	1765
Kurrachee . . .	24 51 2·44	67 3 20·40	50
Madras	13 4 3·75	80 17 21·51	20
Mangalore . . .	12 52 14·14	74 53 9·89	175
Mooltan	30 10 58·70	71 28 54·57	420
Moulmein	16 29 56·49	97 40 8·05	90
Nagarkoil	8 11 25·30	77 28 30·74	110
Peshawur	34 0 17·51	71 35 27·45	1100
Prome	18 49 15·12	95 15 24·91	100
Quetta	30 11 57·29	67 2 58·87	5500
Waltair	17 43 29·31	83 21 30·70	500

The longitudes are from Greenwich, and they require a constant correction, which is usually taken as $-2' \, 30''$, to reduce them to the latest determination of the Madras Observatory relatively to Greenwich.

TABLE II.—List of Arcs with their Distinguishing Numbers and Observed Values.

Name of arc.	No.	Observed value. m. s.	Name of arc.	No.	Observed value. m. s.
Mooltan—Quetta . . .	(1)	17 43·499	Bellary—Bombay . . .	(29)	16 26·867
Kurrachee—Quetta . .	(2)	0 1·603	Bangalore—Bellary . .	(30)	2 37·230
Mooltan—Kurrachee . .	(3)	17 41·976	Bellary—Mangalore . .	(31)	8 19·653
Agra—Deesa	(4)	23 20·370	Mangalore—Bombay . .	(32)	8 7·273
Deesa—Kurrachee . .	(5)	20 40·529	Bangalore—Nagarkoil .	(33)	0 35·708
Deesa—Mooltan . . .	(6)	2 58·582	Nagarkoil—Mangalore .	(34)	10 21·141
Agra—Kurrachee . . .	(7)	44 0·992	Madras—Nagarkoil . .	(35)	11 15·006
Agra—Mooltan . . .	(8)	26 19·053	Madras—Bangalore . .	(36)	10 39·331
Amritsar—Mooltan . .	(9)	13 44·281	Madras—Mangalore . .	(37)	21 36·129
Peshawar—Mooltan . .	(10)	0 27·483	Madras—Bellary . . .	(38)	13 16·567
Amritsar—Peshawar . .	(11)	13 16·776	Madras—Bolarum. . .	(39)	6 54·615
Agra—Amritsar . . .	(12)	12 34·725	Waltair—Madras . . .	(40)	12 16·868
Dehra Dun—Agra . .	(13)	0 7·233	Waltair—Bolarum . .	(41)	19 11·525
Dehra Dun—Amritsar .	(14)	12 41·995	Waltair—Jubbulpore .	(42)	13 28·501
Fyzabad—Agra . . .	(15)	16 27·995	Calcutta—Waltair . .	(43)	20 9·194
Fyzabad—Dehra Dun .	(16)	16 20·704	Calcutta—Jubbulpore .	(44)	33 37·702
Fyzabad—Jubbulpore .	(17)	8 45·040	Calcutta—Fyzabad . .	(45)	24 52·661
Jubbulpore—Agra . . .	(18)	7 43·026	Jalpaiguri—Calcutta . .	(46)	1 30·290
Jubbulpore—Deesa . .	(19)	31 3·393	Jalpaiguri—Fyzabad . .	(47)	26 22·986
Jubbulpore—Bombay .	(20)	28 31·816	Chittagong—Calcutta .	(48)	13 55·145
Jubbulpore—Kalianpur	(21)	9 10·323	Chittagong—Jalpaiguri .	(49)	12 24·816
Agra—Kalianpur . . .	(22)	1 27·319	Akyab—Chittagong . .	(50)	4 14·252
Kalianpur—Bombay . .	(23)	19 21·441	Akyab—Calcutta . . .	(51)	18 9·395
Bombay—Kurrachee . .	(24)	23 12·215	Prome—Akyab	(52)	9 16·262
Bombay—Deesa . . .	(25)	2 31·644	Prome—Chittagong . .	(53)	13 30·472
Jubbulpore—Bolarum .	(26)	5 42·935	Moulmein—Prome . .	(54)	9 38·758
Bolarum—Bombay . .	(27)	22 48·785	Moulmein—Akyab . .	(55)	18 54·974
Bolarum—Bellary . . .	(28)	6 21·943			

Table III. Circuit Errors.

	s.
I	(2) + (3) − (1) = +0·080
II	(4) + (5) − (7) = − ·093
III	(4) + (6) − (8) = − ·101
IV	(6) + (3) − (5) = + ·029
V	(12) + (9) − (8) = − ·047
VI	(11) + (10) − (9) = − ·022
VII	(13) + (12) − (14) = − ·037
VIII	(16) + (13) − (15) = − ·058
IX	(17) + (18) − (15) = + ·071
X	(18) + (22) − (21) = + ·022
XI	(18) + (4) − (19) = + ·003
XII	(21) + (23) − (20) = − ·052
XIII	(20) + (25) − (19) = + ·067
XIV	(25) + (5) − (24) = − ·042
XV	(26) + (27) − (20) = − ·096
XVI	(28) + (29) − (27) = + ·025
XVII	(31) + (32) − (29) = + ·059
XVIII	(35) + (34) − (37) = + ·018
XIX	(38) + (31) − (37) = + ·091
XX	(36) + (30) − (38) = − ·006
XXI	(36) + (33) − (35) = + ·033
XXII	(39) + (28) − (38) = − ·009
XXIII	(40) + (39) − (41) = − ·042
XXIV	(42) + (26) − (41) = − ·089
XXV	(43) + (42) − (44) = − ·007
XXVI	(45) + (17) − (44) = − ·001
XXVII	(46) + (45) − (47) = − ·035
XXVIII	(49) + (46) − (48) = − ·039
XXIX	(52) + (50) − (53) = + ·042
XXX	(54) + (52) − (55) = + ·046
XXXI	(50) + (48) − (51) = + ·002

Table IV. Corrections to Arcs.

Arc.	Correction.	Arc.	Correction.
	s.		s.
(1)	+·029	(29)	+·004
(2)	−·029	(30)	−·003
(3)	−·022	(31)	−·041
(4)	+·045	(32)	−·014
(5)	+·030	(33)	−·010
(6)	+·023	(34)	·000
(7)	−·018	(35)	+·010
(8)	−·033	(36)	−·013
(9)	+·004	(37)	+·028
(10)	+·013	(38)	−·022
(11)	+·013	(39)	·000
(12)	+·010	(40)	+·006
(13)	+·032	(41)	−·036
(14)	+·005	(42)	+·016
(15)	+·012	(43)	−·016
(16)	+·038	(44)	−·007
(17)	−·028	(45)	+·022
(18)	−·031	(46)	+·015
(19)	+·017	(47)	+·002
(20)	−·043	(48)	−·007
(21)	·000	(49)	+·017
(22)	+·009	(50)	−·004
(23)	+·009	(51)	−·009
(24)	−·019	(52)	−·024
(25)	−·007	(53)	+·014
(26)	+·037	(54)	−·011
(27)	+·016	(55)	+·011
(28)	−·013		

The following table, No. V., gives a comparison of the final astronomical values of the longitudinal arcs—as obtained after applying the corrections in Table IV. for the errors of the circuits—with the originally computed geodetic values in terms of the adopted elements of the Earth's figure, and with those values corrected to correspond with Clarke's latest elements of the figure by the formulæ already given on p. 765. The two sets of results are given, both in time and in arc, in the columns headed A − G, and A − G_c. The data under A − G show the extent to which the results are affected both by errors in the geodetic determinations arising from the adopted elements of the Earth's figure and by errors in the astronomical determinations arising from local deflections of the plumb-line; the data under A − G_c may be regarded as well nigh free from the first and showing the second only. The algebraical sum of the quantities A − G is 145″·88, while that of the quantities A − G_c is only 29″·23, showing that the latter quantities are most free from constant error, such as would be caused by employing erroneous axes in the geodetic calculations. Obviously, therefore, it was very necessary to separate the two classes of error. Thus it is to the column A − G_c only that we have to look hereafter for an indication of the probable amounts of the local deflections of the plumb-line on the prime vertical to be met with in the longitudinal arcs.

TABLE V.—Comparison of Astronomical with Geodetic Values.

Number of arc.	Name of arc.	Astronomical value.	Seconds of geodetic value.		A − G.		A − G_c.	
			Original.	Corrected.				
		A.	G.	G_c.	In time.	In arc.	In time.	In arc.
		m. s.	s.	s.	s.	″	s.	″
(1)	Moultan—Quetta . .	17 43·528	43·714	43·526	−0·186	− 2·79	+0·002	+ 0·03
(2)	Kurrachee—Quetta .	0 1·574	1·436	1·426	+0·138	+ 2·07	+0·138	+ 2·07
(3)	Moultan—Kurrachee .	17 41·954	42·278	42·093	−0·324	− 4·86	−0·139	− 2·09
(4)	Agra—Deesa . . .	23 20·415	19·803	19·562	+0·612	+ 9·18	+0·853	+12·80
(5)	Deesa—Kurrachee . .	20 40·559	40·775	40·563	−0·216	− 3·24	−0·004	− 0·06
(6)	Deesa—Moultan . .	2 58·605	58·497	58·466	+0·108	+ 1·62	+0·139	+ 2·09
(7)	Agra—Kurrachee . .	44 0·974	0·578	0·123	+0·396	+ 5·94	+0·851	+12·77
(8)	Agra—Mooltan . . .	26 19·020	18·300	18·023	+0·720	+10·80	+0·997	+14·95
(9)	Amritsar—Moultan .	13 44·285	43·737	43·590	+0·548	+ 8·22	+0·695	+10·43
(10)	Peshawar—Mooltan .	0 27·496	26·192	26·187	+1·304	+19·56	+1·309	+19·64
(11)	Amritsar—Peshawar .	13 16·789	17·545	17·401	−0·756	−11·34	−0·612	− 9·18
(12)	Agra—Amritsar . .	12 34·735	34·563	34·430	+0·172	+ 2·58	+0·305	+ 4·58
(13)	Dehra Dun—Agra . .	0 7·265	9·348	9·346	−2·083	−31·25	−2·081	−31·22
(14)	Dehra Dun—Amritsar	12 42·000	43·911	43·775	−1·911	−28·67	−1·775	−26·63
(15)	Fyzabad—Agra . . .	16 28·007	28·417	28·246	−0·410	− 6·15	−0·239	− 3·59
(16)	Fyzabad—Dehra Dun	16 20·742	19·069	18·897	+1·673	+25·10	+1·845	+27·68
(17)	Fyzabad—Jubbulpore	8 45·012	44·369	44·279	+0·643	+ 9·65	+0·733	+11·00
(18)	Jubbulpore—Agra . .	7 42·995	44·048	43·968	−1·053	−15·80	−0·973	−14·60
(19)	Jubbulpore—Deesa .	31 3·410	3·851	3·535	−0·441	− 6·62	−0·125	− 1·88
(20)	Jubbulpore—Bombay .	28 31·773	32·901	32·615	−1·128	−16·92	−0·842	−12·63
(21)	Jubbulpore—Kalianpur	9 10·323	11·003	10·909	−0·680	−10·20	−0·586	− 8·79
(22)	Agra—Kalianpur . .	1 27·328	26·955	26·940	+0·373	+ 5·60	+0·388	+ 5·82
(23)	Kalianpur—Bombay .	19 21·450	21·898	21·703	−0·448	+ 6·72	−0·253	− 3·80
(24)	Bombay—Kurrachee .	23 12·196	11·725	11·491	+0·471	+ 7·07	+0·705	+10·58
(25)	Bombay—Deesa . .	2 31·637	30·950	30·925	+0·687	+10·31	+0·712	+10·68
(26)	Jubbulpore—Bolarum	5 42·972	43·422	43·365	−0·450	− 6·75	−0·393	− 5·90
(27)	Bolarum—Bombay .	22 48·801	49·479	49·253	−0·678	−10·17	−0·452	− 6·78
(28)	Bolarum—Bellary . .	6 21·930	22·114	22·052	−0·184	− 2·76	−0·122	− 1·83
(29)	Bellary—Bombay . .	16 26·871	27·365	27·203	−0·494	− 7·41	−0·332	− 4·98
(30)	Bangalore—Bellary .	2 37·227	37·374	37·349	−0·147	− 2·21	−0·122	− 1·83
(31)	Bellary—Mangalore .	8 19·612	19·791	19·710	−0·179	− 2·69	−0·098	− 1·47
(32)	Mangalore—Bombay .	8 7·259	7·574	7·495	−0·315	− 4·73	−0·236	− 3·54
(33)	Bangalore—Nagarkoil	0 35·698	35·775	35·769	−0·077	− 1·16	−0·071	− 1·07
(34)	Nagarkoil—Mangalore	10 21·141	21·390	21·291	−0·249	− 3·74	−0·150	− 2·25
(35)	Madras—Nagarkoil .	11 15·016	15·385	15·278	−0·369	− 5·54	−0·262	− 3·93
(36)	Madras—Bangalore .	10 39·318	39·610	39·507	−0·292	− 4·38	−0·189	− 2·84
(37)	Madras—Mangalore .	21 36·157	36·775	36·567	−0·618	− 9·27	−0·410	− 6·15
(38)	Madras—Bellary . .	13 16·545	16·984	16·855	−0·439	− 6·59	−0·310	− 4·65
(39)	Madras—Bolarum . .	6 54·615	54·870	54·803	−0·255	− 3·83	−0·188	− 2·82
(40)	Waltair—Madras . .	12 16·874	16·612	16·493	+0·262	+ 3·93	+0·381	+ 5·73
(41)	Waltair—Bolarum . .	19 11·489	11·482	11·293	+0·007	+ 0·11	+0·196	+ 2·94
(42)	Waltair—Jubbulpore .	13 28·517	28·060	27·926	+0·457	+ 6·86	+0·591	+ 8·87
(43)	Calcutta—Waltair . .	20 9·178	9·684	9·483	−0·506	− 7·59	−0·305	− 4·58
(44)	Calcutta—Jubbulpore	33 37·695	37·744	37·403	−0·049	− 0·74	+0·292	+ 4·38
(45)	Calcutta—Fyzabad .	24 52·683	53·375	53·120	−0·692	−10·38	−0·437	− 6·56
(46)	Jalpaiguri—Calcutta .	1 30·305	30·933	30·917	−0·628	− 9·42	−0·612	− 9·18
(47)	Jalpaiguri—Fyzabad .	26 22·988	24·308	24·034	−1·320	−19·80	−1·046	−15·69
(48)	Chittagong—Calcutta	13 55·138	55·195	55·054	−0·057	− 0·86	+0·084	+ 1·26
(49)	Chittagong—Jalpaiguri	12 24·833	24·262	24·135	+0·571	+ 8·57	+0·698	+10·47
(50)	Akyab—Chittagong .	4 14·248	14·236	14·193	+0·012	+ 0·18	+0·055	+ 0·83
(51)	Akyab—Calcutta . .	18 9·386	9·431	9·249	−0·045	− 0·68	+0·137	+ 2·06
(52)	Prome—Akyab . . .	9 16·238	16·556	16·464	−0·318	− 4·77	−0·226	− 3·39
(53)	Prome—Chittagong .	13 30·486	30·792	30·657	−0·306	− 4·59	−0·171	− 2·57
(54)	Moulmein—Prome . .	9 38·747	38·876	38·781	−0·129	− 1·94	−0·034	− 0·51
(55)	Moulmein—Akyab .	18 54·985	55·432	55·245	−0·447	− 6·71	−0·260	− 3·90

On Local Deflections of the Plumb-line.

The stations of a triangulation are not always well adapted as points for the determination of astronomical latitudes and longitudes, for they are necessarily situated on the most commanding positions that the ground affords, and when these positions are on high mountain ranges, or, indeed, on ground of moderate elevation but with an obvious preponderance of matter in one direction, there is an obvious tendency for the plumb-line to be deflected. But it unfortunately happens that an absolute freedom from apparent causes of disturbance is not always accompanied by an absence of disturbance. From the time when latitude observations taken on the plains of Moscow showed large deflections of the plumb-line which were caused, not by visible irregularities on the surface of the ground, but by variations of density in the hidden masses below the surface, similar results must have been met with in all geodetic surveys; they abound in India, and they have recently been exemplified on a large scale in Europe by Colonel Von Sterneck's pendulum investigations of variations of density under the surface of the earth. Thus, in this Survey, it not unfrequently happens that where the observers have been led by the aspects of the ground to anticipate deflection to have taken place in one direction, the observations show that it has really occurred in the opposite direction. Wherever disturbance takes place, the cause may be either an excess of matter, visible or invisible, in one direction, or a deficiency of matter, visible or invisible, in the opposite direction.

The Himalayan mountain ranges have always been regarded as a very disturbing element in Indian geodesy. In fixing on the station of Kaliana, as the northern extremity of the Great Arc, Everest says: "It has not been deemed prudent to approach too near to the mighty mountain masses of the Himalayan range; and rather than that I preferred to shorten the northern section by selecting Kaliana as the station for observing amplitudes, though it is 69 miles more southerly than Banog, and 57 miles to the south even of the base measured in the Dehra Dun."

It now appears that Everest was quite justified in fixing on that station as the northern extremity of the Great Arc. But meanwhile two sets of calculations have been made to show that the plumb-line at Kaliana is largely deflected by Himalayan attraction. The first is set forth in a paper—in vol. 17. (1847–48) of the 'Memoirs of the Royal Astronomical Society'—by Captain Shortrede, who assumes that the disturbing effect of the Himalayas may be compared to that of a fixed centre of attraction, and taking for his data the differences between the astronomical and the geodetic latitudes and azimuths at Kaliana and at certain Himalayan stations, finds the direction and distance of a centre of attraction which would produce the observed discrepancies, and calculates the meridional deflection of the plumb-line at Kaliana as something ranging between 11″ and 15″.

The second is an elaborate calculation by Archdeacon Pratt, in a paper "On the Attraction of the Himalayan Mountains and of the Elevated Regions beyond them upon the Plumb-line in India," which is published in the 'Philosophical Transactions' for 1855, vol. 145. The Archdeacon dissects the masses of which the Himalayas, and the high table lands of Tibet, and the regions beyond up to the parallel of 50°, are composed; he calculates their attraction and finds that they cause deflections of the plumb-line in the meridian which amount to 28″ at Kaliana, and to 12″ at Kalianpur, and 7″ at Damargida, the next southern stations on the Great Arc. If these calculations are correct, the astronomical latitude of Kaliana should be corrected to 29° 31′ 16″·0, of Kalianpur to 24° 7′ 23″·1, and of Damargardi to 18° 3′ 22″·1; thus the astronomical amplitudes of the two sections would be 5° 23′ 52″·9, and 6° 4′ 1″·0, which exceed the geodetic values by 10″·3 and 7″·6, quantities so large that they cannot be accepted as at all probable.

Archdeacon PRATT's paper in the 'Philosophical Transactions' for 1855, is immediately followed by one by Mr.—afterwards Sir GEORGE—AIRY, Astronomer Royal, who points out that the magnitudes of attractions computed on the theory of gravitation being considerably greater than is necessary to explain the observed discrepancies, is just what we might be prepared to expect when we consider that the crust of the Earth is probably supported by a fluid of greater density than itself, and that great mountain masses such as those to the north of India, necessarily sink into this fluid to a greater depth than the surrounding unelevated portions of the Earth's crust; that this heavy fluid is displaced by light crust, and the positive attraction produced by the elevated masses is diminished by the negative attraction produced by the substitution of light crust for heavy lava. The general conclusion at which Sir GEORGE AIRY arrived was that, supposing the crust of the Earth to be floating in a state of equilibrium, the real disturbance will, in all cases, be less than that found by computing the effect of the mountains on the law of gravitation; near to the elevated region the part which is to be subtracted from the computed effect is a small proportion of the whole; at a distance the part to be subtracted is so nearly equal to the whole that the remainder may be rejected as insignificant, even in cases where the attraction of the elevated country itself would be considerable.

Sir GEORGE AIRY's hypothesis, that the Earth is constituted of a light crust floating on fluid lava, has not met with much acceptance. Eminent mathematicians have subsequently brought forward evidence to show that the Earth must be regarded as more probably solid than fluid to its core; and the investigations of Sir WILLIAM THOMSON (Lord KELVIN) suggest that the Earth's mass must possess such a degree of rigidity as to be inconsistent with a crust of moderate thickness. But as yet theories of the internal constitution of the Earth have thrown little light on the subject of the influence of mountain masses on the plumb-line.

The Indian pendulum observations have, however, thrown much light on the subject. They were made at a number of stations on the Great Arc, from Cape Comorin up to Mussooree on the Himalayas, also at a station on one of the high Himalayan plateaus, at various stations on the coast lines, and also on a few islands, one of which was in the ocean, at a considerable distance from the mainland. The results when reduced to a common temperature, pressure, and origin at the sea-level, and compared with the theoretic vibration numbers computed from the same origin, show the following excesses and defects of observation over theory, as expressed in the daily vibration numbers of a second's pendulum :—[*]

	s.
Average of three island stations	+ 3·25.
,, eight coast stations	+ 0·24.
,, fifteen inland stations under 2000 feet	— 2·27.
,, four stations between 2000 and 7000 feet . . .	— 5·09.
The Himalayan tableland station, 15,400 feet high	— 21·44.

These results show an excess of density under the sea level, and a deficiency above that level which increases to a very notable magnitude at a high altitude on the Himalayas. Thus, therefore, there must be a condensation of the matter of the Earth's crust under ocean beds, and attenuation of the matter under mountains, the crust contracting and condensing wherever it sinks into hollows, and expanding and attenuating when it rises into continents and mountain ranges. Hence it is to be inferred that the matter of which elevations are composed is largely an expansion of the matter in the immediately subjacent strata of the Earth's crust, the masses above and below being closely connected and mutually interdependent; where high elevations exist the strata below having parted with matter are deficient in density; at low elevations there is no appreciable giving off of matter, and the density below is normal; under the sea there is a contraction of matter, and consequently an increase of density. Now in the application of the theory of gravitation to calculate the attraction of mountain masses on the plumb-line, it is tacitly assumed that the Earth's crust under the sea level is of uniform density throughout, whether lying under mountain masses, or under low continents, or under the bed of the sea; the visible masses above the sea level are regarded as wholly unconnected with and independent of the invisible masses below; thus the mountains might consist of just so much extraneous stuff cast on the surface of the Earth by passing meteors or the ruined débris of other worlds, whereas we see they must be portions of the Earth's crust derived from the matter lying immediately below them.

It is interesting to make a rough comparison of the relative magnitudes of the greatest horizontal attraction and the greatest vertical diminution of attraction which has been met with hitherto in the Indian Survey. The greatest meridional attraction has been found at Rajpur, in the Dehra Dun Valley, where it amounts to 46″·16; the greatest attraction on the prime vertical has been found at the Dehra Observatory, where it amounts to about 28″·5, as shown by the electro-telegraphic operations; it may be estimated as 5″ greater at Rajpur; thus the resultant of the two deflections at Rajpur is 57″, and the corresponding horizontal attraction is equal to *gravity* × *tangent deflection,* or ·00027 *g*. The maximum vertical diminution of gravity is* 21″·4, which corresponds to ·0005 *g*, and is thus about twice as great as the horizontal effect.

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[*] See p. 29 of vol. 5, which gives an account of the Indian Pendulum Operations.

It is thus evident that the effect of the attraction of mountain masses on the plumb-line, which may be very large in the immediate vicinity of the mountains, will be reduced at a distance in greater proportion than is assigned by an incomplete application of the law of gravitation, because of the deficiency in the density of the strata under the mountains, which has not hitherto been allowed for. Eventually a point must be reached at which the positive attraction of the matter above will be cancelled by the negative attraction of the deficiency below, and then the mountain masses will have no influence on the plumb-line.

We now proceed to examine the differences between the astronomical and the geodetic data, with a view to getting what information we can regarding the magnitudes of the local deflections. The data for the short arcs of this Survey, whether meridional or longitudinal, give a very close representation of the differences of the local deflections at their extremities, because the triangulation is very accurate, and the geodetic errors of arcs not exceeding 2° or 3° in length, arising from uncertainty in the elements of the Earth's figure, must be very small; therefore any marked differences between the astronomical and the geodetic amplitudes must be almost wholly due to the deflections at the extreme ends of the arcs. But there are no means of determining the actual amount and direction of the deflection at any single point, because it may be so largely influenced by variations of density underground which are not ascertainable. Thus, the differences between the astronomical and the geodetic latitudes in the preceding tables, pages 778 to 790, only give the local deflections on the understanding that there is no deflection at the origin.

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* It must here be pointed out that there is a slight uncertainty in this vibration number, because of the inadequacy of the data for its calculation. In reducing pendulum observations to the sea level, it is necessary to apply two corrections, one for the height of the station, which is always positive, the other for the ratio of the density of the elevated mass to the mean density of the earth, which is always negative. The formula for the double correction which has been employed, for the reasons set forth in chapter ix. of volume 5, is

$$\delta N = \frac{N_0}{a} . H \left(1 - 1{\cdot}005 \frac{3}{4} \frac{\sigma}{\rho}\right),$$

where N_0 is the equatorial vibration number of the pendulum, a the Earth's semi-major axis, H the height above sea level, σ the density of the elevated mass, and ρ the mean density of the Earth. Now the values of both σ and ρ are only known approximately. Here it is assumed that $\sigma = \frac{1}{2} \rho$. Professor YOUNG, who first pointed out the necessity for making an allowance for the density of the elevated mass, assumed that $\sigma : \rho : : 2{\cdot}5 : 5{\cdot}5$. Employing his figures, the deficiency at More, the Himalayan tableland station, would be reduced from 21″·44 to 19″·74. On the other hand, if the density ratio is larger than 1 : 2, as is quite possible, the deficiency in the vibration number will be increased. On any reasonable assumption whatever there will always be a marked deficiency in the density of the strata which lie underneath the elevated mass.

Kalianpur was adopted by EVEREST as the origin, on the assumption that it was free from meridional deflection, the surface of the ground giving no sign of any. To the north, south, and west the same general level prevails for a considerable distance all round; but to the east there is a depression of 200 feet to the plain in which the Sironj base line is situated; thus, while there is apparently no meridional deflection, there may be slight westerly deflection. In section 4 of chapter x. of volume 2 it is shown that the observed azimuth at Kalianpur is certainly 1″·1, and possibly 1″·86, too great, on the evidence of the astronomical azimuths at sixty-three of the principal stations around Kalianpur; the smaller quantity was applied as a correction to the observed azimuth, to obtain the fundamental azimuth for the final geodetic computations. Now that they are completed it is seen that a still larger correction would have brought the final geodetic azimuths into better accordance with the astronomical. Taking 1″·1 as the true excess of the astronomical azimuth at Kalianpur, the zenith is deflected 2″·5 to the east on the prime vertical.

The algebraical mean excess of the whole of the astronomical over the corrected geodetic values of latitude $(A - G_c)$ on the Great Arc, from Punnæ to Kaliana inclusive, which are 54 in number, is — 2″·0. It will be seen that, of the whole 148 astronomical latitudes available for geodetic investigations, there are 90 cases of negative excess to 58 of positive excess; but, if the latitude of Kalianpur is diminished by 2″·0, the whole of the geodetic latitudes will be correspondingly diminished, and this will make the number of positive and negative cases almost exactly equal. Thus it is not improbable that the astronomical latitude is 2″ too great, or, in other words, that the plumb-line at Kalianpur is not deflected to the north under the influence of Himalayan attraction, as has been so often suggested, but is in reality deflected to the south. It seems, therefore, that if we subtract 2″ from each of the negative excesses and add 2″ to each of the positive excesses, we shall have a fairly approximate determination of the meridional deflection at each of the latitude stations.

The stations coming under the influence of Himalayan attraction must first be noticed briefly, and afterwards set aside altogether, as they cannot be employed in a geodetic investigation. Proceeding from east to west, the first station, Ramuapur, is situated on the meridian of 80° 31′, about 54 miles to the south of the Himalayas, and shows a deflection of — 10″·15. Then comes the following group of stations on the Great Arc; Nojli, in the plains, about 30 miles to the south of the Siwaliks, the outer Himalayan range, which rises about 2000 feet above the plains to the south and 1000 feet above the Valley of Dehra Dun to the north, deflection — 12″·86. Amsot is a peak of the Siwaliks 3140 feet high, deflection — 27″·66; the Dehra Dun base line station, deflection — 28″·90; the Dehra Dun Observatory, deflection — 36″·35; Rajpur, at the foot of the ascent to Mussooree, deflection — 46″·16; Mussooree — 35″·12, and Banog — 31″·54, stations on the hill ranges where the northerly attraction is diminished because there is attracting matter to the south. Finally, there is a group of stations to the west; Murree, on a mountain to the south of the

Pir Pinjal range, which lies between India and Kashmir, deflection $-18''\cdot 00$; and two stations on the northern spurs of the Pir Pinjal, with the wide open valley of Kashmir beyond, and which, therefore, show southerly attraction, namely, Gogipatri, deflection $+5''\cdot 03$; and Poshkar, deflection $+16''\cdot 79$.

The nearest stations to the Himalayas which may be included in an investigation of the figure of the Earth must now be noticed. They are four in number, as follows:—

Station.	Latitude.	Longitude.	$A-G_c$.	Distance from Himalayas.
Jarura (Madras meridian) . . .	28° 0′	80° 31′	−5″·00	80 miles south
Kaliána (Great Arc)	29 31	77 42	−5·72	52 ,, ,,
Isanpúr (Rahun meridian) . . .	30 38	76 9	−2·75	65 ,, ,,
Shahpur (Gurhagurh meridian) . .	32 2	75 8	+2·00	40 ,, ,,

It is to be noticed that the southern foot of the Himalayas descends from latitude 33° 50′, on the meridian of 73°, to latitude 29° on the meridian of 81°, and then trends nearly easterly. Thus the apparent diminution of deflection at Isanpur and reversal at Shahpur, as compared with Jarúra and Kaliána, may be due to the circumstance that the Himalayas descend some way to their south, though at a very considerable distance to the east; but speculation on this point is superfluous, as there are so many larger deflections to the south, which cannot possibly be attributed to Himalayan attraction.

Attention must be drawn to group 10 of the Great Arc. It contains one of Lambton's stations, Takalkhera, which Everest rejected, because he calculated that the plumb-line is attracted 5″ northwards at that point by the tablelands of the Mahadeo Hills, which are 20 miles distant and rise about 1600 feet above it. But the group contains four subsequently determined stations whose distances range from 20 to 28 miles to the south of Takalkhera, all which show a still larger amount of northerly attraction. Here, therefore, there must be not only an excess of visible matter above ground in the Mahadeo plateau to the north, but a deficiency of invisible matter underground to the south.

It is only to the operation of underground variations of density that we can look for an explanation of the fact that at Bombay there is apparently large northerly deflection, and at Madras there is large southerly deflection, while there is no visible source of meridional attraction at either place. Whatever the uncertainty about the actual deflection at each place because of the uncertainty at the origin, there is no question that the astronomical amplitude of the arc Bombay—Madras is $16''\cdot 01$ less than the (corrected) geodetic amplitude, mainly because of the local deflections at the two places, the difference being at least twenty times greater than can be due to error in the triangulation. There is no apparent cause to which to attribute these gross deflections. Others of a like nature are presented by the latitude observations at various places, even in the deserts of Rajputana which might be expected to be quite free from local disturbances.

We now proceed to examine the local attractions on longitudinal arcs, as shown in column $A - G_c$ of Table V., page 799. Being deduced from time observations they are necessarily more affected by errors of observation than the local attractions on meridional arcs, but not to any great extent ; for the maximum circuit error of three arcs is $\cdot101^s$, as shown in Table III., which corresponds to a maximum error of $\cdot06^s$, or less than $1''$ of arc, for a single arc. Thus the differences between the observed and the (corrected) geodetic values are a closely approximate indication of the differences between the local attractions on the prime vertical at the extremities of the arcs.

The greatest deflections shown are in the three arcs connecting the station of Dehra Dun, which is situated close under the Himalayas, with Agra to the south, Fyzabad to the east, and Amritsur to the west ; from the mean of the three it is to be inferred that the Himalayan attraction on the prime vertical at Dehra Dun is $28''\cdot5$ easterly. The next greatest deflection is at Peshawur, where it appears from the evidence of the arcs to Mooltan and Amritsur to be about $14''\cdot4$ westerly, which may well be anticipated from the circumstance that the mountain ranges of Afghanistan lie to the west and a broad open valley to the east. The arc, Jalpaiguri-Fyzabad, shows local deflections, aggregating $15''\cdot69$, and there is evidently considerable eastern deflection at Jalpaiguri ; but on combining that arc with the arcs to Calcutta and Chittagong it apparently is about $11''\cdot8$. Deflections of as much as $10''$ to $15''$ are met with on arcs connecting stations which might be expected to be almost wholly free from local attraction, as Agra, Mooltan, and Kurrachee ; thus showing the action of hidden causes under the surface of the Earth in producing deflections of the plumb-line on the prime vertical, such as have already been noticed on the meridian.

Four of the longitude stations are situated on the coasts of the Peninsula, viz., Bombay, Mangalore, Madras, and Waltair. The ocean being on one side and the continent on the other, it is to be supposed that there would be deflection towards the interior of the continent at all four places ; but this is only met with at Waltair, where a range of hills 1600 feet higher than the station lies at a distance of only five miles to the west ; at the three other stations there is a more or less wide expanse of plain before hills are reached. The whole of the arcs from the other stations, which are thirteeen in number—six from Bombay, two from Mangalore, four from Madras, and the arc Madras-Mangalore—show deflection towards the ocean and not towards the interior of the continent. The astronomical latitudes in the Southern Peninsula tell the same tale of deflection towards the ocean. All this corroborates the inferences regarding the contraction and condensation of the strata under the bed of the ocean which have already been derived from the pendulum operations.*

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* This contraction is suggestive of considerable depth below the sea level in the bed of the ocean,

Final Results.

Excluding stations obviously affected by the attraction of the great Himalayan and Tibetan masses, we are in possession of 148 astronomical latitudes and 50 longitudinal arcs which are available for an investigation of the mean figure of the Earth. Thus, the question arises as to the manner in which they can be most advantageously employed.

In his 'Geodesy,' Colonel CLARKE gives his latest determination of the figure of the Earth, in which he employs 14 of the latitude stations and 6 of the longitudinal arcs of the Indian Surveys, in combination with 35 latitude stations of surveys made in other parts of the world. No attempt will be made here to obtain a new value of the figure of the Earth by employing the whole of the Indian data instead of the small portion used by Colonel CLARKE. All that is now aimed at is to set forth the facts to be elicited from the operations in India which will be most serviceable for future mathematical investigations.

The existing data may evidently be treated as they stand, in accordance with either of the various methods which have been employed by AIRY, BESSEL, CLARKE, and others. But in all mathematical treatment of such data it is assumed that the direction of the plumb-line at the astronomical stations is normal, or very closely normal, to the surface, and wherever there is reason to suspect that this may not be the case the station is arbitrarily rejected. Still, as a matter of fact, the paucity of data has frequently led to the employment of stations which are sensibly influenced by local attractions. It is possibly for this reason that some mathematicians have been led to conclude from the data which they have employed that the mean figure of the Earth is not a spheroidal figure, with two axes only, as has been generally accepted on theoretical grounds, but an ellipsoidal figure with three axes, two of which are poles of the equator. This subject has been discussed at length in Archdeacon PRATT'S 'Figure of the Earth,' 4th edition, p. 181, with the conclusion that "local attraction appears to supply a source of correction which makes a resort to so peculiar an hypothesis as an ellipsoidal mean figure unnecessary and untenable."

It is obvious, therefore, that, before mathematical treatment can be advantageously commenced, steps should be taken to diminish to the utmost possible extent the local deflections by which the observations are burdened. This cannot be done by

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near the coast line. The Admiralty charts give the following details on this subject:—At Bombay there is a belt of water of a depth of about 250 feet extending 150 miles westwards, beyond which are soundings of 5,000 to 10,000 feet. At Mangalore a belt of much the same depth extends 50 miles westwards, beyond which are soundings of 6,000 and 7,000 feet. At Madras there are soundings of 5,000 to 10,000 feet within 30 miles of the coast line. At Waltair there is a sounding of 8,000 feet within 50 miles of the coast line. The charts show a line of demarcation between deep and shallow water on the west coast, but there is no such line on the east coast. The shallows may very possibly be recent deposits on ancient contracted beds.

minute surveys of the ground and calculations based on the superficial conditions, because no cognizance can be taken of the existing conditions under ground which may be of quite as great or even greater influence in producing deflections of the plumb-line. Thus, in treating meridional arcs, the only possible way is to combine a number of the astronomical stations within a narrow belt of parallel together, and take the mean latitude of the group, and to do the same with the corresponding geodetic determinations, carrying on the process from south to north as far as the data are available. This gives us combinations of data for a single chain of meridional arcs which will be far more valuable for mathematical treatment than the separate individual initial data, because the mean astronomical latitude of a number of points may certainly be assumed to be far more free from deflection than the latitude of any single point.

When this is done for a number of stations extending over a considerable distance from east to west, the objection arises that the procedure binds us to the assumption that the mean figure of the Earth is spheroidal. But all geodetic calculations of the positions of places on the surface of the Earth have been made hitherto on this assumption; for though an ellipsoidal figure has been suggested and calculated, the several calculations which have been made have placed the poles of the equator in such different positions that it does not appear that any person has yet attempted to calculate terrestrial elements of latitude, longitude, and azimuth on the basis of an ellipsoidal figure. We have to begin by assuming something; we have seen that it is impossible to assume any astronomical arcs of amplitude to be free from error caused by local deflections of the plumb-line, which is our only alternative. It seems obvious that an assumption which is in itself highly probable, and which permits of combinations of large numbers of astronomical observations and a consequent large elimination of the errors caused by local deflections, is more likely than any other to lead to exact and definite results.

Combinations of the latitudes will therefore be presently indicated; but, before doing so, it is necessary to fix the most appropriate lengths of the arcs of amplitude to be given by the combinations. The errors caused by the local deflections are independent of the lengths of the arcs; they must produce much greater influence on short than on long arcs; and therefore it is desirable to fix a relation between the magnitude of the local deflection and the length or amplitude of the arc. Meridional arcs of about 2° in length appear to have been generally regarded hitherto as giving an adequate number of touching points on the Earth's surface to produce a close representation of the figure; but this length seems to have been fixed without sufficient consideration of the magnitudes of the local deflections involved which are liable to be such a disturbing element in the calculations.

The mathematical relations between errors in arcs of amplitude and errors in the lengths of the Earth's axes are given with sufficient approximation by the following formulæ:—

$$d.\Delta\lambda = \frac{1}{a}\{da - 2\,db - 3\,(da - db)\sin^2\lambda\}\,\Delta\lambda,$$

$$d.\Delta \mathrm{L} = -\frac{1}{a}\{da + (da - db)\sin^2\lambda\}\,\Delta \mathrm{L}.$$

If we take the arithmetical mean of the four cases $da = 0$, $db = 0$, $da = db$, and $da = -db$, and put n for the number of degrees in the arc and $1000\,p$ for the axial error in feet, and also put $\lambda = 21°$ the middle latitude of India, we have

$$d.\Delta\lambda'' = 0{\cdot}24\,n.p,$$
$$d.\Delta \mathrm{L}'' = 0{\cdot}15\,n.p.$$

Thus for a meridional arc of 2° and an axial error of 1000 feet, the arc error is only $0''{\cdot}48$, which is but a small fraction of the error often caused by local deflection. A much larger axial error than 1000 feet must therefore be admitted, or the magnitude of the arc must be so considerably increased that the number of touching points on the Earth's surface will be too small to produce a sufficiently close representation of the surface. Let us put 5000 feet as the limit of axial error, and then we obtain the conditions

$$n > \frac{d\,.\,\Delta\lambda}{1{\cdot}2}, \text{ and } n > \frac{d\,.\,\Delta \mathrm{L}}{{\cdot}75},$$

or in other words, when the arc error is

$2''$, n must exceed $1°{\cdot}7$ in a meridional and $2°{\cdot}6$ in a longitudinal arc.
$3''$, ,, ,, $2°{\cdot}5$,, ,, $4°{\cdot}0$,, ,,
$4''$, ,, ,, $3°{\cdot}3$,, ,, $5°{\cdot}3$,, ,,

These conditions have governed the formation of the following table of combinations of latitudes to produce the final meridional arcs of amplitude. The column $A'' - A'$ gives the differences between the values of the astronomical and the (corrected) geodetic arcs of amplitude, as derived from the data in column $A - G_c$; these differences may be assumed to be free from errors caused by local deflections of the plumb-line, and to be mainly due to the errors still latent in CLARKE'S latest determination of the figure of the Earth.

COMBINATIONS of Latitudes and the resulting Arcs of Amplitude.

Chain of triangles and stations.	Number of stations.	Latitudes.		A − G.	A − G$_c$.	Arcs of amplitude.		A″ − A′.
		Astronomical.	Geodetic.			Astronomical.	Geodetic.	
		° ′ ″	° ′ ″	″	″	° ′ ″	° ′ ″	″
Great Arc. Group 1	6	8 12 47·50	8 12 45·18	+ 2·32	+ 1·65			
Great Arc. Group 2	4	9 28 54·35	9 28 51·63	+ 2·72	+ 2·02			
Combination I.	10	8 43 14·24	8 43 11·76	+ 2·48	+ 1·80			
Great Arc. Group 3	3	11 0 3·22	11 0 1·79	+ 1·43	+ 0·72			
Madras. Tiruvendipuram . .	1	11 44 43·40	11 44 37·53	+ 5·87	+ 5·16			
Combination II.	4	11 11 13·27	11 11 10·73	+ 2·54	+ 1·83	2 27 59·03	2 27 58·97	+ 0·03
Mangalore. Mangalore	1	12 52 17·76	12 52 14·60	+ 3·16	+ 2·46			
Madras. Group 1	2	13 2 15·27	13 2 9·31	+ 5·96	+ 5·26			
Great Arc. Group 4	5	13 26 16·56	13 26 16·73	− 0·17	− 0·87			
Mangalore. Group 1	2	14 12 16·24	14 12 19·37	− 3·13	− 3·82			
Madras. Group 2	2	14 14 11·48	14 14 11·85	− 0·37	− 1·06			
Combination III.	12	13 35 5·54	13 35 4·94	+ 0·60	− 0·09	2 23 52·27	2 23 54·21	− 1·92
Great Arc. Group 5	6	14 56 24·50	14 56 22·16	+ 2·34	+ 1·67			
Madras. Group 3	2	15 15 13·21	15 15 16·50	− 3·29	− 3·95			
Mangalore. Group 2	2	15 20 21·47	15 20 23·07	− 1·60	− 2·26			
Madras. Group 4	2	16 10 56·59	16 10 58·29	− 1·70	− 2·34			
Great Arc. Group 6	5	16 11 52·83	16 11 58·05	− 5·22	− 5·86			
Mangalore. Group 3	2	16 35 59·96	16 36 0·35	− 0·39	− 1·02			
Great Arc. Group 7	5	17 8 11·09	17 8 14·73	− 3·64	− 4·24			
Madras. Group 5	2	17 21 57·54	17 22 4·57	− 7·03	− 7·62			
Combination IV.	26	16 4 8·62	16 4 10·86	− 2·24	− 2·88	2 29 3·08	2 29 5·92	− 2·79

COMBINATIONS of Latitudes and the resulting Arcs of Amplitude—(continued).

Chain of triangles and stations.	Number of stations.	Latitudes.		A − G.	A − G$_c$.	Arcs of amplitude.		A″ − A′.
		Astronomical.	Geodetic.			Astronomical.	Geodetic.	
		° ′ ″	° ′ ″	″	″	° ′ ″	° ′ ″	″
Bider Longitudinal Series	7	17 35 54·33	17 35 59·80	− 5·47	− 6·06			
Great Arc. Group 8	2	18 9 0·02	18 9 2·58	− 2·56	− 3·12			
Bombay Longitudinal Series . . .	4	18 27 28·94	18 27 32·51	− 3·57	− 4·11			
Mangalore. Group 4	2	18 28 4·14	18 28 9·62	− 5·48	− 6·02			
Madras. Group 6	2	18 44 45·23	18 44 46·50	− 1·27	− 1·80			
Bombay, Colaba Observatory . .	1	18 53 39·16	18 53 49·32	−10·16	−10·68			
Great Arc. Group 9	4	19 5 10·90	19 5 15·88	− 4·98	− 5·49			
Madras. Group 7	2	19 36 56·79	19 37 3·50	− 6·71	− 7·18			
Mangalore. Group 5	2	20 7 26·05	20 7 31·23	− 5·18	− 5·63			
Madras. Group 8	2	20 35 9·95	20 35 15·55	− 5·60	− 6·01			
Great Arc. Group 10	5	20 48 43·69	20 48 50·69	− 7·00	− 7·40			
Combination V.	33	19 1 16·41	19 1 21·60	− 5·19	− 5·70	2 57 7·79	2 57 10·74	− 2·82
Madras. Group 9	3	21 46 50·00	21 46 56·25	− 6·25	− 6·57			
Mangalore. Group 6	3	22 45 4·63	22 45 6·66	− 2·03	− 2·26			
Bombay. Sonada	1	23 7 15·64	23 7 19·73	− 4·09	− 4·28			
Great Arc. Ládi	1	23 8 39·10	23 8 43·97	− 4·87	− 5·05			
Madras. Group 10	2	23 20 15·69	23 20 10·62	+ 5·07	+ 4·93			
Combination VI.	10	22 39 13·00	22 39 15·37	− 2·37	− 2·60	3 37 56·59	3 37 53·77	+3·10

COMBINATIONS of Latitudes and the resulting Arcs of Amplitude—(continued).

Chain of triangles and stations.	Number of stations.	Latitudes.		A−G.	A−G$_c$.	Arcs of amplitude.		A″−A′.
		Astronomical.	Geodetic.			Astronomical.	Geodetic.	
		° ′ ″	° ′ ″	″	″	° ′ ″	° ′ ″	″
Calcutta-Karachi longitudinal	7	24 1 5·85	24 1 4·87	+0·98	+0·97			
Great Arc. Origin. Kalianpur	1	24 7 11·10	24 7 11·10					
Bombay. Group 1	3	24 19 51·48	24 19 58·82	−7·34	−7·31			
Madras. Group 11	2	24 21 3·46	24 21 2·37	+1·09	+1·12			
Mangalore. Group 7	2	24 25 17·32	24 25 19·71	−2·39	−2·35			
Great Arc. Group 11	2	25 21 24·02	25 21 21·21	+2·81	+2·96			
Mangalore. Group 8	2	25 41 34·87	25 41 37·11	−2·24	−2·04			
Madras. Group 12	3	25 51 35·16	25 51 30·25	+4·91	+5·21			
Bombay. Group 2	3	26 10 38·68	26 10 39·22	−0·54	−0·27			
Mangalore. Group 9	3	26 40 41·71	26 40 42·59	−0·88	−0·53			
Combination VII.	28	25 2 12·59	25 2 12·81	−0·22	−0·11	2 22 59·59	2 22 57·44	+2·49
Great Arc. Group 12	2	27 3 17·51	27 3 22·92	−5·41	−5·00			
Madras. Group 13	2	27 7 45·40	27 7 42·81	+2·59	+3·00			
Bombay. Group 3	2	27 34 58·52	27 34 55·30	+3·22	+3·72			
Madras. Jarúra	1	27 59 50·22	27 59 55·78	−5·56	−5·00			
Mangalore. Group 10	3	28 6 2·78	28 6 2·93	−0·15	+0·43			
Great Arc. Group 13	2	28 17 25·90	28 17 28·63	−2·73	−2·11			
Combination VIII.	12	27 42 4·43	27 42 5·32	−0·89	−0·37	2 39 51·84	2 39 52·51	−0·26
Bombay. Group 4	3	28 55 52·01	28 55 51·12	+0·89	+1·29			
Mangalore. Group 11	2	29 28 16·94	29 28 17·46	−0·52	+0·32			
Great Arc. Kaliána	1	29 30 47·98	29 30 54·54	−6·56	−5·72			
Bombay. Group 5	2	30 22 27·93	30 22 30·13	−2·20	−1·18			
Mangalore. Group 12	2	30 30 13·89	30 30 17·26	−3·37	−2·33			
Mangalore. Group 13	3	31 39 4·06	31 39 2·15	+1·91	+3·20			
Combination IX.	13	30 9 2·59	30 9 3·39	−0·80	+0·10	2 26 58·16	2 26 58·07	+0·47

From the preceding combinations we obtain the following table of what appear to be the most trustworthy results at present derivable from the operations of the Indian Survey :—

RESULTING MERIDIONAL ARCS.

Combinations.	Astronomical latitudes.	Astronomical amplitudes.	Arc-lengths in feet.
	° ′ ″	° ′ ″	
I.	8 43 14·24		
		2 27 59·03	894,947·2
II.	11 11 13·27		
		2 23 52·27	870,416·6
III.	13 35 5·54		
		2 29 3·08	902,015·2
IV.	16 4 8·62		
		2 57 7·79	1,072,166·9
V.	19 1 16·41		
		3 37 56·59	1,319,028·9
VI.	22 39 13·00		
		2 22 59·59	865,705·7
VII.	25 2 12·59		
		2 39 51·84	968,481·6
VIII.	27 42 4·43		
		2 26 58·16	890.617·5
IX.	30 9 2·59		

The arc-lengths in feet are deduced from the original geodetic amplitudes which are converted into feet by the usual formulæ.

For the Longitudinal Arcs the treatment is simpler. As we assume the figure of the Earth to be spheroidal, and, therefore, the length of an arc on any given parallel of latitude to be exactly proportional to its amplitude, we have simply to form the longest arcs on the most convenient parallels of latitude which we can obtain from combinations of the given arcs. The whole of the arcs having been made consistent by the mathematical treatment which has been described, it does not matter which of them are employed in forming a combination; but it will be convenient to take those which are nearest to the selected parallels. The combinations have the immense advantage of being free from all errors arising from local deflections of the plumb-line, excepting those which occur at the extreme eastern and western stations of the arcs of parallel, those occurring at the intermediate stations becoming eliminated. Thus the magnitudes of the deflections which remain and cannot be eliminated will be reduced from large and significant fractions of small arcs to comparatively small and insignificant fractions of large arcs; at least this will happen in three out of the four cases to be now brought forward, the exception being the arc between Madras and Mangalore, which is necessarily a short one, the length being curtailed by the Indian Ocean.

The following table gives the adopted longitudinal arcs and their combinations. It is arranged to show the excesses of the astronomical amplitudes over both the original geodetic values and those values corrected to accord with the latest determined lengths of the Earth's axes, as a preliminary to obtaining estimates of the most probable values of the magnitudes of the local deflections of the plumb-line, in accordance with what has been previously done for the meridional arcs. It will be seen that what was of comparatively little importance for the meridional determinations is of considerable importance for the longitudinal.

LONGITUDINAL Arcs and their Combinations.

Names of arcs and combinations.	Amplitudes.		A−G.	$A-G_c$.	
	Astronomical.	Geodetic.		In time.	In arc.
First Arc.—Madras-Mangalore. Mean latitude 12° 58′ 8″·95.					
	m. s.	m. s.	s.	s.	″
I. Madras-Mangalore	21 36·157	21 36·775	−0·618	−0·410	− 6·15
Second Arc.—Moulmein-Bombay Combination. Mean latitude 17° 41′ 52″·99.					
Moulmein-Akyab	18 54·985	18 55·432	−0·447	−0·260	− 3·90
Akyab-Calcutta	18 9·386	18 9·431	−0·045	+0·137	+ 2·06
Calcutta-Waltair	20 9·178	20 9·684	−0·506	−0·305	− 4·58
Waltair-Bolarum	19 11·489	19 11·482	+0·007	+0·196	+ 2·94
Bolarum-Bombay	22 48·801	22 49·479	−0·678	−0·452	− 6·78
II. Moulmein-Bombay	99 13·839	99 15·508	−1·669	−0·684	−10·26
Third Arc.—Chittagong-Kurrachee Combination. Mean latitude 23° 35′ 40″·99.					
Chittagong-Calcutta . . .	13 55·138	13 55·195	−0·057	+0·084	+ 1·26
Calcutta-Jubbulpore . . .	33 37·695	33 37·744	−0·049	+0·292	+ 4·38
Jubbulpore-Deesa	31 3·410	31 3·851	−0·441	−0·125	− 1·87
Deesa-Kurrachee	20 40·559	20 40·775	−0·216	−0·004	− 0·06
III. Chittagong-Kurrachee . . .	99 16·802	99 17·565	−0·763	+0·247	+ 3·71
Fourth Arc.—Jalpaiguri-Quetta Combination. Mean latitude 28° 21′ 37″·34.					
Jalpaiguri-Fyzabad	26 22·988	26 24·308	−1·320	−1·046	−15·69
Fyzabad-Agra	16 28·007	16 28·417	−0·410	−0·239	− 3·59
Agra-Mooltan	26 19·020	26 18·300	+0·720	+0·997	+14·96
Mooltan-Quetta	17 43·528	17 43·714	−0·186	+0·002	+ 0·03
IV. Jalpaiguri-Quetta	86 53·543	86 54·739	−1·196	−0·286	− 4·29

Thus we obtain the following combination data for longitudinal arcs, as we already have done for meridional arcs :—

RESULTING LONGITUDINAL ARCS.

Number of Arc.	Mean latitude.	Astronomical amplitude.	Arc-lengths, in feet.
	° ′ ″	° ′ ″	
I.	12 58 8·95	5 24 2·36	1,923,108
II.	17 41 52·99	24 48 27·59	8,635,432
III.	23 35 40·99	24 49 12·03	8,311,387
IV.	28 21 37·34	21 43 23·15	6,987,444

The first arc is too short to be employed in a geodetic investigation, because of the magnitude of the error in the astronomical amplitude, apparently caused by local deflections of the plumb-line towards the strata under the bed of the ocean at each extremity of the arc ; the rule which has been adopted to govern the relations between the amplitude of an arc and the magnitude of the deflections requires that for such an error as we have here, apparently 6″·15, the length of the arc should be at least half as much again as it is.

The second arc is good, though probably influenced to some extent by deflection towards the strata under the ocean at Bombay ; but this is of little significance in an arc of so great length.

The third arc is best of all, as it is apparently least affected by local deflections of the plumb-line, and is of considerable length.

The fourth arc is not so good as the smallness of the sum of the local deflections suggests; it is burdened with considerable local deflection at Jalpaiguri, its eastern extremity, and possibly also at Quetta, its western extremity, which is situated on an elevated plateau 5500 feet high, and is surrounded by mountains which rise to a much higher altitude on the east than on the west; moreover, the geodetic connection between Mooltan and Quetta rests on a long single chain of secondary triangles, and is, therefore, much weaker than that of any of the other longitude stations, all of which are more or less immediately connected with the principal triangulation.

Thus the results which may be put forward as essentially representing India's contribution to geodesy consist of eight meridional arcs extending from latitude 8° 43′ 14″ to latitude 30° 9′ 3″, and resting—not as usual on latitude observations taken at one more station than the number of arcs, but—on latitude observations taken at 148 stations, giving an average of over 16 stations for each of the fundamental latitudes; there are also two excellent longitudinal arcs on the parallels of 17° 42′ and 23° 36′ each exceeding 24° in amplitude and 8,300,000 feet in length.

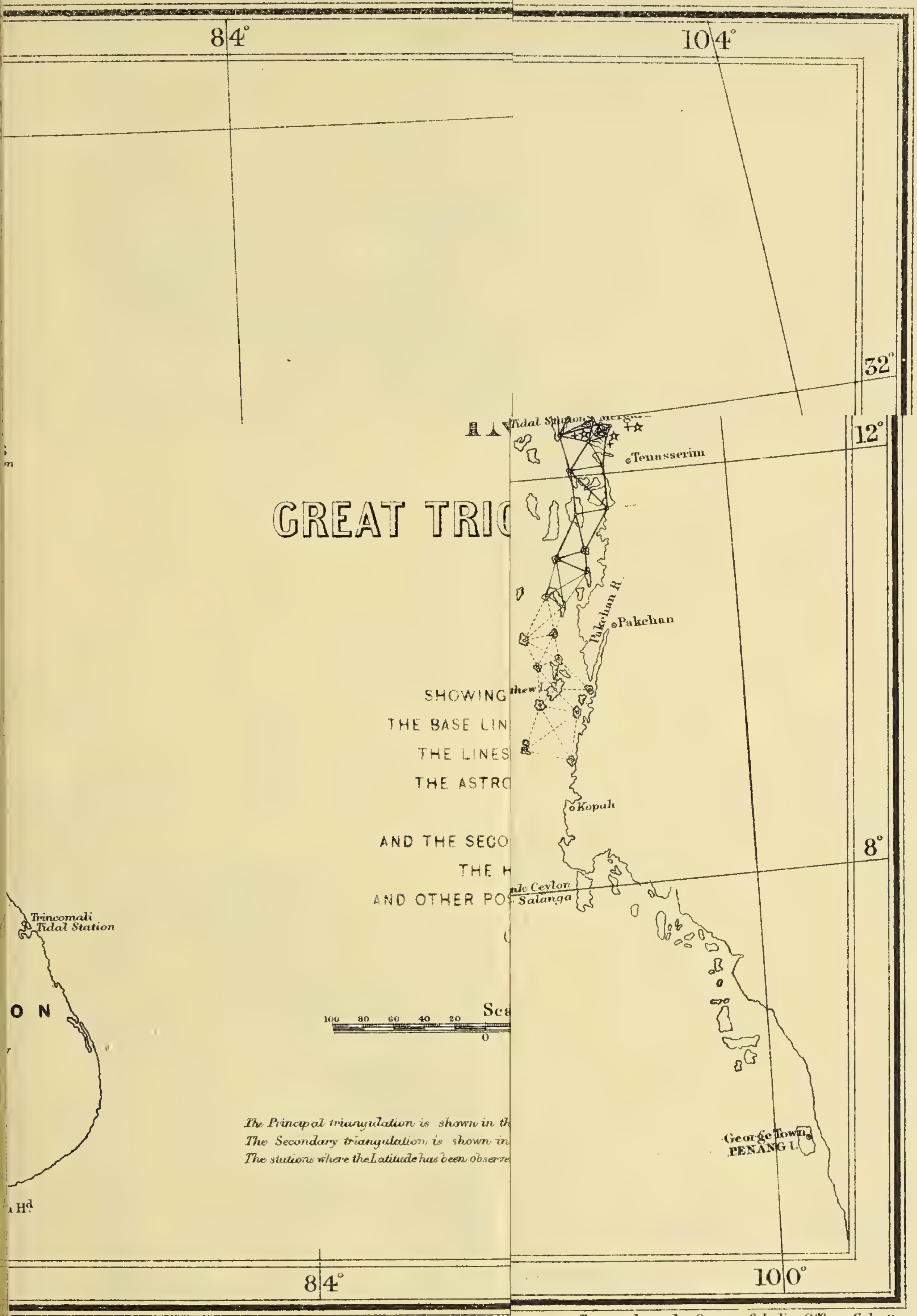
In all mathematical investigations of the figure of the Earth which have yet been made, individual results have invariably been employed and never any combinations of results for the purpose of diminishing errors caused by local deflections of the plumb-line. This has almost certainly arisen from the circumstance that the astronomical data have been too few to be otherwise employed. But the Indian Survey furnishes abundant data, far exceeding what has ever been put forward hitherto by any other survey; and these data not only indicate the general presence of large local disturbances of the plumb-line and make it more prominent than appears to have been generally recognized hitherto, but they admit of and suggest, they almost necessitate, a system of combination of results which will have the happy result of greatly diminishing the influence of deflections of the plumb-line in future investigations of the Earth's figure.

The eight meridional and the two longest longitudinal arcs here given, are believed to be the most valuable contribution to geodetic science that has yet been made. They are the result of operations which have been carried on in India during a period of over ninety years, with more or less vigour at different times, but always with the cordial support and approval of the Government of India.

Description of Plate.

The accompanying Index Chart of the Great Trigonometrical Survey of India shows the whole of the Principal Triangulation, the Stations at which Astronomical Latitudes have been observed, and the Electro-Telegraphic Longitudinal Arcs; also the Pendulum Stations. It also shows the Azimuth Stations, the lines of the Spirit Levelling Operations, and the Secondary Triangulation to the Himalayan Mountains and various points external to the Principal; but it omits for the sake of clearness all the large amount of internal Secondary Triangulation which has been executed.

SECOND EDITION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Engraved at the Survey of India Offices, Calcutta.

UILLIER, C. I. E., R. E., Surveyor-General of India.

1893.

SECOND EDITION

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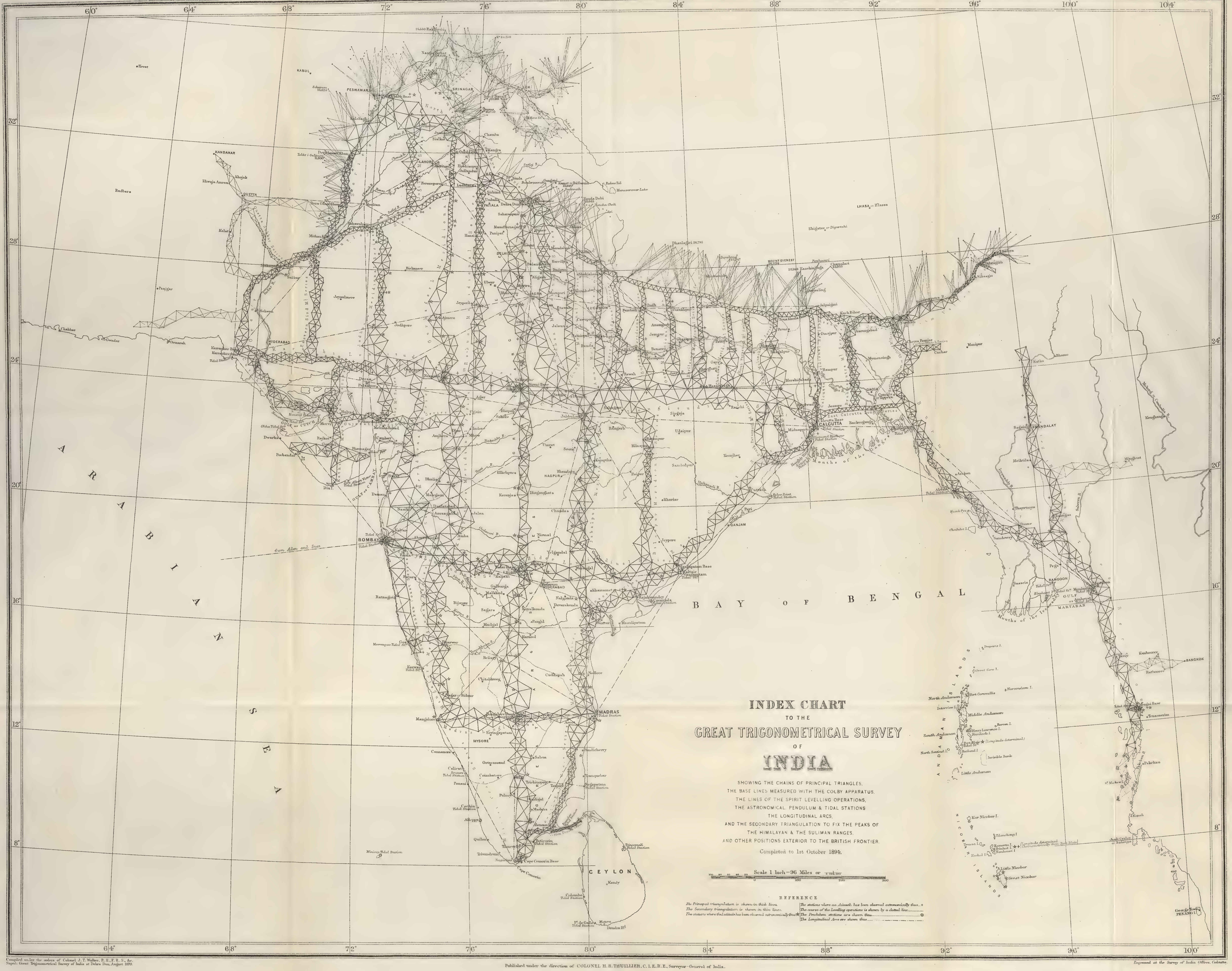
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PESHAWAR
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KANDAHAR
QUETTA
LAHORE
PATIALA
DELHI
AGRA
GWALIOR
LUCKNOW
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MOUNT EVEREST
LHASA or Hlassa
CALCUTTA
DACCA
MANDALAY
RANGOON
BOMBAY
NAGPUR
GANJAM
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MYSORE
CEYLON
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A R A B I A N   S E A
B A Y   o f   B E N G A L
A N D A M A N   I S L A N D S
N I C O B A R   I S L A N D S
from Aden and Suez

INDEX CHART
TO THE
GREAT TRIGONOMETRICAL SURVEY
OF
INDIA

SHOWING THE CHAINS OF PRINCIPAL TRIANGLES,
THE BASE LINES MEASURED WITH THE COLBY APPARATUS,
THE LINES OF THE SPIRIT LEVELLING OPERATIONS,
THE ASTRONOMICAL, PENDULUM & TIDAL STATIONS
THE LONGITUDINAL ARCS,
AND THE SECONDARY TRIANGULATION TO FIX THE PEAKS OF
THE HIMALAYAN & THE SULIMAN RANGES,
AND OTHER POSITIONS EXTERIOR TO THE BRITISH FRONTIER.

Completed to 1st October 1894.

Scale 1 Inch = 96 Miles or $\frac{1}{6\,082,560}$

REFERENCE

The Principal triangulation is shown in thick lines.
The Secondary triangulation is shown in thin lines.
The stations where the Latitude has been observed astronomically thus ☆
The stations where an Azimuth has been observed astronomically thus .... +
The course of the Levelling operations is shown by a dotted line ....
The Pendulum stations are shown thus .... ⊛
The Longitudinal Arcs are shown thus ...... — · — · — · —

Compiled under the orders of Colonel J. T. Walker, R. E., F. R. S., &c.
Supdt: Great Trigonometrical Survey of India at Dehra Dun, August 1870.

Published under the direction of COLONEL H. R. THUILLIER, C. I. E., R. E., Surveyor-General of India.
October 1893.

Engraved at the Survey of India Offices, Calcutta.

Reg. No. 672, Royal Society.—Aug. 95.—950.

XXII. BAKERIAN LECTURE.—*On the Laws of Connexion between the Conditions of a Chemical Change and its Amount.*—III. *Further Researches on the Reaction of Hydrogen Dioxide and Hydrogen Iodide.*

By A. VERNON HARCOURT, *M.A., F.R.S., Pres. C.S., Student of Christ Church, and Lee's Reader in Chemistry, and* WILLIAM ESSON, *M.A., F.R.S., Fellow of Merton College, and Deputy Savilian Professor of Geometry in the University of Oxford.*

Received March 15,—Read May 9, 1895.

NEARLY thirty years ago we laid before the Royal Society the results of our inquiries into two cases of gradual chemical change, viz.: (1) the reaction of hydrogen permanganate and hydrogen oxalate, and (2) the reaction of hydrogen dioxide and hydrogen iodide.*

We have continued at intervals our investigation into the latter reaction, and have obtained some further results, which we desire now to communicate to the Royal Society.

The manner in which our observations were made is fully described in the previous paper, to which we hope the reader will refer; but it may be convenient after so long an interval again to describe briefly our mode of observing and of calculating the rate of change. The vessel employed consisted of a tall glass cylinder, 12 × 3 inches, round which, about $2\frac{1}{2}$ inches from the top, a fine line had been etched. The cylinder was closed by a caoutchouc stopper, through which passed (1) an inverted funnel-tube in the centre, (2) a thermometer, (3) a short tube, $1 \times \frac{1}{2}$ inch, giving access to the interior.

Into the cylinder were poured water and measured quantities of solutions of all the reacting substances except hydrogen dioxide; the temperature was brought to the desired degree; and more water was added till the upper surface of the liquid coincided with the line round the cylinder. Then a measure of hydrogen dioxide was brought in. Large bubbles of carbonic acid were sent from the funnel-tube through the liquid to act as a stirrer. To provide for the ready addition from time to time of equal quantities of sodium thiosulphate, by which the change is measured, without materially altering by these additions the total volume of liquid, single drops of a strong solution of the salt were collected in tubes about 6 inches long, which could be dipped into the cylinder through the short tube. These drops, which were found to be exceedingly uniform, were brought one at a time into the liquid whenever it showed the blue colour of iodized starch. Each drop removed the iodine which had been formed since the change of colour was previously observed, and, subsequently, so much of the iodine which was continually being formed as it was capable of reacting with. As soon as the small portion of thiosulphate was exhausted, iodine was no longer removed, the liquid became blue again, and the moment of change was noted. Thus were observed the successive intervals required for the performance of a known fraction of the total change, and from these the rate of the change was inferred. The time that elapses between two successive appearances of the blue colour becomes continually greater as the amount of peroxide in the solution diminishes, and, finally, the last measure of thiosulphate requires for conversion into tetrathionate more iodine than the residual peroxide can furnish, and the blue colour does not return. After many hours the excess of thiosulphate was determined by means of a dilute solution of iodine; one or more drops of thiosulphate were then brought into the liquid and determined in the same manner. These observations give the value of one drop and of the remainder of the last drop in terms of the solution of iodine; calling the first of these d, the second r, and the number of drops added between the first and last appearances of the blue colour n, r/d is the fraction of a drop which remained unacted upon, and $(d-r)/d$ is the fraction of a drop acted upon by the last portion of peroxide. Thus the amount of peroxide originally taken is $n+(d-r)/d$, and of this in each observed interval an unit disappeared, the unit being the mass of peroxide which reacts with the mass of thiosulphate dissolved in one drop.

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* 'Philosophical Transactions,' vol. 156, p. 193, and vol. 157, p. 117. Also 'Journal of the Chemical Society,' vol. 20, p. 460.

These observations furnish the means of measuring the time required for a definite amount of chemical change under known conditions. The time required varied because one condition was continually varying, namely, the amount of peroxide in the liquid.

If y is the amount of peroxide at a time t, y' at a time t', the relation connecting these quantities is

$$y' = ye^{-\alpha(t'-t)},$$

in which α is the fraction of the peroxide which disappears in an unit of time. Each observation furnishes a value of α, and the mean of the values so obtained from a set of observations is taken as the true value under the conditions of each experiment.

In our former paper the effect of varying two of the conditions of the reaction is established, namely, that the amount of change is directly proportional (1) to the amount of dioxide, and (2) to the amount of iodide in one volume of the solution. Since the amount of change is also directly proportional to the number of volumes, or total volume, of the solution, and to the time during which the change proceeds, we expressed the whole amount of change under any conditions of the reaction by the equation

$$\Sigma = ditv \,.\, f(a, b, c, \ldots),$$

where d represents the amount of dioxide, i the amount of iodide in an unit volume of the solution, t the time during which the change proceeds, v the number of unit volumes or total volume of the solution, and $a, b, c, \ldots$ other conditions of the change.

Of these other conditions two only are necessary conditions of the change, temperature, and the presence of water; but the addition to the solution of other substances which play no part in the final reaction affects the rate at which the change proceeds, and thus a very large number of observations may be made of the influence which different substances exercise upon the rate of one chemical change, furnishing a basis for the comparison and classification of such substances.

In all the experiments here recorded one of the reacting substances has been either hydrogen dioxide, or sodium dioxide, and the other either hydrogen iodide, sodium iodide, or potassium iodide. Beside these, water has always been present, and, as a rule, in preponderating quantity. Other substances added in different series of experiments have been hydrogen sulphate, hydrogen chloride, sodium hydrogen carbonate, sodium chloride, sodium sulphate. None of these substances, as far as we know, any more than the water which they replace, participates directly in the reaction. We have also made sets of observations at different temperatures with the view of determining the relation between this condition of the reaction and the rate of change.

As in our former paper, we express the conditions of each experiment by stating the temperature and the amount of each ingredient in 1 cub. centim. of the solution; and in order to present at once known masses and atomic or molecular proportions, we use the symbol H to express one-millionth of a gram of hydrogen, and the other chemical symbols italicized to express the corresponding atomic masses; for example H^2SO^4 stands for 98 millionths of a gram, or ·098 milligram of hydrogen sulphate. The actual volume of solution used (which was rather less than 1 litre) and the actual weights of the substances dissolved, are only material as furnishing the data for calculating the ingredients of an unit volume. The rate of change, being the ratio of actual to potential change, is the same in a litre and in a cubic centimetre.

It has been shown in the previous paper that if $y_0\,(H^2O^2 + 2HI = I^2 + 2H^2O)$ is the amount of potential change at a time t_0, and $y_1\,(H^2O^2 + 2HI = I^2 + 2H^2O)$ is the amount of potential change at a time t_1, the quantities y_0, y_1, t_0, t_1, are connected by the equation given above,

$$y_1 = y_0\, e^{-\alpha(t_1 - t_0)},$$

$y_0 - y_1$ being the number of units of actual change which have taken place during the interval $t_1 - t_0$. If the interval $t_1 - t_0$ is diminished without limit, the ratio $\frac{y_0 - y_1}{t_1 - t_0}$ is equal to αy_0. That is to say, the amount of change occurring at every moment is in a constant ratio to the remaining amount of potential change.

The substances used in most of our sets of observations were (1) a solution containing hydrogen dioxide, made by dissolving sodium dioxide in a slight excess of dilute hydrogen sulphate ; (2) a solution of potassium iodide or hydrogen iodide ; and (3) dilute hydrogen sulphate. Of the last-named acid a relatively large quantity was generally taken, varying from 1 cub. centim. to 60 cub. centims. of concentrated sulphuric acid.

Variation of Hydrogen Sulphate.

The following observations were made in order to ascertain the influence on the rate of change of variations in the amount of hydrogen sulphate. At the head of Table I. the absolute quantities of each substance are stated as numbers of millionth-gram molecules per cub. centim., n being the quantity varied in successive experiments, and z the unknown fraction of the potassium iodide taken, which, in each experiment, undergoes the decomposition

$$\mathrm{KI} + \mathrm{H^2SO^4} = \mathrm{HI} + \mathrm{KH\,SO^4}.$$

The temperature of the solutions was in every case 30° C. The numbers represented as y_0 are the numbers of drops of thiosulphate required to reduce all the iodine formed in the course of each experiment by the measure of hydrogen dioxide taken. The intervals between successive observations are expressed in minutes ; and α is the fraction of the hydrogen peroxide present at any moment, which would be decomposed if the amount present at that moment were kept constant for one minute.

TABLE I.

$(38{\cdot}1n - 3{\cdot}64z)\,H^2SO^4$, $3{\cdot}64\,\{(1 - z)\,KI + zHI\}$, $3{\cdot}64zKHSO^4$, $(55400 - 65n)\,H^2O$. Temperature 30° C.

$n = 5$, $y_0 = 11{\cdot}2$.		$n = 6$, $y_0 = 11{\cdot}64$.		$n = 7$, $y_0 = 11{\cdot}7$.		$n = 8$, $y_0 = 11{\cdot}7$.		$n = 9$, $y_0 = 11{\cdot}67$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.
5′·32	·0175	4′·47	·0201	3′·98	·0224	3′·62	·0245	3′·32	·0270
5·88	·0175	4·96	·0199	4·44	·0221	3·96	·0247	3·63	·0271
6·52	·0177	5·50	·0199	4·90	·0222	4·44	·0245	4·05	·0270
7·45	·0175	6·19	·0199	5·53	·0220	4·96	·0246	4·53	·0270
8·51	·0175	7·06	·0199	6·30	·0221	5·67	·0245	5·20	·0269
10·05	·0175	8·19	·0200	7·28	·0222	6·58	·0245	6·02	·0269
12·17	·0175	9·75	·0200			7·90	·0244	7·22	·0269
15·47	·0175	12·08	·0201			9·70	·0247	8·91	·0270
21·30	·0176	15·88	·0202			12·87	·0244	11·82	·0269
		23·80	·0200			18·85	·0245	17·38	·0270
		47·45	·0199			36·15	·0245	33·95	·0269
Mean rate ·0175		Mean rate ·0200		Mean rate ·0222		Mean rate ·0245		Mean rate ·0270	

TABLE I.—(continued).

$n = 10$, $y_0 = 11·19$.		$n = 11$, $y_0 = 11·7$.		$n = 12$, $y_0 = 11·16$.		$n = 15$, $y_0 = 11·15$.		$n = 16$, $y_0 = 11·17$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.
3′·42	·0302	2′·78	·0321	2′·65	·0354	2′·18	·0429	2′·02	·0464
3·83	·0301	3·02	·0324	2·93	·0353	2·44	·0426	2·21	·0469
4·33	·0301	3·35	·0324	3·32	·0348	2·70	·0428	2·49	·0463
4·95	·0302	3·75	·0325	3·75	·0348	3·03	·0432	2·78	·0470
5·82	·0303	4·28	·0325	4·30	·0350	3·50	·0430	3·20	·0469
7·20	·0303	5·00	·0323	5·05	·0351	4·13	·0429	3·82	·0462
9·10	·0299	5·97	·0323	6·17	·0349	5·02	·0429	4·65	·0462
12·50	·0301	7·38	·0324	7·88	·0349	6·42	·0429	5·90	·0464
20·25	·0301	9·74	·0323	(2) 28·83*	·0348	8·91	·0428	8·18	·0462
		14·35	·0322	55·62	·0355	14·70	·0427	13·32	·0464
		27·41	·0323			47·24	·0430	41·28	·0466
Mean rate ·0301		Mean rate ·0323		Mean rate ·0350		Mean rate ·0429		Mean rate ·0465	

$n = 17$, $y_0 = 11·22$.		$n = 18$, $y_0 = 11·29$.		$n = 20$, $y_0 = 5·11$.		$n = 25$, $y_0 = 5·09$.		$n = 30$, $y_0 = 5·06$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.
1′·92	·0486	1′·83	·0507	3′·67	·0592	2′·83	·0773	2′·31	·0951
2·10	·0490	1·95	·0525	4·75	·0586	3·67	·0762	2·98	·0948
2·31	·0497	(2) 4·32	·0561	6·58	·0589	5·05	·0774	4·15	·0953
2·62	·0495	2·77	·0533	11·03	·0582	8·45	·0769	7·01	·0946
3·00	·0497	3·25	·0532	39·50	·0585			30·38	·0951
3·53	·0495	3·91	·0536						
4·27	·0498	4·99	·0531						
5·47	·0494	6·83	·0531						
7·50	·0495	10·70	·0537						
12·26	·0489	28·58	·0522						
34·80	·0492								
Mean rate ·0493		Mean rate ·0531		Mean rate ·0587		Mean rate ·0769		Mean rate ·0950	

It will be seen that approximately from $n = 5$ to $n = 12$ the values of α increase in arithmetical progression with a mean difference of ·00252 ; from $n = 15$ to $n = 20$ the values of α increase in arithmetical progression with a mean difference of ·00313 ; and from $n = 20$ to $n = 30$ the values of α increase in arithmetical progression with a mean difference of ·00363. The most probable values of α calculated on the hypothesis that there are three such series, are given in Table II., the three formulæ being

---

* "(2)" prefixed to an interval means that, an observation having been missed, the interval is that in which two units of change occurred.

$$\alpha = \cdot00250\, n + \cdot0047 \quad \ldots\ldots\ldots \quad (1),$$

$$\alpha = \cdot00312\, n - \cdot0037 \quad \ldots\ldots\ldots \quad (2),$$

$$\alpha = \cdot00367\, n - \cdot0147 \quad \ldots\ldots\ldots \quad (3).$$

Table II.

n.	α.		Differences $(\alpha' - \alpha)/(n' - n)$.	
	Found.	Calculated.	Found.	Calculated.
5	·0175	·0172		
6	·0200	·0197	·0025	·0025
7	·0222	·0222	·0022	·0025
8	·0245	·0247	·0023	·0025
9	·0270	·0272	·0025	·0025
10	·0301	·0297	·0031	·0025
11	·0323	·0322	·0022	·0025
12	·0350	·0347	·0027	·0025
13·5		·0384		·0025
13·5		·0384		
15	·0429	·0431		·0031
16	·0465	·0462	·0036	·0031
17	·0493	·0493	·0028	·0031
18	·0531	·0525	·0038	·0032
20	·0587	·0587	·0028	·0031
20	·0587	·0587		
25	·0769	·0770	·0036	·0037
30	·0950	·0954	·0036	·0037

In other sets of observations on the effects of varying the mass of hydrogen sulphate, and of other substances, in unit volume, the amounts of potassium iodide have been different, and sometimes hydrogen iodide or sodium iodide has been used instead of potassium iodide. For the comparison of the rates it is convenient to reduce them in all cases to what they would have been with an unit of iodide of whatever kind.

If i denotes the number of millionth-gram molecules of iodide and s the number of millionth-gram molecules of hydrogen sulphate in a cub. centim., the three formulæ become,

$$\alpha = i\,\{4730 + 18\,(s - 190\cdot5)\}\,10^{-6} \quad \ldots\ldots \quad (4),$$

$$\alpha = i\,\{10550 + 22{\cdot}5\,(s - 514)\}\,10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5),$$

$$\alpha = i\,\{16130 + 26{\cdot}5\,(s - 762)\}\,10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

(4) expresses the relation from $s = 190$ to $s = 514$; (5) from $s = 514$ to $s = 762$; (6) from $s = 762$ to $s = 1143$.

In these equations, and the corresponding equations which follow, the three constants have the following significance. Taking (4) as an example, the number 4730 depends on the nature of the reacting substances and on the temperature of the solution, it may also vary with the value of i (as is shown subsequently, p. 842) but not with the value of s. The number 18 also depends on the nature of the reacting substances and the temperature, but not on the values of i or s; it is the increment of rate with a millionth-gram molecule of iodide per cub. centim. due to each additional millionth-gram molecule of hydrogen sulphate. The number 190·5 is the minimum number of such molecules of hydrogen sulphate taken in any experiment of this series. It appears therefore that the increment in the rate of change due to each unit-substitution of sulphuric acid for water is constant till a certain ratio of acid to water is reached; at this point the increment suddenly rises and remains constant until another ratio of acid to water is reached, after which it again rises and then remains constant as far as our experiments proceeded.

In the most dilute solution, where $s = 190$, the number of water-molecules for every molecule of hydrogen sulphate was 289; where the increment of the rate first changes, at $s = 514$, the number of water-molecules was 106; where the second change occurs, at $s = 762$, it was 71, and in the last experiment it was 46.

These results, which are confirmed by two subsequent sets of observations, accord with a view which seems on other grounds to be probable, and which may be stated as follows. When a drop of sulphuric acid is mixed with a relatively large volume, such as 500 cub. centims. of water, the liquid consists of a mixture of water with whatever hydrate of the acid contains the largest number of molecules of water. If the addition of acid is continued slowly drop by drop, each drop increases the proportion of this first hydrate and diminishes the proportion of water, till a point is reached at which the liquid consists of the hydrate. After this point, a new order of events begins; a second hydrate is formed with a larger proportion of acid, its amount increasing and that of the first hydrate decreasing, till the liquid consists of the second hydrate; then begins the formation of a third hydrate, and so on. Thus, the change would proceed regularly, but discontinuously, through a number of successive hydrates, each gradually replacing its predecessor, till finally the liquid consisted of a mixture of the n^{th} hydrate, having the minimum proportion of water, with sulphuric acid. The influence of such hydrates upon the rate of a chemical change occurring in their presence may probably not be exactly in proportion to the ratio of acid to water in their composition. If so, when in our experiments, the amount of water or of one hydrate was decreased and that of another hydrate was increased in a regular arithmetical progression, it was to be expected that the successive rates would also increase in arithmetical progression, with an increment dependent upon the relative influence of the hydrate that was diminished in quantity and of that which was increased. So where a third hydrate was progressively replacing the second; but the increment of the rate due to the replacement of H^2SO^4, m H^2O, by H^2SO^4, n H^2O, might differ from that due to the replacement of H^2SO^4, n H^2O, by H^2SO^4, p H^2O.

It will be seen that our experimental results accord with this hypothesis, and are inconsistent with the view that the gradual addition of sulphuric acid to water causes a continuous change in the composition of the liquid.

A second set of observations, made at a lower temperature, the conditions and results of which are given in Table III., shows that the value of the increment of rate, caused by successive replacements of water by hydrogen sulphate, changes abruptly at certain points. The position of these points, as of those assumed in the previous set of experiments, cannot be exactly determined. Small changes in the origin and inclination of the straight lines shown on Diagram 1, which are consistent with the position of the experimental points, materially alter the positions of the intersections. Thus, the existence of two hydrates is much more nearly established than their composition. It is, however, interesting to find that our results are perfectly consistent with the supposition that at the three temperatures at which we have experimented, 16°, 20°, and 30°, the composition of the two hydrates is the same. Their composition is probably not far removed from H^2SO^4, 106 H^2O, and H^2SO^4, 71 H^2O, and it may have some significance that these numbers are to one another in the ratio of 3 to 2.

TABLE III.

$54{\cdot}5nH^2SO^4$, $12{\cdot}2\ HI$, $(55500 - 93n)H^2O$. Temperature 16° C.

$n = 1$, $y_0 = 14{\cdot}02$.		$n = 2$, $y_0 = 6{\cdot}5$.		$n = 2$, $y_0 = 6{\cdot}24$.		$n = 3$, $y_0 = 6{\cdot}55$.		$n = 4$, $y_0 = 6{\cdot}39$.	
Intervals.	a.	Intervals.	a.	Intervals.	a.	Intervals.	a.	Intervals.	a.
6′·68	·0111	10′·57	·0158	11′·13	·0157	8′·27	·0200	6′·83	·0249
7·20	·0111	12·77	·0157	13·58	·0156	9·95	·0200	8·32	·0247
7·83	·0111	16·02	·0157	17·38	·0155	12·40	·0200	10·43	·0248
8·57	·0111	21·38	·0157	23·65	·0156	16·67	·0198	14·15	·0247
9·47	·0111	32·45	·0157	38·20	·0155	25·07	·0198	22·05	·0246
10·57	·0111			106·10	·0157	51·28	·0202	51·57	·0248
12·00	·0111								
13·83	·0111								
16·23	·0112								
Mean rate ·0111		Mean rate ·0157		Mean rate ·0156		Mean rate ·0200		Mean rate ·0247	

TABLE III.—(continued).

$n = 6$, $y_0 = 6·4$.

Intervals.	α.
4′·98	·0341
6·02	·0340
7·58	·0340
10·27	·0338
15·73	·0342
37·00	·0339
Mean rate	·0340

$n = 6$, $y_0 = 4·77$.

Intervals.	α.
6′·97	·0337
9·12	·0338
13·22	·0339
24·55	·0339
Mean rate	·0338

$n = 8$, $y_0 = 6·4$.

Intervals.	α.
3′·87	·0439
4·68	·0438
5·95	·0434
8·05	·0433
12·47	·0432
29·00	·0433
Mean rate	·0434

$n = 8$, $y_0 = 6·15$.

Intervals.	α.
4′·11	·0433
4·94	·0437
6·39	·0432
8·87	·0431
14·60	·0430
46·96	·0433
Mean rate	0·433

$n = 10$, $y_0 = 6·45$.

Intervals.	α.
3′·15	·0535
3·80	·0532
4·75	·0535
6·48	·0528
9·93	·0528
22·00	·0532
Mean rate	·0532

$n = 10$, $y_0 = 6·2$.

Intervals.	α.
3′·30	·0533
4·04	·0528
5·11	·0532
7·07	·0530
11·46	·0528
33·79	·0531
Mean rate	·0530

$n = 12$, $y_0 = 6·4$.

Intervals.	α.
3′·20	·0640
4·03	·0640
5·42	·0643
8·38	·0643
19·52	·0643
Mean rate	·0642

$n = 12$, $y_0 = 6·18$.

Intervals.	α.
3′·32	·0644
4·25	·0643
5·88	·0641
9·53	·0641
28·84	·0643
Mean rate	·0642

$n = 14$, $y_0 = 6·44$.

Intervals.	α.
2′·2	·0767
2·7	·0752
3·35	·0762
4·55	·0755
7·00	·0753
15·52	·0764
Mean rate	·0759

$n = 14$, $y_0 = 6·12$.

Intervals.	α.
2′·87	·0757
3·66	·0761
5·11	·0757
8·49	·0752
29·57	·0755
Mean rate	·0756

$n = 16$, $y_0 = 6·42$.

Intervals.	α.
1′·87	·0904
2·28	·0893
2·84	·0902
3·87	·0893
5·96	·0894
13·57	·0894
Mean rate	·0897

$n = 16$, $y_0 = 6·25$.

Intervals.	α.
1′·93	·0902
2·37	·0893
3·03	·0885
4·23	·0869
6·65	·0883
18·13	·0887
Mean rate	·0886

$n = 16$, $y_0 = 7$.

Intervals.	α.
2′·01	·0908
2·49	·0895
3·24	·0889
4·48	·0906
7·76	·0893
Mean rate	·0898

$n = 17$, $y_0 = 6·28$.

Intervals.	α.
2′·18	·0964
2·77	·0962
3·80	·0955
6·07	·0951
15·67	·0968
Mean rate	·0960

$n = 18$, $y_0 = 6·5$.

Intervals.	α.
(2) 4′·43	·1021
3·32	·1014
5·02	·1017
10·74	·1023
Mean rate	·1019

TABLE III.—(continued).

$n = 18$, $y_0 = 6·35$.		$n = 18$, $y_0 = 6·23$.		$n = 18$, $y_0 = 6·98$.		$n = 20$, $y_0 = 6·53$.		$n = 20$, $y_0 = 6·29$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α	Intervals.	α.
′		′		′		′		′	
2·02	·1026	2·08	·1017	2·17	·1035	1·72	·1161	1·77	·1184
2·60	·1005	2·63	·1024	2·78	·1019	2·15	·1161	2·24	·1184
3·48	·1019	3·63	·1019	2·98	·1030	2·85	·1167	3·07	·1180
5·48	·1012	5·85	·1014	6·90	·1019	13·48	·1156	4·88	·1186
13·17	·1023	16·33	·1014					12·70	·1184
Mean rate ·1017		Mean rate ·1018		Mean rate ·1025		Mean rate ·1161		Mean rate ·1183	

It will be seen that approximately from $n = 1$ to $n = 8$ the values of α are in arithmetical progression with a mean difference ·0046, from $n = 10$ to $n = 14$ the values of α are in arithmetical progression with a mean difference ·0056, and from $n = 14$ to $n = 20$ the values of α are in arithmetical progression with a mean difference of ·0068. The most probable values of α calculated on this hypothesis are given in the third column of Table IV., the formulæ from $n = 1$ to $n = 8$, from $n = 10$ to $n = 14$, and from $n = 14$ to $n = 20$, being severally,

$$\alpha = {\cdot}0046\,n + {\cdot}0064 \quad \ldots\ldots\ldots \quad (7),$$

$$\alpha = {\cdot}0057\,n - {\cdot}004 \quad \ldots\ldots\ldots \quad (8),$$

$$\alpha = {\cdot}0067\,n - {\cdot}0180 \quad \ldots\ldots\ldots \quad (9).$$

TABLE IV.

n.	α.		Differences $(\alpha' - \alpha)/(n' - n)$.	
	Found.	Calculated.	Found.	Calculated.
1	·0111	·0110		
2	·0156	·0156	·0045	·0046
3	·0200	·0202	·0044	·0046
4	·0247	·0248	·0047	·0046
6	·0339	·0340	·0046	·0046
8	·0433	·0432	·0047	·0046
9·46		·0499		·0046
9·46		·0499		
10	·0531	·0530		·0057
12	·0642	·0644	·0055	·0057
14	·0757	·0758	·0057	·0057
14	·0757	·0758		
16	·0894	·0892	·0067	·0067
17	·0960	·0959	·0069	·0067
18	·1020	·1026	·0060	·0067
20	·1172	·1160	·0076	·0067

If i and s denote the number of HI and H^2SO^4 in a cubic centimetre, the three formulæ become

$$\alpha = i\,\{902 + 6{\cdot}92\,(s - 54{\cdot}5)\}\,10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10),$$

$$\alpha = i\,\{4090 + 8{\cdot}57\,(s - 515)\}\,10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11),$$

$$\alpha = i\,\{6210 + 10{\cdot}08\,(s - 763)\}\,10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12).$$

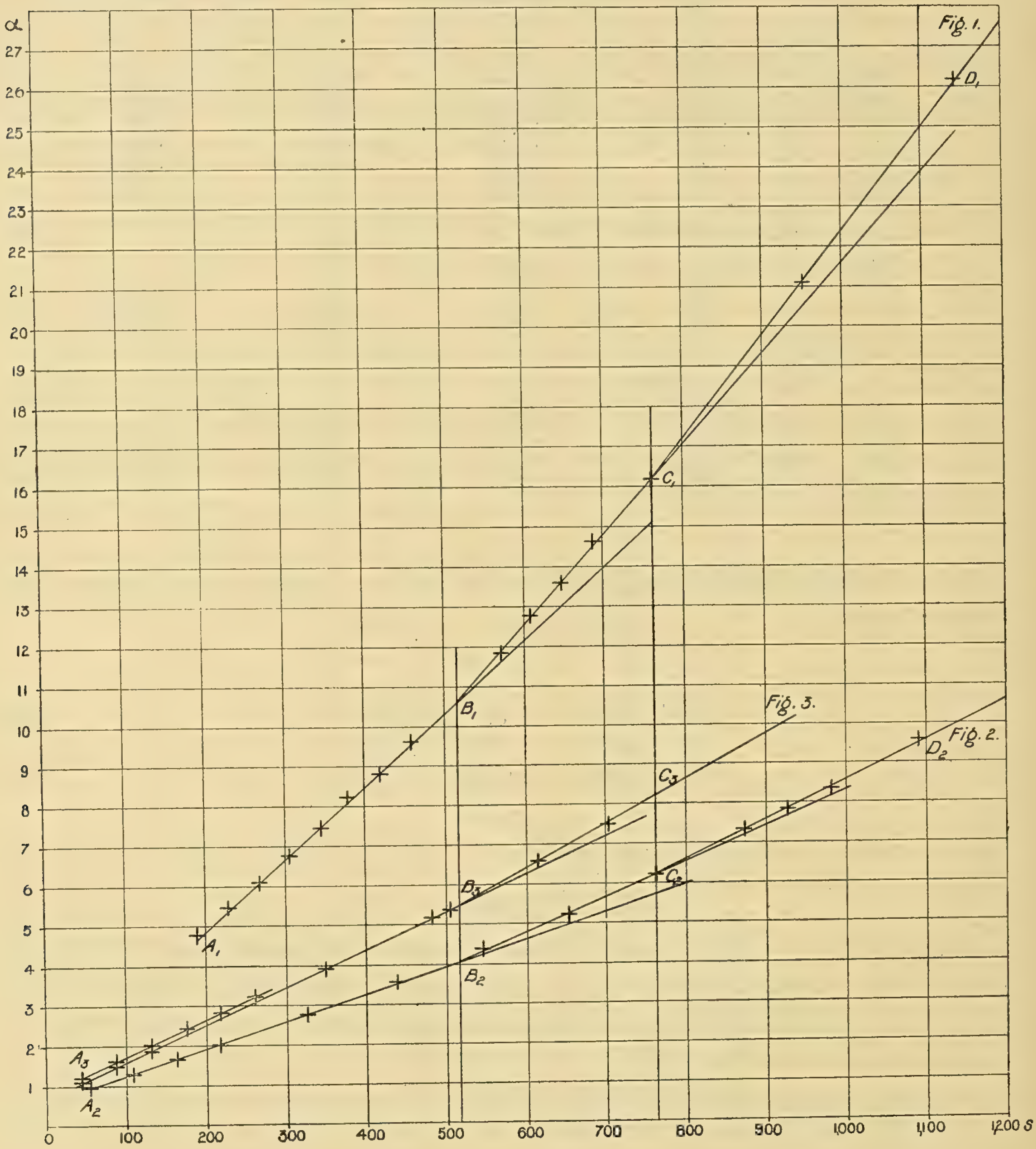
(10) expresses the relation from $s = 54$ to $s = 515$, (11) from $s = 515$ to $s = 763$, (12) from $s = 763$ to about $s = 1100$.

If these three equations are compared with the corresponding three, (4), (5), and (6), deduced from the observations with a liquid of higher temperature, it will be seen that the increment in the rate of change due to an unit of hydrogen sulphate undergoes the same abrupt transitions at the same values of s, viz., $s = 515$ and 763 in this case, and $s = 514$ and 762 in the former case.

Also the ratios of the corresponding values of the increment per unit of hydrogen sulphate between and above these points are very nearly the same in the two sets, namely, $18 : 6{\cdot}92 = 2{\cdot}61$; $22{\cdot}5 : 8{\cdot}57 = 2{\cdot}62$; $26{\cdot}5 : 10{\cdot}08 = 2{\cdot}63$; the mean ratio being 2·62. Thus a rise of temperature of 14° augments in all three cases the increment of the rate of change per unit of hydrogen sulphate in the ratio 2·62 : 1.

This result will be found to agree absolutely with the ratio deduced from experiments on the effect of increase of temperature upon the rate of change (p. 868), according to which the ratio should be $\left(\frac{273 + 30}{273 + 16}\right)^{20\cdot38} = 2\cdot62$.

Diagram 1.



On Diagram 1, figs. 1, 2, and 3, the units of hydrogen sulphate are measured along the horizontal axis and the units of actual change per thousand of potential change, per minute, along the vertical axis. The ordinate at each point is found by dividing the experimental rate by the number of units of iodide and, for convenience of scale, multiplying by one thousand. The lines are those expressed by the equations (4), (5), (6), &c., multiplied by $1000/i$. For example, the position of the point D_1 denotes that in a solution of the temperature 30°, containing per cubic centim. HI and 1143 H^2SO^4 and a constant amount of hydrogen dioxide corresponding to 1000 units of potential change, the number of units of actual change in one minute would be 26·1. The line A_1B_1 is that expressed by the equation $\alpha = \{4730 + 18\,(s - 190{\cdot}5)\}\,10^{-3}$, the ordinate of the lowest point of which, when $s = 190{\cdot}5$, is 4·73. It will be seen that the straight lines A_1B_1, B_1C_1, C_1D_1, and the corresponding lines in figs. 2 and 3, pass very close to the experimental points and that no continuous curve would pass as near these points. Of these lines, A_1B_1 represents the law of variation of the rate of change with hydrogen sulphate when the units of the latter vary between 190 and 514, B_1C_1 when the units vary between 514 and 762, C_1D_1 when the units vary between 762 and 1143. Similarly A_2B_2 represents the law of variation of the rate of change at 16° when the units of hydrogen sulphate vary from 54 to 515, B_2C_2 when the units vary from 515 to 763, C_2D_2 when the units vary from 763 to 981. The points of abrupt transition of value of the increment of the rate of change per unit of hydrogen sulphate are in the one case B_1 and C_1; in the other B_2 and C_2.

As the intervals to be observed in the last three or four sets of the series at 16° C. (Tables III. and IV.) were rather short for accurate determination, the sets were repeated with one-third the amount of hydrogen iodide previously used. The conditions and results are given in the following Table :—

TABLE V.

54·5 n H^2SO^4, 4·07 HI, $(55500 - 93\,n)$ H^2O. Temperature 16° C.

$n = 14$, $y_0 = 7{\cdot}14$.		$n = 16$, $y_0 = 7{\cdot}01$.		$n = 18$, $y_0 = 7{\cdot}1$.		$n = 20$. $y_0 = 7{\cdot}01$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α,
5·96	·0253	5·16	·0298	4·43	·0343	3·88	·0396
7·07	·0252	6·14	·0297	5·27	·0340	4·64	·0393
8·62	·0251	7·59	·0293	6·34	·0344	5·72	·0389
11·02	·0251	9·67	·0297	8·19	·0341	7·36	·0389
15·20	·0252	13·62	·0297	11·44	·0340	10·40	·0388
25·01	·0252	23·18	·0297	19·05	·0339	17·62	·0390
				70·31	·0341		
Mean rate .	·0252	Mean rate .	·0297	Mean rate .	·0341	Mean rate .	·0391

Comparing the rates in this and the previous series, it is seen that they agree, the effect of reducing the amount of iodide to one-third being to reduce the rate in the same proportion. With 14, 16, 18, and 20 proportions of hydrogen sulphate the rates are,

in the first case	·0757,	·0894,	·1020,	·1172;
in the second case	·0252,	·0297,	·0341,	·0391.
The top row, divided by 3, become	·0252,	·0298,	·0340,	·0391.

Table VI. gives the conditions and results of a fourth series of experiments, in which smaller proportions of hydrogen sulphate were taken, rates convenient for observation being maintained by using more iodide and a higher temperature.

TABLE VI.

7·62 nH^2SO^4, 14·22 HI, (55400 − 15n) H^2O. Temperature 30° C.

$n = 0$, $y_0 = 17$.		$n = 1$, $y_0 = 17{\cdot}7$.		$n = 2$, $y_0 = 15{\cdot}65$.		$n = 3$, $y_0 = 17{\cdot}8$.		$n = 4$, $y_0 = 17{\cdot}86$.	
Intervals.	a.	Intervals.	a.	Intervals.	a.	Intervals.	a.	Intervals.	a.
′		′				′		′	
2·90	·0209	(2) 4·92	·0244	2·43	·0271	1·93	·0300	1·82	·0316
2·97	·0217	2·71	·0243	2·63	·0269	2·07	·0296	1·90	·0320
3·30	·0209	2·94	·0240	2·79	·0272	2·22	·0294	2·01	·0324
3·53	·0210	3·10	·0244	3·06	·0269	2·35	·0299	2·17	·0321
3·82	·0209	3·40	·0242	3·32	·0270	2·56	·0294	2·30	·0318
4·16	·0209	3·68	·0243	3·62	·0272	2·72	·0299	2·53	·0320
4·59	·0207	4·07	·0240	4·08	·0268	2·95	·0300	2·75	·0320
5·08	·0207	4·52	·0242	4·58	·0269	3·33	·0292	2·99	·0323
5·62	·0209	5·02	·0243	5·17	·0271	3·65	·0295	3·36	·0318
6·40	·0209	5·77	·0241	6·00	·0272	(2) 8·65	·0299	3·75	·0319
7·26	·0212	6·71	·0240	7·20	·0270	5·45	·0292	4·27	·0319
		7·92	·0243	8·97	·0270	6·44	·0294	4·95	·0318
		9·77	·0245	11·95	·0268	7·98	·0292	5·85	·0320
		12·91	·0244	17·41	·0272	10·17	·0301	7·18	·0321
		19·00	·0243			14·63	·0302	9·40	·0319
		37·00	·0240			27·12	·0299	37·17	·0324
Mean rate ·0210		Mean rate ·0242		Mean rate ·0270		Mean rate ·0297		Mean rate ·0320	

The rates are very nearly in arithmetical progression, as the following comparison shows:—

TABLE VII.

n.	α.	α'.	Differences, $\alpha_n - \alpha_{n-1}$.	Difference per unit, HI and H^2SO^4.
0	·0210	·0213		
1	·0242	·0241	·0032	·0000295
2	·0270	·0268	·0028	·0000258
3	·0297	·0296	·0027	·0000249
4	·0320	·0323	·0023	·0000212

The rates given under α' are in arithmetical progression, with a difference ·00276. The actual differences, which follow, show however a fall in value which is not likely to be accidental, and may mean that when such small quantities of sulphuric acid are added as 1, 2, 3, 4 molecules to about 7000 molecules of water, the increment of the rate with each addition is nearly constant, but shows a slight decrease. Probably the main effect is a constant increment, but some slight secondary effect of the presence of sulphuric acid, in very dilute solutions, causes a further acceleration, which, with larger quantities, is no longer discernible.

Tables VIII., VIII.A, and IX. give the results of two further series of experiments, in which the rate of change was determined in presence of quantities of sulphuric acid such that the ratio of the number of molecules of water to one molecule of sulphuric acid varied from 830,000, where no sulphuric acid was added beyond the small quantity present in the solution of hydrogen dioxide, to 77. In the first series of experiments the amount of sulphuric acid was increased until the proportion of water molecules was reduced to 209. The earlier experiments of this series are placed in a separate table (VIII.A), and are reserved for subsequent discussion, because in this case also the effect of the first additions of very small quantities of acid differs from that of subsequent additions. The second series (Table IX.) was a continuation of the first, except that the mass of hydrogen iodide per cubic centimetre was reduced from 22·8 to 11·4 HI, in order to prevent the rate of change with the larger proportions of sulphuric acid becoming too great for accurate observation.

TABLE VIII.—$(\cdot 067 + 43\cdot 85n)\, H^2SO^4$, $22\cdot 82\, HI$, $(55474 - 81n)\, H^2O$.
Temperature, 20° C.

$n = 1$, $y_0 = 8\cdot 485$.		$n = 2$, $y_0 = 8\cdot 29$.		$n = 3$, $y_0 = 8\cdot 115$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.
4′·60	·0272	3′·50	·0367	2′·88	·0457
5·27	·0272	4·00	·0369	3·28	·0461
6·13	·0274	4·72	·0367	3·92	·0455
7·43	·0271	5·68	·0369	4·75	·0458
9·27	·0272	7·20	·0368	6·07	·0458
12·35	·0274	9·87	·0367	8·45	·0457
18·75	·0274	15·58	·0369	14·18	·0451
41·00	·0272	40·63	·0367	49·93	·0455
Mean rate . .	·0273	Mean rate . .	·0368	Mean rate . .	·0457

$n = 4$, $y_0 = 8\cdot 84$.		$n = 5$, $y_0 = 8\cdot 495$.		$n = 6$, $y_0 = 6\cdot 83$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.
2′·48	·0551	1′·97	·0636	2′·18	·0727
2·87	·0551	2·23	·0642	2·57	·0732
3·38	·0555	2·63	·0636	3·20	·0725
4·22	·0549	3·17	·0634	4·15	·0730
5·43	·0555	3·95	·0637	6·00	·0727
7·90	·0549	5·28	·0638	10·83	·0730
14·22	·0551	8·08	·0634		
		17·33	·0638		
Mean rate . .	·0551	Mean rate . .	·0637	Mean rate . .	·0729

TABLE VIIIA.—$(\cdot067 + 8\cdot8n)\,H^2SO^4$, $22\cdot82\,HI$. Temperature 20° C.

$n = 0$, $y_0 = 8\cdot88$.		$n = 1$, $y_0 = 7\cdot35$.		$n = 3$, $y_0 = 5\cdot67$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.
7′·18	·0166	7′·40	·0198	8′·25	·0235
8·13	·0167	8·70	·0197	10·30	·0234
9·40	·0167	10·47	·0198	13·57	·0234
11·22	·0166	13·27	·0197	20·10	·0233
13·77	·0166	18·02	·0196	38·98	·0234
17·90	·0166	28·23	·0196		
28·60	·0167	67·13	·0201		
45·53	·0167				
Mean rate . .	·0167	Mean rate . .	·0197	Mean rate . .	·0234

TABLE IX.—$(\cdot067 + 43\cdot85n)\,H^2SO^4$, $11\cdot41\,HI$, $(55537 - 88\cdot5n)\,H^2O$.
Temperature 20° C.

$n = 1$, $y_0 = 6\cdot7$.		$n = 2$, $y_0 = 5\cdot95$.		$n = 3$, $y_0 = 6\cdot25$.		$n = 5$, $y_0 = 6\cdot92$	
Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.
12′·72	·0127	10′·35	·0178	7′·83	·0222	4′·87	·0321
15·24	·0127	12·80	·0176	9·55	·0221	5·80	·0319
18·88	·0127	16·87	·0173	12·25	·0219	7·08	·0321
24·89	·0127	23·68	·0175	16·73	·0220	9·28	·0318
36·39	·0127	40·88	·0176	26·78	·0219	13·20	·0317
69·70	·0127			73·70	·0218	22·98	·0320
Mean rate .	·0127	Mean rate .	·0176	Mean rate .	·0220	Mean rate .	·0319

$n = 8$, $y_0 = 6\cdot49$.		$n = 11$, $y_0 = 7\cdot145$.		$n = 14$, $y_0 = 5\cdot86$.		$n = 16$, $y_0 = 6\cdot295$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.
3′·75	·0446	2′·57	·0587	2′·50	·0749	2′·02	·0858
4·53	·0444	3·00	·0592	3·05	·0756	2·43	·0860
5·68	·0444	3·67	·0588	4·02	·0747	3·13	·0848
7·57	·0446	4·68	·0591	5·77	·0747	4·22	·0853
11·50	·0446	6·48	·0591	10·28	·0750	6·70	·0856
24·83	·0448	10·68	·0590				
		35·08	·0588				
Mean rate .	·0446	Mean rate .	·0589	Mean rate .	·0749	Mean rate .	·0856

In the preceding tables the number of millionth-gram molecules of water per cub. centim. was determined by weighing a known volume of the liquid after each set of observations. The total mass thus found, less the masses of the other substances, was the mass of water, and this divided by the volume and by 18, the molecular weight of water, and multiplied by a million, gave the required number. It was found that these numbers decreased very nearly in arithmetical progression with the increase of sulphuric acid, and this relation is assumed in the formulæ $(55474 - 81n)\, H^2O$ at the head of Table VIII., and $(55537 - 88{\cdot}5n)\, H^2O$ at the head of Table IX.

In the following table the results given in Table VIII. are compared with a series in arithmetical progression.

TABLE X.

n.	α.		Differences $(\alpha' - \alpha)/(n' - n)$.	
	Found.	Calculated.	Found.	Calculated.
1	·0273	·0275		
2	·0368	·0366	·0095	·0091
3	·0457	·0457	·0089	·0091
4	·0551	·0548	·0094	·0091
5	·0637	·0639	·0086	·0091
6	·0729	·0730	·0092	·0091

The calculated values of α are deduced from the formula

$$\alpha = {\cdot}01844 + {\cdot}00909n \quad . \; . \; . \; . \; . \; . \; . \quad (13),$$

which for i units of iodide and s H^2SO^4 in one cub. centim. becomes

$$\alpha = i\,\{1206 + 9{\cdot}08\,(s - 43{\cdot}85)\}10^{-6} \quad . \; . \; . \; . \; . \; . \; . \quad (14).$$

This formula is represented graphically on Diagram 1, by the straight line above A_3B_3 and parallel to it.

The coefficient of s, $9{\cdot}08 \times 10^{-6}$, when multiplied by the factor $\left(\frac{273 + 30}{273 + 20}\right)^{20{\cdot}38}$ which (p. 868, *et seq.*) expresses the effect of an increase of temperature from 20° to 30°, gives 18×10^{-6}, which according to equation (4) is the coefficient of s for similar concentrations at 30°.

Table XI. gives a comparison of the results shown on Table IX. with two series in arithmetical progression.

TABLE XI.

n.	α.		Differences $(\alpha' - \alpha)/(n' - n)$.	
	Found.	Calculated.	Found.	Calculated.
1	·0127	·0130		
2	·0176	·0175	·0049	·0045
3	·0220	·0221	·0044	·0046
5	·0319	·0312	·0049	·0045
8	·0446	·0450	·0043	·0046
11	·0589	·0587	·0048	·0046
11·75	..	·0621	..	·0045
11·75	..	·0621	..	
14	·0749	·0748	..	·0056
16	·0856	·0861	·0053	·0056

The calculated values of α between $n = 1$ and $n = 11{\cdot}75$, and between $n = 11{\cdot}75$ and $n = 16$ are deduced from the equations

$$\alpha = \cdot 01297 + \cdot 00457\,(n - 1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15),$$

$$\alpha = \cdot 0621 + \cdot 00564\,(n - 11{\cdot}75) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16).$$

The value of α common to these two formulæ is ·0621, which corresponds to $n = 11{\cdot}75$, or $s = 515$ (represented by the point B_3 in Diagram 1, fig. 3), and to the hydrate, H^2SO^4, 106 H^2O.

Stated for millionth-gram molecules of iodide and hydrogen sulphate the formulæ become

$$\alpha = i\,\{1139 + 9{\cdot}13\,(s - 43{\cdot}85)\}\,10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17),$$

$$\alpha = i\,\{5440 + 11{\cdot}27\,(s - 515)\}\,10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18).$$

These formulæ are represented by the straight lines A_3B_3, B_3C_3, fig. 3, Diagram 1.

The coefficient of s in (17), namely 9·13, agrees closely with the coefficient of s in (14) which is 9·08. The ratios of the coefficients of s in (17) and in the corresponding formula (4) for the series at 30° is 1·97, and the ratio of the coefficients of s in (18) and in (5) is 1·98. The ratio for a change of temperature from 20° to 30° given by the formula $\left(\frac{273 + 30}{273 + 20}\right)^{20{\cdot}38}$ is 1·98. Similarly, a comparison of the coefficients of s in (17) and (18) with the corresponding values at 16° in (10) and (11) gives the ratios

1·32 and 1·315, agreeing in both cases with the ratio calculated from the temperature formula $\left(\frac{273 + 20}{273 + 16}\right)^{20\cdot38} = 1\cdot32$.

There has still to be considered the effect of the small quantities of hydrogen sulphate in the presence of which the observations were made of which the details have been given in Table VIIIA. Their results may be summarized as follows:—

TABLE XII.

n.	α.	Differences $(\alpha'-\alpha)/(n'-n)$.
0	·0167	
1	·0197	·0030
3	·0234	·0018

The increment of rate between the first pair of determinations, due to an addition of 8·8 H^2SO^4 per cub. centim., and the corresponding diminution of water, is ·003, giving for the increment per unit, or coefficient of s, $14\cdot9 \times 10^{-6}$. A comparison of these results with those obtained with similarly small proportions of hydrogen sulphate (Table VII.) shows that both at 20° and at 30° the first small addition of hydrogen sulphate causes a much greater increment of the rate than subsequent small additions, the actual coefficient of s for values of s between 0 and 7·62 being at 30° $29\cdot5 \times 10^{-6}$, which, multiplied by the temperature-factor $\left(\frac{273 + 20}{273 + 30}\right)^{20\cdot38}$ gives the same number as the observations at 20°, namely, $14\cdot9 \times 10^{-6}$.

The coefficient of s between the values of s 8·9 and 26·5, is scarcely higher than it is over the range of values of s from 43·8 to 515 (see equation (15) and line A_3B_3, Diagram 1). At 30° the minimum and first constant value of the coefficient of s appears only to be reached when the proportion of acid is rather larger. More experiments with such very dilute solutions at different temperatures would probably yield interesting results.

Variation of Hydrogen Chloride.

The influence of hydrogen chloride upon the rate of change was next investigated. In Table XIII. are recorded the conditions and the rates of change found in a series of sets of observations in which the number of HCl per cub. centim. were varied from 0 to 355.

TABLE XIII.

$71·1\ n\ HCl$, $14·22\ HI$. Temperature, 30° C.

$n = 0$, $y_0 = 7·51$.	
Intervals.	α.
6·90	·0207
7·90	·0211
9·60	·0208
12·18	·0206
16·23	·0207
(2) 76·60	·0208
Mean rate .	·0208

$n = 0$, $y_0 = 6·73$.	
Intervals.	α.
(2) 16·53	·0213
(2) 26·57	·0215
21·22	·0215
40·30	·0214
Mean rate .	·0214

$n = 1$, $y_0 = 7·41$.	
Intervals.	α.
3·65	·0397
4·30	·0395
5·18	·0395
6·55	·0393
8·80	·0395
13·37	·0401
31·37	·0394
Mean rate .	·0396

$n = 2$, $y_0 = 6·11$.	
Intervals.	α.
3·85	·0567
4·93	·0567
6·85	·0565
11·47	·0559
Mean rate .	·0564

$n = 3$; $y_0 = 6·3$.	
Intervals.	α.
2·35	·0735
2·81	·0743
3·63	·0730
4·91	·0736
7·61	·0750
19·85	·0738
Mean rate .	·0739

$n = 4$, $y_0 = 6·45$.	
Intervals.	α.
1·88	·0895
2·21	·0916
2·70	·0946
3·80	·0906
5·75	·0912
12·95	·0904
Mean rate .	·0913

$n = 4$, $y_0 = 7·72$.	
Intervals.	α.
1·50	·0925
1·78	·0906
2·12	·0906
2·58	·0923
3·43	·0914
5·03	·0912
9·60	·0908
Mean rate .	·0913

$n = 5$, $y_0 = 5·78$.	
Intervals.	α.
(2) 6·79	·108
4·17	·107
7·67	·107
Mean rate .	·1073

On Table XIV. the series of rates thus found are compared with numbers calculated on the hypothesis that the increase of rate is proportional to the increase of acid, from the formula

$$\alpha = \cdot0397 + \cdot017\,(n - 1)\ .\ .\ .\ .\ .\ .\ .\ .\ .\ . \qquad (19).$$

TABLE XIV.

n.	α.		Differences $(\alpha' - \alpha)/(n' - n)$.	
	Found.	Calculated.	Found.	Calculated.
0	·0211			
1	·0396	·0397	·0185	
2	·0564	·0567	·0168	·0170
3	·0739	·0737	·0175	·0170
4	·0913	·0907	·0174	·0170
5	·1073	·1077	·0160	·0170

Putting i for the number of HI and c for the number of HCl per cub. centim. the equation which represents the variation of the rate with the amount of hydrogen chloride over the range of these observations is

$$\alpha = i\{2800 + 16{\cdot}8\,(c - 71{\cdot}1)\}10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20).$$

The increment of rate per unit of hydrogen chloride between 0 and 71, or up to 0·071 normal, is, as with hydrogen sulphate, greater than that due to subsequent additions of acid which were extended up to 355 HCl per cub. centim. or 0·355 normal. Probably if the rates of change were determined with quantities of hydrogen chloride between 0 and 71 HCl it would be found that only the first ten or twenty millionth-gram molecules per cub. centim. caused a greater acceleration, and that after this point, which may be that at which hydrogen chloride combines with the maximum quantity of water, the acceleration increases in the same ratio as with the subsequent additions of acid. Whether in more concentrated solutions the acceleration due to an increment of hydrogen chloride changes at certain points, as the total amount present and its ratio to the amount of water increases, we have not at present investigated.

Comparing the influence upon the rate of the presence of hydrogen sulphate and of hydrogen chloride respectively, it will be seen that, molecule for molecule, the two acids are nearly equivalent, instead of one molecule of hydrogen sulphate being equivalent to two molecules of hydrogen chloride as in combination with bases In a solution at 30° containing in each cub. centim., besides water, 300 H^2SO^4, 4 HI, and 1 H^2O^2, all these quantities being kept constant, iodine would be formed at the rate of ·0268 I^2 per cub. centim. per minute. This amount would be increased to ·034 I^2 by bringing into the liquid another 100 H^2SO^4 per cub. centim., the increment for each H^2SO^4 in presence of 4 HI being over this range of concentration 72×10^{-6} (equation 4); it would be increased to ·0335 I^2 by adding in each cub. centim. 100 HCl, the increment for each HCl in presence of 4 HI being $67{\cdot}2 \times 10^{-6}$ (equation 20).

Variation of Iodide.

In our former paper on the same subject ('Phil. Trans.,' 1867, pp. 132–135), an account is given of experiments which prove that the rate at which the change under investigation proceeds, varies directly with the amount of iodide, when relatively small quantities of potassium iodide are present in solutions containing relatively large quantities of hydrogen sulphate or hydrogen chloride. It is also pointed out that this relation cannot hold good exactly unless the iodide added has no secondary effect upon the rate of change. Doubtless, every substance in the liquid influences the rate of change, whether itself capable of undergoing change as the reaction proceeds or not. Water promotes the change as little as any other substance with which we have experimented, for the rate of change is generally increased and never decreased by the substitution in a given volume of any of these substances for water. Thus an addition (or substitution for water) of hydrogen iodide, like an addition of hydrogen chloride or hydrogen sulphate, causes an increment in the rate.

The equation $\alpha = \alpha_1 + ak$ gives the rate of change when, α_1 being the rate under any fixed conditions, those conditions have been varied by bringing into the solution a units of a substance, one unit of which causes the increment, or additional rate, k. With different quantities of hydrogen sulphate, or of hydrogen chloride, and different small quantities of potassium or sodium iodide, the equation takes the form $\alpha = i\,(\alpha_1 + sk)$, or $\alpha = i\,(\alpha_1 + ck)$, and similarly with different quantities of hydrogen iodide or, in presence of sodium hydrogen carbonate, of potassium or sodium iodide, the equation becomes $\alpha = i\,(\alpha_1 + ik)$.

When, however, the variation made in a set of observations is of a neutral iodide in an acid solution, in which the result is the replacement of a small proportion of hydrogen sulphate or chloride by hydrogen iodide and the addition of a small quantity of a neutral salt, the quantity ik becomes so small relatively to α_1 as to be negligible, and the equation is simplified to $\alpha = i\alpha_1$.

To illustrate the simple relation which is found to exist in such a case between the amount of iodide and the rate, we reproduce the results of one series of experiments given in our previous paper.

The amounts of potassium iodide taken were, it will be seen, in arithmetical progression from $1{\cdot}82\,KI$ to eight times that quantity. The corresponding rates are also in arithmetical progression, as is shown by comparing them with numbers calculated from the equation $\alpha = {\cdot}01347n$.

TABLE XV.

$(381{\cdot}3 - 1{\cdot}82z)\,HCl$, $1{\cdot}82\,\{(n-z)\,KI + zHI\}$, $1{\cdot}82zKCl$. Temperature, 30° C.

n.	α.	
	Found.	Calculated.
1	·0136	·0135
2	·0268	·0269
3	·0404	·0404
4	·0538	·0539
5	·0672	·0673
6	·0804	·0808
7	·0948	·0943
8	·1080	·1078

The equation which gives the rate per unit of iodide is $\alpha = {\cdot}0074i$. The maximum variation in the *conditions* of the change, as distinct from the variation of iodide, was, when $n = 8$, the substitution per cub. centim. of 14·6 out of 380 HCl by 14·6 HI and the addition of 14·6KCl. Such a change of conditions does not produce a measurable effect.

The following experiment, which gives a similar result, was made in a different manner. The amount of iodide was gradually increased during a set of observations by adding to the liquid single drops of a strong solution of sodium iodide. When three or four observations, giving two or three intervals, had been made, another drop of iodide was brought in; the following interval was occupied with mixing the liquid and adjusting the temperature, and the moment of the next observation of the change of colour was the starting point of the short series of observations which served to determine the rate of change with the new proportion of iodide. In the following Table n represents the number of drops thus added.

TABLE XVI.

546 H^2SO^4, ·266*n* *NaI*. Temperature, 16° C.

n.	*α*.		*α*/*n*.	
	Found.	Calculated.	Found.	Calculated.
1	·00121	·00118	·00121	·00118
2	·00241	·00236	·00120	„
3	·00358	·00354	·00119	„
4	·00474	·00472	·00118	„
5	·00592	·00590	·00118	„
6	·00711	·00708	·00118	„
7	·00825	·00826	·00118	„
8	·00944	·00944	·00118	„
10	·01180	·01180	·00118	„
12	·01413	·01416	·00118	„
16	·01880	·01888	·00117	„
28	·03250	·03304	·00116	„

It will be seen that the rates vary directly with the amounts of iodide, the numbers in the third column being calculated from the equation $\alpha = {\cdot}00118n$, or, per unit of iodide, $\alpha = {\cdot}00444i$.

When, however, hydrogen iodide is taken instead of potassium iodide, and in larger quantities, the effect of this acid can be determined like the corresponding effects of hydrogen sulphate and of hydrogen chloride, the second term of the equation $\alpha = i\,(\alpha_1 + ik)$ becoming easily measurable.

In the set of observations whose conditions and results are given in the following table, the amount of hydrogen iodide in each cub. centim. was varied from 14·5 to 43·5 *HI*.

TABLE XVII.

7·25*n* *HI*. Temperature, 30° C.

$n = 2$. $y_0 = 7{\cdot}75$.		$n = 3$. $y_0 = 7{\cdot}95$.		$n = 4$. $y_0 = 8{\cdot}41$.		$n = 5$. $y_0 = 7{\cdot}73$.		$n = 6$. $y_0 = 7{\cdot}7$.	
Intervals.	*a*.	Intervals.	*α*.	Intervals.	*α*.	Intervals.	*α*.	Intervals.	*α*.
6′·45	·0214	3′·90	·0344	2′·47	·0513	2′·00	·0692	1′·57	·0879
7·52	·0214	4·43	·0351	2·88	·0502	2·35	·0684	1·82	·0881
8·93	·0214	5·32	·0345	3·38	·0502	2·82	·0681	2·17	·0881
11·05	·0214	6·47	·0348	3·95	·0516	3·50	·0678	2·68	·0881
14·45	·0214	8·38	·0348	5·05	·0509	4·53	·0689	3·51	·0883
21·30	·0212	11·85	·0349	6·83	·0508	6·67	·0684	5·12	·0883
		20·60	·0349	10·53	·0509	12·52	·0689	9·62	·0881
				24·17	·0509				
Mean rate	·0214	Mean rate	·0348	Mean rate	·0508	Mean rate	·0685	Mean rate	·0881

Comparing the rates thus found, it is evident that they are not in arithmetical progression; but such a series of rates is deducible from the equation $\alpha = i(\alpha_1 + ik)$, where α_1 represents what the rate would be with an unit of iodide, if the iodide were only a substance capable of undergoing change, and k represents the addition to this rate which is caused by the first and each additional *HI* per cubic centimetre as an ingredient of the solution promoting the change. The best value for α_1 in this case is found to be ·00118, and that for k ·0000194. The values of α in the following table are calculated accordingly from an equation

$$\alpha = n\,(\cdot 00856 + \cdot 00102\,n) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (21),$$

which, since $n = 7{\cdot}25\,i$, becomes for an unit of iodide

$$\alpha = i\,(1180 + 19{\cdot}4\,i)\,10^{-6} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (22).$$

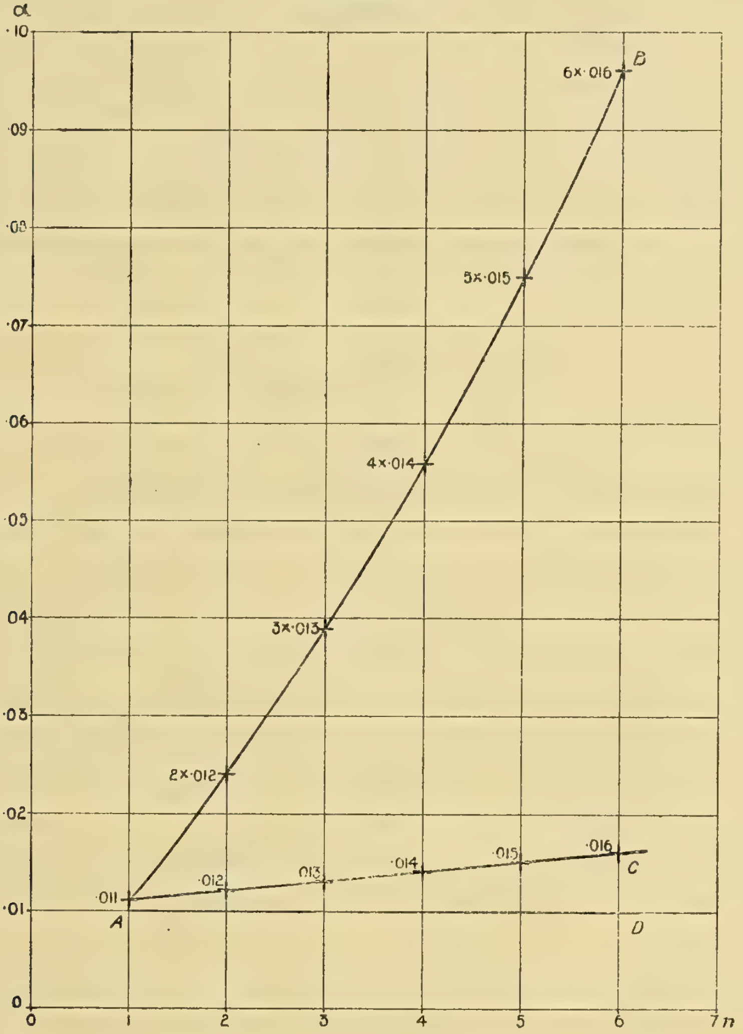
If that part of the increase of rate which is due to hydrogen iodide being one of the substances undergoing change, and which is therefore directly proportional to the amount of iodide in the liquid, is separated from the other part by reducing all the rates to what they would have been with only one proportion of iodide so acting, the effect of hydrogen iodide as an accelerant will be, so to say, isolated. In the fourth column are given the rates divided by the corresponding proportions of iodide. These numbers are in arithmetical progression and correspond to the previous series representing the accelerating effects of hydrogen sulphate and hydrogen chloride.

TABLE XVIII.

n.	α.		α/n.		$\alpha'/n' - \alpha/n$.	
	Found.	Calculated.	Found.	Calculated.	Found.	Calculated.
1	..	·0096	..	·0096		
2	·0214	·0212	·0107	·0106		·0010
3	·0348	·0349	·0116	·0116	·0009	·0010
4	·0508	·0506	·0127	·0126	·0011	·0010
5	·0685	·0684	·0137	·0137	·0010	·0011
6	·0881	·0882	·0147	·0147	·0010	·0010

The relation between the amount of hydrogen iodide in a given volume of the liquid and the rate of change may also be exhibited graphically. The equation of the curve AB, Diagram 2, which is similar to equation (21), but shows more clearly the numerical relations, is $\alpha = n\,(\cdot 01 + \cdot 001\,n)$. It expresses the rates of change with $n \times 6{\cdot}01$ *HI* per cub. centim. at 34°.* The line A D is parallel to the base where $\alpha = \cdot 01$ and represents what may be regarded in all cases as the initial rate, namely, the rate with one proportion of iodide (6·08 i) reacting with hydrogen dioxide, but not otherwise affecting the rate of change. But hydrogen iodide has a further influence upon the rate ; it adds to the initial rate a small fraction, which for one proportion of hydrogen iodide in the case chosen is ·001, each additional proportion of hydrogen iodide adding the same amount. Thus, if hydrogen iodide could be added to the solution, as hydrogen sulphate and hydrogen chloride may, without increasing the amount of iodide present, the successive rates would be those represented by the line AC, of

Diagram 2.



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* If in the quantity $(1180 + 19{\cdot}4\,i)$ of equation (22) n is substituted for i, which $= \frac{1180}{194}$, or 6·08 i, the quantity becomes $(1180 + 118\,n)$; and if the same substitution is made outside the bracket, equation (22) becomes $\alpha = n\,(\cdot 00718 + \cdot 000718\,n)$. To raise the numbers within the bracket to $(\cdot 01 + \cdot 001)$ the rate must be multiplied by 1·39, which would result from raising the temperature from 30° to 34°.

which the equation is $\alpha = \cdot 01 + \cdot 001\, n$. But since the amount of iodide present is also increased, and the rate varies directly with the amount of iodide, the rates represented by AC must be multiplied by the number of proportions of hydrogen iodide taken, that is, by the values of n, giving the curve AB.

We may, perhaps, be allowed to illustrate the two-fold increase in the rate of change due to additions of hydrogen iodide by a simile. Some children are in a room playing at blind-man's buff. The child whose eyes are bandaged runs about among the others trying to catch one. It is a difficult task while the players are few; but as the number increases, the chance, or rate, of catching increases also. Not, however, directly with the number, as one might at first suppose, but in a greater degree. The joining in of more players has other incidental results. The blind-man is stimulated to greater efforts, and those who would escape impede one another. Thus each new-comer increases the chance of someone else being caught as well as adding the chance that he may be caught himself.

Variation of Sodium Hydrogen Carbonate.

Of the many substances and classes of substances whose influence upon the change it might be interesting to try, we have only made systematic trial of one other, namely, sodium hydrogen carbonate. We chose this substance on two grounds: (1) it does not contain any element which is not already present in all the solutions with which we have worked, since water, carbonic acid, and sodium thiosulphate, are indispensable; (2) it seemed unlikely to undergo that double decomposition with sodium iodide which causes some uncertainty as to the substances actually present in the solution when a neutral iodide and hydrogen sulphate or chloride come together.

The amount of carbonic acid in the solutions was not determined; it may be considerably greater in presence of the sodium bicarbonate than in other solutions, and the relative effect of the acid may be greater; and these two causes may contribute to the unexpected result that the molecule $NaHCO^3$ accelerates the change more than H^2SO^4 or HCl.

On the other hand, the presence in the solution of such an amount of carbonic acid as dissolves in the slightly acid solutions we have generally used, is, in these solutions, without effect upon the rate. It was important to establish this point, especially in connexion with the inquiry into the effect of varying the temperature of the solution, since an increase of temperature also reduces the amount of carbonic acid dissolved. The following table gives the conditions and results of two sets of observations, one made in the ordinary manner, using large bubbles of carbonic acid to stir the liquid, the second being a repetition of the first, except that in place of carbonic acid, first nitrogen and then air were used. It will be seen that the intervals are very nearly the same in the two cases.

TABLE XIX.

203 H^2SO^4, 2·91 *KI*. Temperature, 17° C.

Gas passed through the solution.		
Carbonic acid.	Nitrogen.	Air.
Intervals. Set I.	Intervals. Set II.	
′	′	
3·43	3·37	
3·57	3·58	
3·72	3·65	
3·87	3·88	
4·05	3·95	
4·20	4·20	
4·40	4·40	
4·73	4·62	
4·87	4·87	
5·13	5·12	
5·45	5·38	
5·75	5·75	
6·17	..	6·12
6·48	..	6·50
7·03	..	7·02

In Table XX. are given the conditions and results of twelve sets of observations in which the amount of sodium hydrogen carbonate present was varied.

TABLE XX.—25·2 *n* $NaHCO^3$, 8·59 NaI. Temperature, 15° C.

$n = 1$, y_0 = 7·32.	
Intervals.	α.
13·73	·0107
16·12	·0107
19·67	·0106
24·93	·0105
33·80	·0106
52 35	·0108
133·75	·0106
Mean rate .	·0106

$n = 1$, y_0 = 6·35.	
Intervals.	α.
16·03	·0107
19·33	·0107
24·23	·0108
32·97	·0107
52·17	·0106
126·42	·0107
Mean rate .	·0107

$n = 2$, y_0 = 6·2.	
Intervals.	α.
9·87	·0184
11·40	·0187
14·65	·0185
20·42	·0183
32·75	·0185
97·25	·0184
Mean rate .	·0185

$n = 2$, y_0 = 6·74.	
Intervals.	α.
8·42	·0191
10·05	·0191
12·47	·0190
(2) 40·27	·0190
44·70	·0191
Mean rate .	·0191

$n = 3$, y_0 = 4·51.	
Intervals.	α.
9·28	·0270
12·37	·0271
18·75	·0271
40·10	·0270
Mean rate .	·0271

$n = 4$, y_0 = 5·14.	
Intervals.	α.
6·10	·0355
7·78	·0355
10·82	·0355
17·82	·0354
Mean rate .	·0355

$n = 5$, y_0 = 4·94.	
Intervals.	α.
5·12	·0442
6·63	·0442
9·40	·0443
16·43	·0441
Mean rate .	·0442

$n = 6$, y_0 = 4·45.	
Intervals.	α.
4·65	·0547
6·28	·0546
9·73	·0540
21·45	·0546
Mean rate .	·0545

$n = 6$, y_0 = 7·46.	
Intervals.	α.
2·70	·0532
3·14	·0535
3·80	·0532
4·75	·0532
6·37	·0535
9·80	·0531
21·58	·0532
Mean rate .	·0533

$n = 7$, y_0 = 7·5.	
Intervals.	α.
2 28	·0627
2·69	·0621
3·24	·0619
4·07	·0619
5·40	·0624
8·28	·0618
17·80	·0621
Mean rate .	·0621

$n = 8$, y_0 = 4·69.	
Intervals.	α.
3·33	·0721
4·43	·0714
6·50	·0714
12·55	·0714
Mean rate .	·0716

$n = 9$, y_0 = 7·41.	
Intervals.	α.
1·83	·0793
2·15	·0789
(2) 5·90	·0778
4·35	·0798
6·78	·0791
15·65	·0789
Mean rate .	·0790

The rates thus found with from 25·2 $NaHCO^3$ per cub. centim. to nine times that quantity, form a series in arithmetical progression. They are compared in the following table with a series calculated from the equation

$$\alpha = \cdot 00165 + \cdot 00863\, n \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (23).$$

TABLE XXI.

n.	α.		Differences $(\alpha' - \alpha)/(n' - n)$.	
	Found.	Calculated.	Found.	Calculated.
1	·0106	·0103		
2	·0188	·0189	·0082	·0086
3	·0271	·0275	·0083	·0086
4	·0355	·0362	·0084	·0087
5	·0442	·0448	·0087	·0086
6	·0539	·0534	·0097	·0086
7	·0621	·0621	·0082	·0087
8	·0716	·0707	·0095	·0086
9	·0790	·0793	·0074	·0086

If the number of NaI and of $NaHCO^3$ per cub. centim. are represented by i and b the equation becomes

$$\alpha = i\,(192 + 40b) \times 10^{-6} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (24).$$

Substitution of Atoms and Groups.

Instead of varying the conditions of the change by taking more or less of some ingredients of the solution, it was possible to make a smaller variation, and thus, perhaps, to obtain more instructive results, by substituting salts with the same metal or acid radicle, one for another, in the proportion of their molecular masses. If, for instance, $n\ NaI$ were substituted in each cub. centim. for $n\ KI$, the actual substitution, of which the effect could be measured, would be that of so many sodium-atoms for the same number of potassium-atoms; and if $n\ NaCl$ were similarly substituted for $n\ NaHCO^3$, the actual exchange would be of chlorine-atoms for the same number of the monad radicles HCO^3. Although we are now inclined to think that what affects the character of a solution is the nature and number of the molecules, not of the atoms, in unit volume, and that therefore to make such a substitution is to make two changes at once, instead of the one change of only adding or only subtracting, we give the conditions and results of the experiments made from the above point of view.

In the first set, the solutions of potassium iodide and of sodium iodide, of which equal quantities were taken, were found not to be equivalent; but by comparing one with another the two sets made with each iodide, the effects of each can be inferred, and thus the effect of substituting one for the other in molecular proportions.

TABLE XXII.

57·2 $NaHCO^3$, m (10·42 KI), n (8·07 NaI). Temperature, 17° C.

m.	n.	α.	α/i.
1	0	·0273	·00262
0	1	·0216	·00267
3	0	·0850	·00272
0	3	·0669	·00276

Assuming the values of α from $m = 1$ to $m = 3$, and from $n = 1$ to $n = 3$, to increase, as in other cases, in arithmetical progression, the increment in the rate, with unit of iodide, per unit of KI is $4·8 \times 10^{-6}$, and per unit of NaI is $5·6 \times 10^{-6}$, at a temperature of 17°. Hence the effect of substituting NaI for KI, or Na for K, is to increase the rate, with unit of iodide, by the small quantity $·8 \times 10^{-6}$.

The next series was made by substituting for a part of the hydrogen chloride used in the initial set of observations, the corresponding molecular proportion of sodium chloride, a substitution which was readily effected by adding to successive solutions, made up with the same measure of hydrogen chloride, one, two, and three, smaller measures of a solution of sodium carbonate.

The conditions of these sets of observations, and the rates that were found, are given in the following Table, and are compared with a series of rates calculated from the formula,

$$\alpha = ·0384 - ·00534\, n,$$

which per unit of iodide and unit-substitution of $NaCl$ for HCl becomes

$$\alpha = i\,(3690 - 6·14 \times 83·5\, n)\,10^{-6}.$$

TABLE XXIII.

$(535 - 83{\cdot}5\,n)\,HCl$, $83{\cdot}5\,n\,NaCl$, $10{\cdot}42\,KI$. Temperature, 16° C.

n.	α.		$(\alpha - \alpha') / (n' - n)$.	
	Found.	Calculated.	Found.	Calculated.
0	·0385	·0384	·0058	·0053
1	·0327	·0331	·0048	·0054
2	·0279	·0277	·0056	·0053
3	·0223	·0224		

The effect of substituting in unit volume $NaCl$ for HCl, in presence of an unit of iodide at 16°, is, therefore, a decrement in the rate of $6{\cdot}14 \times 10^{-6}$. This may be compared with the effect of removing (instead of neutralizing) the same amount of hydrogen chloride, a number obtainable from equation (20) by applying the temperature-correction. The number thus found is $-6{\cdot}18 \times 10^{-6}$, showing that the influence of sodium chloride in a liquid containing hydrogen chloride is inappreciable. In the set of observations, in which the maximum quantity of sodium carbonate was taken, corresponding to a replacement of 250 out of 535 HCl by 250 $NaCl$, the observed rate was ·0223. If 250 out of 535 HCl had been omitted, and no sodium chloride added, the rate, so far as it can be inferred from observations made at 30°, would have been ·0223.

It appears, however, from the experiments which follow, that the influence upon the rate of sodium chloride, and doubtless, therefore, of other salts, is far greater in presence of sodium hydrogen carbonate than it is in presence of hydrogen chloride.

The substitution of which it was proposed to measure the influence, in a solution containing excess of sodium hydrogen carbonate, was that of chlorine for iodine, the proportions of sodium chloride and sodium iodide being so varied in successive experiments that the mass of sodium present in the liquid might be always the same. The conditions and results of these sets of observations are given in the following table. The effect of this substitution, according to the scheme given in the first two columns, is represented by the equation,

$$\alpha = n\,(26360 - 639n) \times 10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25),$$

which per unit of iodide becomes

$$\alpha = i\,(5780 - 30{\cdot}7i) \times 10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26).$$

From the coefficient of i being negative, it appears that the rate of change is increased more by the presence of sodium chloride than by the presence (apart from its participation in the change) of sodium iodide. A substitution of iodide for chloride in molecular proportions causes in this respect a retardation; but, absolutely, each salt accelerates. To show their separate effect, equations (25) and (26) may be written

$$\alpha = n\,(22485 + 774m + 135n) \times 10^{-6} \text{ and } \alpha = i\,(4931 + 37{\cdot}2c + 6{\cdot}5i) \times 10^{-6}.$$

TABLE XXIV.

92·6 $NaCO^3$, m (4·56 $NaCl$), n (4·56 NaI). Temperature 19°.

m.	n.	α.		α/n.		$\alpha/n - \alpha'/n'$.	
		Found.	Calculated.	Found.	Calculated.	Found.	Calculated.
3	2	·0503	·0502	·0251	·0251		
2	3	·0729	·0733	·0243	·0244	·0008	·0007
1	4	·0951	·0952	·0238	·0238	·0005	·0006
0	5	·1160	·1158	·0232	·0232	·0006	·0006

From equation (26) it appears that the effect of substituting, in unit volume and in presence of an unit of iodide, $NaCl$ for NaI, or Cl for I, is to add, at a temperature of 19°, $30{\cdot}7 \times 10^{-6}$ to the rate. The effect of adding NaI in presence of sodium hydrogen carbonate has been shown to be, at a temperature of 17°, an addition of $5{\cdot}6 \times 10^{-6}$, which would become, at a temperature of 19°, an addition of $6{\cdot}5 \times 10^{-6}$. From which it follows that the effect of adding $NaCl$ is $37{\cdot}2 \times 10^{-6}$. It is singular that the effect of $NaCl$ should be so much greater than that of NaI. Both salts have, in this case, a marked and measurable effect upon the rate, whereas, in presence of hydrogen chloride, the effect of the same small quantities of both is quite inappreciable.

The last comparison made from the substitution point of view was between $NaHCO^3$ and NaI, or HCO^3 and I. In this case, as the following table shows, the effect of the substitution of NaI for $NaHCO^3$ is to diminish the rate with unit of iodide by ·00022/5·68 = 39 millionths.

TABLE XXV.

(114·4 − 5·68n) $NaHCO^3$, 5·68$nNaI$. Temperature 19°.

n.	α.	α/i.	$(\alpha/i - \alpha'/i')\,/\,(n' - n.)$
1	·0283	·00499	
3	·0774	·00455	·00022

If the substitution here made is divided into an increase of sodium iodide and a diminution of sodium hydrogen carbonate, we can infer from these observations the effect upon the rate of the presence of $NaHCO^3$ at 19°. The increment due to an addition of NaI at this temperature has been shown to be $6\cdot5 \times 10^{-6}$, and hence the decrement due to a subtraction of $NaHCO^3$ is $39 + 6\cdot5 = 45\cdot5$ millionths.

Influence on the Rate of the Presence in the Solution of Various Substances.—Summary.

In the tables which follow are put together the increments of the rate, with unit of iodide at the actual temperature, caused by the presence in each cubic centimetre of one millionth-gram molecule of each of the substances we have taken, such quantity of such substance being substituted for its own bulk of water. The three sets of numbers given under H^2SO^4 probably belong, as has been explained, to different hydrates of that substance. The numbers given are the number of millionths added to the rate—or fraction disappearing per minute—in consequence of the presence of each millionth-gram molecule per cubic centimetre. The degree of dilution which we thus adopt for comparison, may also be expressed by saying that each solution contains, per litre, a milligram-molecule of the substance, or that it is a millinormal solution.

TABLE XXVI.

t.	H^2SO^4.	H^2SO^4.	H^2SO^4.	HCl.	HI.	$NaHCO^3$.
	54 to 515.	515 to 763.	763 to 1143.	71 to 355.	7 to 44.	25 to 227.
° 15	..	..	..	..	..	40
16	6·92	8·57	10·08			
20	9·10	11·27				
30	18·00	22·50	26·50	16·8	19·4	

TABLE XXVII.—In presence of $NaHCO^3$.

t.	KI.	NaI.	$NaCl$.
	10 to 31.	10 to 31.	0 to 14.
° 17	4·8	5·6	
19	..	..	37·2

In a solution containing hydrochloric acid the addition of sodium chloride, even in the relatively large quantity of 250 *NaCl* per cubic centimetre did not affect the rate. We have only determined the influence of salts in presence of an excess of sodium hydrogen carbonate.

In the following table the increments given above have been reduced to their several values at 15°. The principles and method according to which this reduction has been made are explained in the section which follows on the variation of temperature.

TABLE XXVIII.—Temperature, 15°.

H^2SO^4.	H^2SO^4.	H^2SO^4.	HCl.	HI.	$NaHCO^3$.
54 to 515.	515 to 763.	763 to 1143.	71 to 355.	7 to 44.	25 to 227.
6·38	7·98	9·4	5·74	5·72	40

TABLE XXIX.—In presence of $NaHCO^3$. Temperature, 15°.

KI.	NaI.	$NaCl$.
10 to 31.	10 to 31.	0 to 14.
4·15	4·84	27·7

Ionic Dissociation.

During the interval between the publication of our former papers and the present time the hypothesis of the decomposition or dissociation[*] of compounds into their ions, by the act of solution, has met with general acceptance.

According to this hypothesis the dilute solutions of hydrogen chloride and potassium iodide, &c., which we have studied, contain a mixture of those substances with ions of hydrogen, chlorine, potassium, and iodine. The observation of the rate of change in solutions of different degrees of concentration might, perhaps, have furnished data regarding the extent of this ionic dissociation and its relation to dilution. But our results do not seem to show any effect of progressive dilution beyond the necessary consequence that every addition of water diminishes proportionally the mass of each acid or salt in unit volume.

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[*] In terminology there is no appeal against usage; but whether nature recognizes any such distinction—corresponding apparently to that between divorce and judicial separation—as the term dissociation is used to express, may well be doubted. A substance which was one, and which under changed conditions becomes two or more, is not decomposed in a different way, nor do the newly-formed substances exist in a different state, according to the greater or less facility with which these substances can be brought under the conditions under which they will unite again.

For example, the concentration of the potassium iodide in the experiments given in Table XV. was varied eightfold, from ·0146 normal to ·00182. Comparing unit-volumes of these two solutions, the more dilute solution contained, according to the hypothesis, less than one-eighth of the mass of potassium iodide in the stronger solution, since of that which remained a larger fraction was dissociated; and, correspondingly, the total mass of ions in unit volume of the more dilute solution was more than one-eighth of that present in the same volume of the stronger solution. If then the rate of interaction of ions is different from that of the molecules from which they are formed—if, for example, the time needed for the change, $n\text{H}^2\text{O}^2 + 2\text{H}'' + 2\text{I}'' = n(2\text{H}^2\text{O} + \text{I}^2)$, were different from that needed for $n(\text{H}^2\text{O}^2 + 2\text{HI} = 2\text{H}^2\text{O} + \text{I}^2)$—the total rate due to molecular and ionic interaction would vary with the amount of ionic dissociation. But it is not so; the rate with the more concentrated solution is ·108, the rate with the solution diluted eight-fold is ·0136, just one-eighth as great, the variation being simply proportional to the mass of substance acting. All our observations show the same fact, though some of the solutions were of more than normal (gram-molecule per litre) strength, and were diluted twenty- or thirty-fold, and others were varied from decinormal to much less than centinormal strength.

It seems to follow that the acids and salts which have been the subject of these experiments are either, (1) so near complete dissociation in solutions of normal strength that no great increase in the proportion of ions to molecules is caused by further dilution, or (2) that not much dissociation has yet taken place in solutions of less than centinormal strength, or (3) that the ions interact at the same rate, and accelerate chemical change in the same degree, as the molecules from which they are formed.

Reactions of Hydrogen Dioxide with Potassium Iodide and Iodine.

Our previous paper concludes with the record of a number of sets of observations which showed that in a mixture of hydrogen iodide, with equal or with larger molecular proportions of potassium iodide, the rate of change is not proportional to the amount of hydrogen dioxide present at any moment.

We have shown that this result must occur whenever the rate observed is dependent upon more than one gradual change. If, for example, hydrogen dioxide converted potassium iodide into potassium iodate, and potassium iodate acted on hydrogen iodide or, in presence of carbonic acid and water, upon potassium iodide, then iodine would be formed both in this manner and directly by the interaction of hydrogen dioxide and hydrogen iodide. Now one at least of the changes $\text{KI} + 3\text{H}^2\text{O}^2 = 3\text{H}^2\text{O} + \text{KIO}^3$ and $\text{KIO}^3 + 6\text{HI} = \text{KI} + 3\text{H}^2\text{O} + 3\text{I}^2$ must happen gradually. Otherwise the whole of the hydrogen dioxide would be decomposed and the corresponding quantity of iodine formed at once. If the change $KI + 3H^2O^2 = 3H^2O + KIO^3$ happened at once, the rate of change might be normal, that is, proportional to the amount of iodate remaining at any moment, for potassium iodide being in excess all hydrogen peroxide would have disappeared; but if this change were gradual, then the rate of the observed change, that is, the formation of iodine, would be abnormal, unless the proportion of HI and KI molecules, and the rate of action of H^2O^2 upon each, were such that the rate of formation of iodine and of decomposition of hydrogen dioxide were the same by the single and by the double reaction. Such a coincidence could only happen in one of our sets of observations, in which the amount of potassium iodide was varied, and was very unlikely to happen at all. We have not further examined these subsidiary reactions, though it would be of interest to do so. Some other similar cases have been well investigated by Mr. PENDELBURY and Miss M. SEWARD ('Roy. Soc. Proc.,' vol. 45, p. 396).

In our mode of observing the rate of change we have brought into the solution a drop of sodium thiosulphate as quickly as possible after the appearance of the blue colour due to free iodine. Our object in doing so has been to keep constant the amount of iodide with which the hydrogen dioxide was reacting; but the practice proved to be also necessary, in certain cases at least, for another reason, namely, that iodine may act upon sodium or hydrogen dioxide. Such an action, irregularly diminishing from time to time the amount of the substance whose rate of disappearance was being measured, would throw our results into confusion. As a rule it has not occurred, and we believe such a change does not happen at all in very dilute acid solutions of iodine. But in presence of sodium hydrogen carbonate it may happen; and in view of the result we are about to describe we think it likely that some of the discrepancies which may be noticed in the observations and rates given on Tables XX. and XXI. may be due to this cause.

With the view of reducing to a minimum, in comparing the effect upon the rate of different quantities of sodium iodide, that part of the acceleration which is due to the increase of the saline ingredients of the liquid, a solution was prepared containing per cub. centim. 686 $NaHCO^3$ and ·211 NaI. To this the usual measure of dioxide was added, and after a considerable lapse of time, the liquid being fully charged with carbonic acid, no change of colour had occurred. A second proportion of iodide was added; still no iodine was liberated. Some solution of iodine was dropped into the liquid; on stirring, the blue colour at once disappeared. Instead of liberating, the dioxide was now fixing iodine. Or rather, both actions were going on simultaneously, but with such unequal velocities that the iodine formed was kept down, as when sodium thiosulphate is present, to an infinitesimal quantity, insufficient to impart a perceptible blue tinge to the solution of starch. On repeating the experiment with rather less of the bicarbonate and more iodide we found that the addition of dioxide produced a pale blue colour which did not deepen. When a drop of a dilute solution of sodium thiosulphate was added, the colour became fainter or disappeared, but soon returned. When a drop of dilute chlorine water was added, the colour deepened, but soon faded to its original paleness. This stability of colour the liquid retained as long as the dioxide lasted.

The explanation of this rather striking phenomenon seems to be as follows. The formation of iodine by the interaction of dioxide and iodide takes place as in other cases, $2NaI + H^2O^2 + 2CO^2 = NaHCO^3 + I^2$; but the removal of iodine, probably according to the equation $2NaHCO^3 + H^2O^2 + I^2 = 2NaI + 2H^2O + 2CO^2 + 0^2$, takes place also. The number of occurrences of these changes in unit time is directly proportional in both cases to the amount of dioxide present, and also, in the first case, to the amount of iodide, in the second, to the amount of iodine. The rate of each change is also influenced by the amount of sodium hydrogen carbonate present, and the temperature of the solution. When first sodium dioxide is added to the solution, and there at once converted into hydrogen dioxide and sodium hydrogen carbonate,—$Na^2O^2 + 2H^2O + 2CO^2 = H^2O^2 + 2NaHCO^3$,—the dioxide reacts with the iodide, and iodine is formed. The amount of iodine increases, at first more quickly, then more and more slowly, until—the rate of its reconversion into iodide increasing proportionally with its increase—it is being removed as fast as it is formed. Thenceforward the amount of iodine in the liquid, and therefore the depth of the blue colour, is constant; the two changes exactly balance and undo one another. The only permanent result is the decomposition of the dioxide, which proceeds just twice as fast (except for a minute diminution in the amount of iodide present) as it would if the appearance, and practically the formation, of iodine were prevented by an addition of thiosulphate. As the dioxide diminishes, iodine is formed more slowly, and is reduced more slowly, always in the same proportion. When the balance is upset by an addition of thiosulphate or of chlorine water, iodine is formed more quickly than it is reduced, or reduced more quickly than it is formed, until its amount, and the colour of the liquid, is the same as before.

Variation of Temperature.

Tables XXX. to LIV. contain the results of experiments upon the effect of varying the temperature of the solution, in which the change takes place, and of calculations relating to them. In all the sets of experiments, except those recorded in Table XXXVIII., the volume of the solution was measured at the same temperature, and not at the temperature at which the experiment was made, so that a cub. centim. contained at different temperatures different quantities of the substances which affect the rate of change. But the rates at different temperatures cannot be compared unless the change has taken place with the same amounts of substances in each unit of volume. A correction has, therefore, to be applied to the rates actually found, to reduce them to the rates which would have been obtained if the solution had been brought at each temperature to the same volume after the introduction of the various substances. This correction can be applied from existing data for all the substances except water. The experiments recorded in Table XXXVIII. were made in solutions brought at each temperature to the same volume, containing thus in each cub. centim. a constant amount of hydrogen sulphate and iodide, but a continually diminishing quantity of water as the temperature was increased. It will be seen hereafter that the effect of this diminution, if any, is so small as not to affect the rate by an amount exceeding the limits of possible experimental error.

From the series of rates in Tables XXX., XXXVI., &c., a new series of rates was obtained, for which the quantities of substances in each unit volume of the solution at any temperature were taken to be the same as at zero. This new series was calculated in the following way.

Let the volume of the solution which is V_0 at zero be $V_0(1+\mu)$ at $t°$, and the number of units of the substances introduced into the solution be, of iodide I, of other substances not undergoing change (including water), X_1, X_2 . . . , and let a be the rate at $t°$ with unit of iodide, b the increment of this rate per unit of iodide, b_1, b_2 . . . the increment of this rate per unit of each of the other substances; then the rate α actually found is equal to

$$\frac{\mathrm{I}}{V_0(1+\mu)}\left\{a + b\frac{\mathrm{I}}{V_0(1+\mu)} + b_1\frac{X_1}{V_0(1+\mu)} + b_2\frac{X_2}{V_0(1+\mu)} + \ldots\right\} \quad . \quad . \quad (27),$$

and the rate α' which would have been found with the same amounts of substances per unit volume as there were in the solution at a temperature zero is

$$\frac{\mathrm{I}}{V_0}\left\{a + b\frac{\mathrm{I}}{V_0} + b_1\frac{X_1}{V_0} + b_2\frac{X_2}{V_0} + \ldots\right\} \quad . \quad . \quad . \quad . \quad (28),$$

from (27),

$$\alpha(1+\mu)^2 = \frac{\mathrm{I}}{V_0}\left\{a(1+\mu) + b\frac{\mathrm{I}}{V_0} + b_1\frac{X_1}{V_0} + \ldots\right\} = \alpha' + \frac{a\mu\mathrm{I}}{V_0},$$

or,

$$\alpha' = \alpha(1+\mu)^2 - a\mu i, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29),$$

i being the amount of iodide in unit volume of the solution when its temperature is zero. The quantity μ was determined by a series of observations of the expansion in solutions of the same composition as those used in the experiments; the quantity a is known for 30° from the equation (22), p. 842, and for other temperatures by interpolation from the series of experimental rates, the term $a\mu i$ in which it occurs being so small that the approximation is sufficiently exact.

The following table contains the results of a set of experiments at different temperatures in solutions containing 203 H^2SO^4 and 7·28 HI. The values of t are the corrected temperatures corresponding to the actual readings of our thermometer at 0°, 5°, 10°, &c.

TABLE XXX.

$(203 - 7{\cdot}28z)\, H^2SO^4,\ 7{\cdot}28\,\{(1 - z)\, KI + zHI\},\ 7{\cdot}28z\, KHSO^4$. Temperature, t° C.

$t = 0$, $y_0 = 17{\cdot}3$.

Intervals.	α
14·27	·00418
14·77	·00429
(2) 31·38	·00446
17·87	·00438
19·62	·00433
21·45	·00433
23·65	·00432
26·37	·00432
Mean rate	·00433

$t = 4{\cdot}96$, $y_0 = 17{\cdot}2$.

Intervals.	α.
9·70	·00617
10·33	·00617
10·95	·00622
11·78	·00620
12·63	·00621
13·70	·00622
15·03	
Mean rate	·00620

$t = 10{\cdot}11$, $y_0 = 17{\cdot}25$.

Intervals.	α.
6·63	·00900
7·08	·00898
7·52	·00902
Mean rate	·00900

$t = 15{\cdot}08$, $y_0 = 13{\cdot}25$.

Intervals.	α.
6·1	·0129
6·6	·0129
7·23	·0129
Mean rate	·0129

$t = 15{\cdot}08$, $y_0 = 9{\cdot}2$.

Intervals.	α.
8·97	·0128
10·10	·0129
11·52	·0130
13·72	·0128
16·92	·0126
21·03	·0129
Mean rate	·0128

$t = 20{\cdot}03$, $y_0 = 9{\cdot}04$.

Intervals.	α.
7·17	·0185
8·30	·0185
9·73	·0186
12·02	·0184
15·37	·0185
21·43	·0186
35·73	·0188
Mean rate .	·0186

$t = 20{\cdot}34$, $y_0 = 9{\cdot}25$.

Intervals.	α.
6·17	·0185
6·87	·0188
7·92	·0187
Mean rate .	·0187

$t = 25{\cdot}09$, $y_0 = 5{\cdot}25$.

Intervals.	α.
8·23	·0257
10·43	·0257
14·30	·0257
22·82	·0258
Mean rate .	·0257

$t = 30{\cdot}05$, $y_0 = 5{\cdot}23$.

Intervals.	α.
5·83	·0363
7·57	·0357
10·33	·0359
16·57	·0359
46·68	·0359
Mean rate .	·0359

$t = 35{\cdot}05$, $y_0 = 6{\cdot}23$.

Intervals.	α.
3·57	·0490
4·27	·0497
5·37	·0503
7·38	·0503
12·30	·0483
33·75	·0497
Mean rate .	·0495

$t = 40{\cdot}05$, $y_0 = 7{\cdot}72$.

Intervals.	α.
2·03	·0684
2·32	·0695
2·80	·0686
3·43	·0695
4·54	·0689
6·66	·0689
12·59	·0691
Mean rate .	·0690

$t = 45{\cdot}01$, $y_0 = 8{\cdot}09$.

Intervals.	α.
1·38	·0955
1·60	·0951
1·87	·0957
2·33	·0940
2·95	·0948
4·13	·0946
6·85	·0951
25·23	·0953
Mean rate .	·0950

$t = 50{\cdot}18$, $y_0 = 8$.

Intervals.	α.
1·00	·133
1·18	·131
1·37	·133
1·70	·131
2·20	·131
3·08	·131
5·33	·130
Mean rate .	·131

The following table gives the series of rates, corrected by the equation (29), p. 856, corresponding to those in the preceding table. The ingredients in a cubic centimetre of the solution were at zero,

$$(203{\cdot}3 - 7{\cdot}29z)\,H^2SO^4, \quad 7{\cdot}29\{(1 - z)\,KI + zHI\}, \quad 7{\cdot}29zKHSO^4,$$

and the values of α are calculated from those in Table XXX so as to represent the rates which would have been found if the solution had been made up at the temperature of each set of observations, so as to contain these amounts per cubic centimetre.

Table XXXI.

t.	α.
0·00	·00433
4·96	·00621
10·11	·00902
15·08	·01294
20·34	·01871
25·09	·02590
30·05	·03630
35·05	·05010
40·05	·07010
45·01	·09700
50·18	·13400

To facilitate the investigation of the law of connexion between α and t, a calculation was made, by interpolation, of the rates at degrees of temperature expressed in whole numbers. The following table gives these rates, their logarithms to the base 10, and the successive differences of the logarithms.

Table XXXII.

t.	α.	Log α.	Δ log α.
0	·00433	$\bar{3}$·636	
5	·00622	$\bar{3}$·794	·158
10	·00896	$\bar{3}$·952	·158
15	·01282	$\bar{2}$·108	·156
20	·01832	$\bar{2}$·263	·155
25	·02570	$\bar{2}$·410	·147
30	·03620	$\bar{2}$·559	·149
35	·05000	$\bar{2}$·699	·140
40	·07000	$\bar{2}$·845	·146
45	·09700	$\bar{2}$·987	·142
50	·13270	$\bar{1}$·123	·136

It was at first thought that $\Delta \log \alpha$ was constant, and the values of this expression over the range of temperature from zero to 20° are not far from constant; but an examination of the values from zero to 50° shows that they continually diminish as the temperature increases. As a first approximation to the law of connexion between α and t the values of $\Delta \log \alpha$ may be assumed to be in arithmetical progression; so that we may put $\Delta \log \alpha_x = a - bx$. The most probable values of a and b are ·16005 and ·0025, which give for the series $\Delta \log \alpha$, ·16005, ·15755, ·15505, ·15255, ·15005, ·14755, ·14505, ·14255, ·14005, ·13755, and for the corresponding series of values of $\log \alpha$ $\bar{3}\cdot635$, $\bar{3}\cdot795$, $\bar{3}\cdot953$, $\bar{2}\cdot108$, $\bar{2}\cdot260$, $\bar{2}\cdot410$, $\bar{2}\cdot558$, $\bar{2}\cdot703$, $\bar{2}\cdot845$, $\bar{2}\cdot985$, $\bar{1}\cdot123$. These values are fairly concordant with the experimental values, and the formula

$$\log \alpha = \bar{3}\cdot6354 + \cdot1613x - \cdot00125x^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

would give the rates at intermediate temperatures with sufficient accuracy, $5x$ being the number of degrees counted from zero, C. This formula, however, would evidently fail for higher degrees of temperature, for the difference between successive values of $\log \alpha$ would vanish when $x = \cdot16005 \div \cdot0025$, or at a temperature of 320°, and the rate would be equal to the rate at zero when $x = \cdot1613 \div \cdot00125$, or at a temperature of 645°. The rates on this hypothesis would first increase and then diminish, so that the formula can only be regarded as convenient for calculating the rates at different temperatures ranging from zero to 50°.

A function of x, the successive differences of which closely resemble the successive differences of $\log \alpha$, is $u_x = \log (c + x)$, when c is considerably larger than x; the second differences of this function slowly diminish as x increases, and if a series of approximate values of this function were given for values of x ranging from zero to 10°, it might easily be supposed that the successive differences formed an arithmetical progression, and if the assumption $\Delta u_x = a - bx$ were made and the most probable values of a and b found, the values of u_x calculated on this hypothesis would not differ sensibly from the actual values.

Assuming, then, that

$$\log \alpha_x = m \log \left(\frac{c + x}{c}\right) + \log \alpha_0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31),$$

the problem is to determine the values of m and c from the experimental numbers. First, for these substitute the more orderly series of numbers calculated from the formula $\Delta \log \alpha_x = a - bx$, then suppose that these agree with the series of numbers calculated in a similar way from $m\Delta \log (c + x) = a - bx$; it is clear that the three series $\Delta \log \alpha_x$, $m\Delta \log (c + x)$, $a - bx$ will agree with a high degree of approximation for a value of x equidistant from its first and last values.

Taking this value of x to be 5, the assumption gives, from the values printed above,

$$m\Delta \log (c+5) = {\cdot}15005 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32),$$

$$m\Delta \log (c+6) = {\cdot}14755 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33).$$

Now

$$\begin{aligned}\Delta \log (c+6) &= \log \frac{c+7}{c+6} \\ &= \log \left(1 + \frac{1}{c+6}\right) = \log \left\{1 + \frac{1}{(c+5)\left(1+\frac{1}{c+5}\right)}\right\} \\ &= \log \left\{1 + \frac{1}{c+5}\left(1 - \frac{1}{c+5} + \frac{1}{(c+5)^2} - \dots\right)\right\} \\ &= \log \left[\frac{c+6}{c+5}\left\{1 - \frac{1}{(c+5)(c+6)} + \dots\right\}\right] \\ &= \log \frac{c+6}{c+5} - \frac{\log e}{(c+5)(c+6)} \text{ approximately} \\ &= \Delta \log (c+5) - \frac{\log e}{(c+5)(c+6)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34),\end{aligned}$$

so that

$$\frac{m \log e}{(c+5)(c+6)} = {\cdot}0025 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35),$$

and (32),

$$m\,\Delta \log (c+5) = {\cdot}15005 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36).$$

Therefore

$$(c+5)(c+6) \log \frac{c+6}{c+5} = 60{\cdot}02 \log e = 26{\cdot}066 \quad . \quad . \quad . \quad . \quad (37).$$

This equation is approximately satisfied by $c = 54{\cdot}52$, which gives for the value of the expression on the left hand, 26·065.

It is probable then that the series of values of $\log \alpha$ is satisfied by an equation of the form

$$\log \alpha_x = m \log \frac{c+x}{c} + \log \alpha_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38),$$

c having the value 54·52 ; but since $5x = t$, this is the same as

$$\log \alpha_t = m \log \frac{272{\cdot}6 + t}{272{\cdot}6} + \log \alpha_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39),$$

or,

$$\frac{\alpha_t}{\alpha_0} = \left(\frac{272{\cdot}6 + t}{272{\cdot}6}\right)^m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40).$$

This equation implies that no chemical change will take place when $t = -\,272{\cdot}6$, a temperature at once recognised as almost identical with that usually assumed as the temperature of the absolute zero, viz., — 273° C. At this point molecules are at rest, and hence gases have no tension and chemical change ceases. Thus, within the limits of experimental errors, it may be assumed that the zero of chemical change coincides with the zero of absolute temperature. If we call the absolute temperature at the freezing point T_0, the equation of connexion of the amount of chemical change with temperature may be written in the form

$$\frac{\alpha_T}{\alpha_{T_0}} = \left(\frac{T}{T_0}\right)^m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41),$$

T being the absolute temperature corresponding to t degrees on the ordinary scale. In the subsequent discussion of experimental results T_0 expresses the absolute temperature 273° C. The form of the equation shows that the relation between the amount of chemical change at a given temperature and the absolute temperature is independent of the units in which each of these quantities is measured. It will be seen subsequently that the number m, which remains constant in a series of experiments at different temperatures with the same kind of solution, varies with the nature of the main ingredient of the solution but not with the amount of that ingredient.

The differential equation corresponding to the integral equation is

$$\frac{d\alpha}{\alpha}\bigg/\frac{dT}{T} = m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42),$$

so that if α, α' are two rates corresponding to two temperatures very near to each other,

$$\frac{\alpha' - \alpha}{\alpha}\bigg/\frac{T' - T}{T} = m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43);$$

this relation implies that the increment of each unit of chemical change, due to a small rise of temperature, is proportional to the increment of each unit of temperature; or the increase of each unit of chemical change per unit increment of each unit of temperature is constant at all temperatures.

These statements are illustrated graphically on Diagram 3, p. 862, by the curve APP′. The value of m in the formula $\alpha/\alpha_0 = (T/T_0)^m$, which has been calculated from the experimental numbers in a manner explained subsequently, is 20·38. The curve APP′ represents the above formula with this value of m. The rate at zero C. is taken as the unit rate. The rate at any temperature t° C., or 273° + t°, counting from the absolute zero, is calculated from the formula

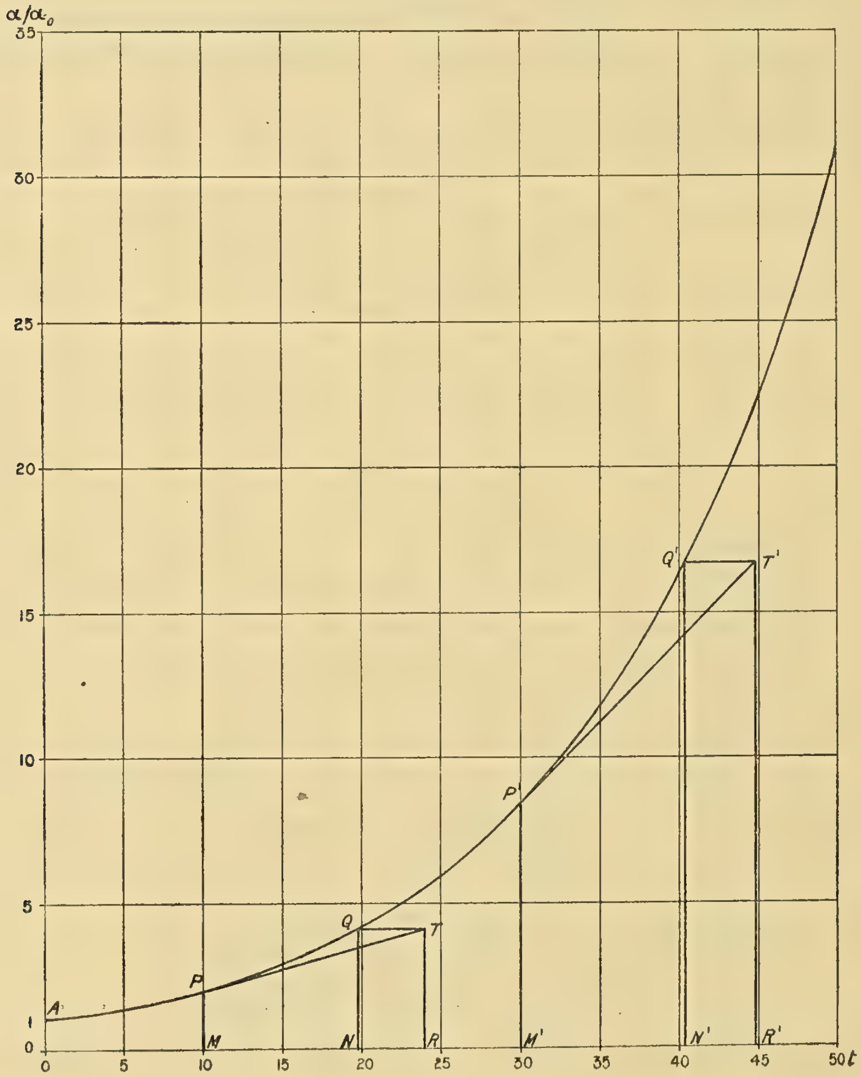
$$\alpha = (T/T_0)^{20 \cdot 38}, \text{ or } \{(273+t)/273\}^{20 \cdot 38} \quad . \quad . \quad . \quad . \quad . \quad . \quad (44).$$

The following table gives the values of α corresponding to the values of t :

TABLE XXXIII.

t.	α.
0	1·00
5	1·44
10	2·08
15	2·97
20	4·23
25	5·96
30	8·38
35	11·67
40	16·22
45	22·39
50	30·77

Diagram 3.



The amount of chemical change at zero being 1·00, the amount at 10° is 2·08, represented by MP, *i.e.*, the rate is rather more than doubled by a rise of temperature of 10° from zero to 10°; if the increase of chemical change per unit of temperature were supposed to remain constant at this point P, *i.e.*, if the relation between chemical change and temperature were represented by the tangent PT at P instead of the curve PP′, the amount of change would be doubled when the temperature had been increased by $T/m = 283/20·38$ or 13·9° C., represented by MR. The actual increase of temperature MN, which doubles the rate at P, is obtained by drawing TQ parallel to MR, to meet the curve in Q, and QN parallel to PM, to meet MR in N. It is seen that MN is rather less than 10°, the amount of change at 20° being 4·23 units. A similar construction is made for the change at 30°, represented by $M'P' = 8·38$ units; P′T′ is the tangent at P′, $R'T' = 2M'P'$, $M'R' = 14·9°$; the change being actually doubled by an increase of temperature, represented by M′N′, of a little more than 10°, the change at 40° being 16·22 units. Also the ratio MR : M′R′ is equal to the ratio of the absolute temperatures at P and P′, viz., 13·9 : 14·9 = 283 : 303, and the increments of each unit of change per degree centigrade at P and P′, measured by $\frac{RT - MP}{MR \,.\, MP}$ or $\frac{1}{MR}$ and by $\frac{R'T' - M'P'}{M'R' \,.\, M'P'}$ or $\frac{1}{M'R'}$, are inversely as the absolute temperatures at P and P′, viz., are in the ratio 303 : 283; in other words, the increase of unit change per degree centigrade is $\frac{1}{MR}$, or ·072, at 10°, and $\frac{1}{M'R'}$, or ·067, at 30°.

It is also convenient to express the relation between chemical change and temperature in the form in which it originally presented itself, viz.,

$$\log \alpha - \log \alpha_0 = m (\log T - \log T_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45),$$

and to state this relation concisely by saying that the logarithmic increment of chemical change due to increase of temperature varies as the logarithmic increment of absolute temperature. This mode of stating the law has the advantage that the graphic representation of the relation is a straight line, as is shown in Diagram 4, p. 864. The divisions of the horizontal axis in this mode of representation are the logarithms of T/T_0, and the divisions of the vertical axis are the logarithms of α/α_0. Fig. 1, Diagram 4, represents in this manner the same relation as that represented by the curve in Diagram 3. The numbers are calculated from the formula

$$\log (\alpha/\alpha_0) = 20·38 \log (T/T_0), \text{ or } 20·38 \log \{(273 + t)/273\} \quad . \quad . \quad (46),$$

and are given in the following table :—

Diagram 4.

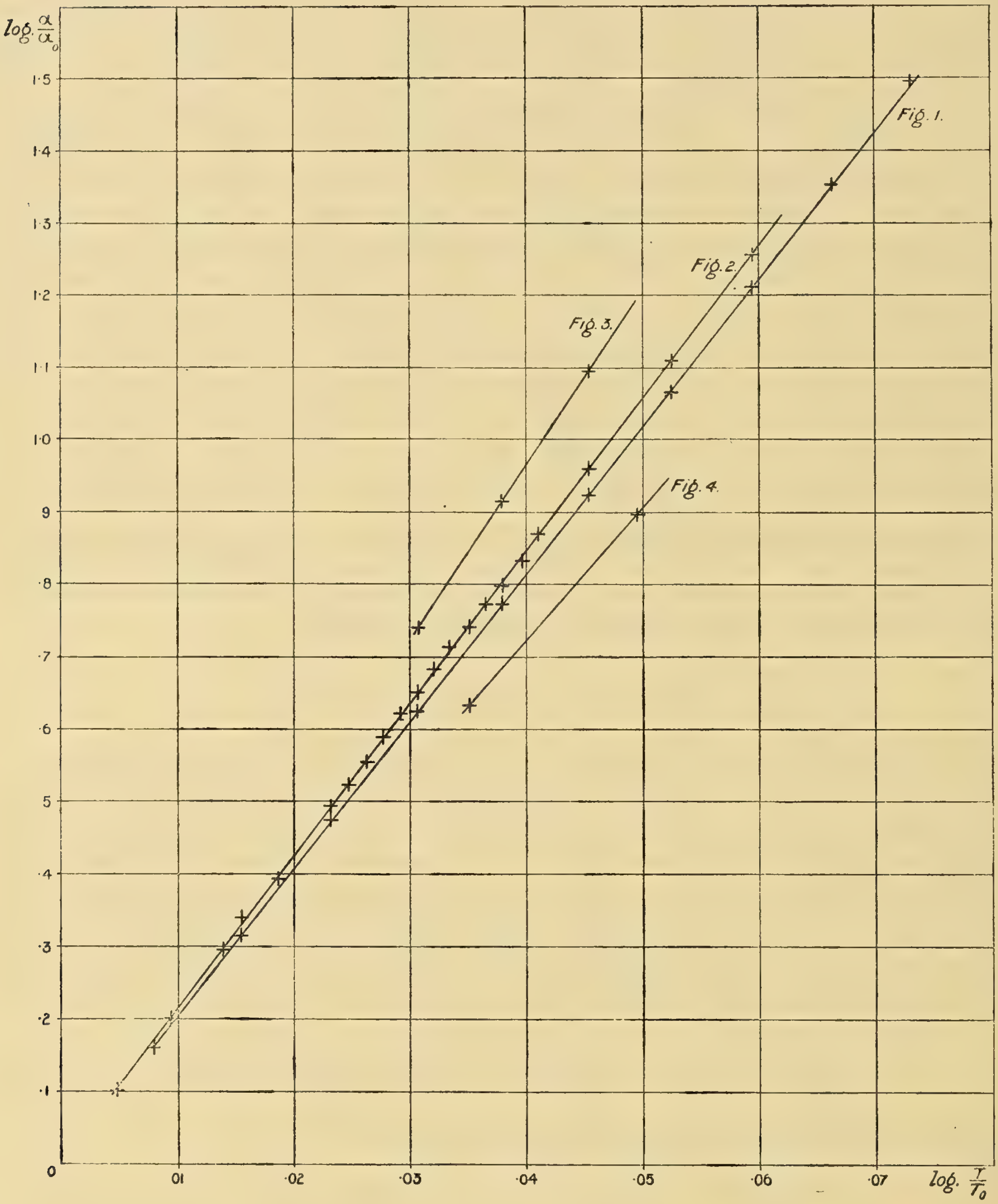

log. α/α₀
1·5
1·4
1·3
1·2
1·1
1·0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
·01
·02
·03
·04
·05
·06
·07
log. T/T₀
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.

TABLE XXXIV.

t.	Log (T/T_0).	Log (α/α_0).
0	·0000	0·000
5	·0079	0·160
10	·0156	0·318
15	·0232	0·473
20	·0307	0·626
25	·0380	0·775
30	·0453	0·923
35	·0524	1·067
40	·0594	1·210
45	·0663	1·350
50	·0730	1·488

Assuming, then, as a highly probable expression of the law of connexion between the chemical change which we have studied and temperature, at least for the range from zero to 50°, the equation

$$\log(\alpha/\alpha_0) = m \log(T/T_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47),$$

a determination was made of the most probable values of m and $\log \alpha_0$ for the series of numbers in Table XXXI. These were found to be $m = 20\cdot38$, $\log \alpha_0 = \bar{3}\cdot635$. The values of $\log \alpha$ and α given by these are compared in the following table with the values found by experiment.

TABLE XXXV.

$(203\cdot3 - 7\cdot29z) H^2SO^4$, $7\cdot29 \{(1-z) KI + zHI\}$, $7\cdot29 zKHSO^4$.

t.	log α.		α.	
	Found.	Calculated.	Found.	Calculated.
0	$\bar{3}\cdot636$	$\bar{3}\cdot635$	·00433	·00432
5	$\bar{3}\cdot794$	$\bar{3}\cdot796$	·00622	·00625
10	$\bar{3}\cdot952$	$\bar{3}\cdot953$	·00896	·00898
15	$\bar{2}\cdot108$	$\bar{2}\cdot108$	·01282	·01284
20	$\bar{2}\cdot263$	$\bar{2}\cdot261$	·01830	·01820
25	$\bar{2}\cdot410$	$\bar{2}\cdot410$	·02570	·02570
30	$\bar{2}\cdot559$	$\bar{2}\cdot558$	·03620	·03610
35	$\bar{2}\cdot699$	$\bar{2}\cdot703$	·05000	·05040
40	$\bar{2}\cdot845$	$\bar{2}\cdot845$	·07000	·07000
45	$\bar{2}\cdot987$	$\bar{2}\cdot985$	·09700	·09670
50	$\bar{1}\cdot123$	$\bar{1}\cdot123$	·13300	·13300

The calculated values of α agree within the limits of experimental error with the values determined by experiment. For the range of temperature from zero to 50° the law is established, and the assumption that the zero of the particular chemical change here considered coincides with the absolute zero of temperature is justified.

To confirm this result another set of experiments was made with the same amount of hydrogen sulphate and half the amount of potassium iodide, the results of which are recorded in the following table.

TABLE XXXVI.

$(203 - 3{\cdot}64z)\,H^2SO^4,\ 3{\cdot}64\,\{(1 - z)\,KI + zHI\},\ 3{\cdot}64zKHSO^4$. Temperature t° C.

$t = 20{\cdot}34$, $y_0 = 17$.		$t = 30{\cdot}05$, $y_0 = 13$.		$t = 40{\cdot}05$, $y_0 = 8$.		$t = 50{\cdot}18$, $y_0 = 4$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.
6′·48	·00933	4′·43	·0181	3′·83	·0348	4′·25	·0678
7·00	·00923	4·85	·0179	4·43	·0348	6·07	·0669
7·50	·00918	5·23	·0182	5·22	·0349		
Mean rate .	·00925	Mean rate .	·0181	Mean rate .	·0348	Mean rate .	·0673

The values of α in this table were corrected for expansion as in the previous case, and values corresponding to whole numbers of degrees of temperature obtained by interpolation. The most probable value of m is, as before, 20·38, and of log α_0, $\bar{3}{\cdot}335$.

The following table exhibits the agreement of the experimental and calculated numbers.

TABLE XXXVII.

$(203{\cdot}3 - 3{\cdot}645z)\,H^2SO^4,\ \ 3{\cdot}645\,\{(1 - z)\,KI + zHI\},\ \ 3{\cdot}645zKHSO^4$.
Temperature t° C.

t.	Log α.		α.	
	Found.	Calculated.	Found.	Calculated.
0		$\bar{3}{\cdot}335$		·0022
20	$\bar{3}{\cdot}957$	$\bar{3}{\cdot}961$	·0091	·0091
30	$\bar{2}{\cdot}261$	$\bar{2}{\cdot}258$	·0182	·0181
40	$\bar{2}{\cdot}547$	$\bar{2}{\cdot}550$	·0352	·0355
50	$\bar{2}{\cdot}833$	$\bar{2}{\cdot}829$	·0681	·0675

A third set of experiments was made with $468 H^2SO^4$ and $4·72 KI$, at different temperatures, the results of which are recorded in the following table.

TABLE XXXVIII.

$(468 - 4·72z) H^2SO^4$, $4·72 \{(1 - z) KI + zHI\}$, $4·72z KHSO^4$. Temperature t°.

$t = 10$, $y_0 = 7·95$.		$t = 15$, $y_0 = 8·82$.		$t = 20$, $y_0 = 9$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.
11′·77	·0114	7′·40	·0163	5′·02	·0234
13·65	·0114	8·39	·0163	5·74	·0232
16·18	·0114	9·71	·0163	6·59	·0234
19·75	·0114	11·59	·0163	7·85	·0232
		14·15	·0164	9·59	·0232
		18·50	·0164	12·33	·0233
63·38	·0114	26·93	·0163	17·39	·0233
		48·63	·0164	29·82	·0233
Mean rate .	·0114	Mean rate .	·0163	Mean rate .	·0233

$t = 20$, $y_0 = 8·73$.		$t = 25$, $y_0 = 8·89$.		$t = 30$, $y_0 = 10·74$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.
5′·22	·0232	3′·64	·0327	2′·11	·0463
5·96	·0232	4·15	·0326	2·37	·0457
6·90	·0233	4·77	·0328	2·64	·0461
8·29	·0231	5·68	·0327	3·04	·0455
10·27	·0231	(2) 16·17	·0324	3·52	·0457
13·48	·0231	13·03	·0325	4·17	·0459
19·70	·0231	22·93	·0327	5·15	·0459
37·60	·0232			6·80	·0457
				10·00	·0455
				18·56	·0460
Mean rate .	·0232	Mean rate .	·0326	Mean rate .	·0458

The experimental values of α in this table require no correction for expansion, the volume was made up to the same amount at each temperature, so that the only variable ingredient was water, less of which was contained in an unit volume at the higher temperatures than at the lower. No sensible departure from the law, due to this cause, can be detected. The most probable value of m is 20·35 and of log α_0

$\bar{3}\cdot740$. The following table exhibits the agreement of the experimental and calculated numbers.

TABLE XXXIX.

$(468 - 4\cdot72z)\ H^2SO^4,\ 4\cdot72\ \{(1 - z)\ KI + zHI\},\ 4\cdot72zKHSO^4$. Temperature $t°$ C.

t.	log α.		α.	
	Found.	Calculated.	Found.	Calculated.
0		$\bar{3}\cdot740$		·0055
10	$\bar{2}\cdot057$	$\bar{2}\cdot057$	·0114	·0114
15	$\bar{2}\cdot212$	$\bar{2}\cdot212$	·0163	·0163
20	$\bar{2}\cdot366$	$\bar{2}\cdot365$	·0233	·0232
25	$\bar{2}\cdot514$	$\bar{2}\cdot514$	·0326	·0326
30	$\bar{2}\cdot661$	$\bar{2}\cdot662$	·0458	0459

The number m does not sensibly differ from the value 20·38 in all these three sets of experiments in which the main ingredient of the solution was hydrogen sulphate, although the ratio of the amount of H^2SO^4 to KI varies from 99·2 to 27·9. Assuming the chemical change at zero centigrade in each set to be the unit change for that set, and the value of m to be 20·38 for each set, the amount of chemical change, expressed in terms of the unit change, at a given temperature will be the same in all the three sets. In other words, if α_t, α_0 are the rates at $t°$ and zero in any of the three sets, the ratio $\alpha_t : \alpha_0$ will be the same as the corresponding ratio in the other two sets; the theoretical value of this ratio being $\{(273 + t)/273\}^{20\cdot38}$, or $(T/T_0)^{20\cdot38}$.

In the following Table the values of log (α/α_0) and of α/α_0, corresponding to each value of t and log (T/T_0), are given for the three sets of experiments, and also the calculated values of 20·38 log (T/T_0) and of $(T/T_0)^{20\cdot38}$ compared with the values of log (α/α_0) and of α/α_0 determined by experiment.

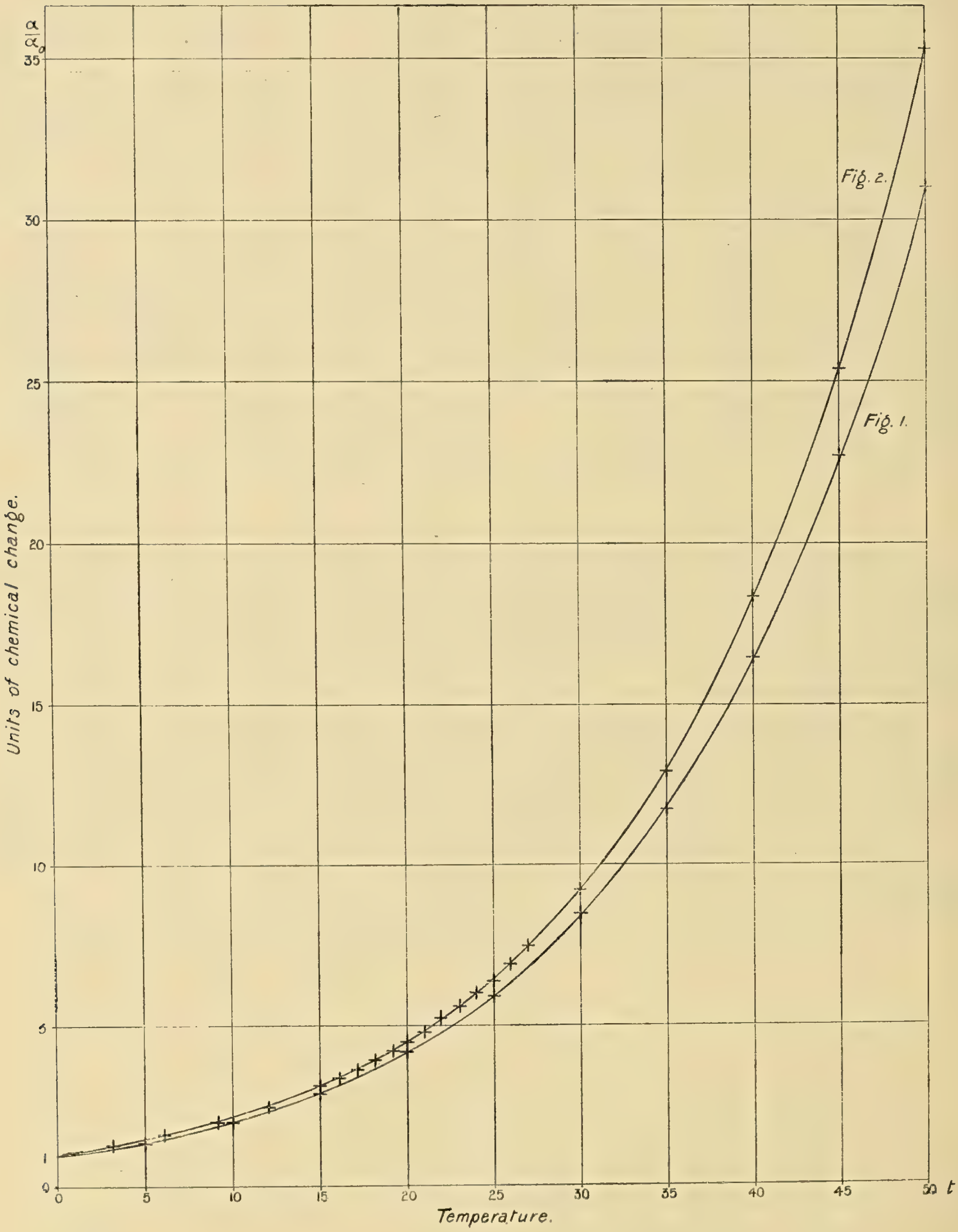
TABLE XL.

t.	$\log (T/T_0)$	$\log (\alpha/\alpha_0)$.				α/α_0.			
		Found.			Calculated.	Found.			Calculated.
		1.	2.	3.		1.	2.	3.	
0	·0000	0·001	..	..	0·000	1·002	..	..	1·000
5	·0079	0·159	..	..	0·160	1·442	..	..	1·445
10	·0156	0·317	..	0·316	0·318	2·075	..	2·070	2·080
15	·0232	0·473	..	0·473	0·473	2·972	..	2·972	2·972
20	·0307	0·628	0·622	0·627	0·626	4·246	4·188	4·237	4·227
25	·0380	0·775	..	0·775	0·775	5·957	..	5·957	5·957
30	·0453	0·924	0·926	0·922	0·923	8·395	8·435	8·356	8·376
35	·0524	1·064	..	..	1·067	11·590	..	..	11·670
40	·0594	1·210	1·212	..	1·210	16·220	16·290	..	16·220
45	·0663	1·352	..	..	1·350	22·490	..	..	22·390
50	·0730	1·491	1·498	..	1·488	30·970	31·480	..	30·770

These numbers agree with one another and with the calculated values within the limits of experimental error. They are represented graphically on Diagram 5, fig. 1, p. 870, in the manner described on p. 861, and on Diagram 4, fig. 1, p. 864 in the manner described on p. 863, the points determined by experiment being in each case represented by crosses.

In all the sets of observations of the rates at different temperatures which we have thus far recorded and discussed the medium in which the change has taken place has been dilute sulphuric acid. The observations which follow show that in presence of hydrochloric acid the value of m is different, but that it is constant for all the cases in which the rates of change in this medium at different temperatures have been determined.

Diagram 5.


$\frac{\alpha}{\alpha_0}$
35
30
25
20
15
10
5
1
0
Units of chemical change.
Fig. 2.
Fig. 1.
0
5
10
15
20
25
30
35
40
45
50
t
Temperature.

The following Tables, XLI.–XLIV., give the conditions and results of these observations:—

TABLE XLI.

$(381 - 7{\cdot}28z)\,HCl$, $7{\cdot}28\,\{(1 - z)\,KI + zHI\}$, $7{\cdot}28zKCl$. Temperature $t°$ C.

$t = 10{\cdot}11$, $y_0 = 17{\cdot}2$.		$t = 15{\cdot}08$, $y_0 = 10{\cdot}2$.		$t = 20{\cdot}34$, $y_0 = 6{\cdot}2$.		$t = 25{\cdot}09$, $y_0 = 2{\cdot}2$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.
4′·52	·0132	5′·50	·0188	6′·47	·0272	16′·17	·0374
4·87	·0131	6·10	·0189	7·88	·0270	47·50	·0377
5·17	·0132	6·90	·0188	10·07	·0270		
5·60	·0130						
6·00	·0131						
6·48	·0132						
Mean rate .	·0131	Mean rate .	·0188	Mean rate .	·0271	Mean rate .	·0375

$t = 18{\cdot}2$, $y_0 = 17{\cdot}2$.		$t = 19{\cdot}28$, $y_0 = 10{\cdot}2$.		$t = 20{\cdot}34$, $y_0 = 6{\cdot}2$.		$t = 21{\cdot}32$, $y_0 = 17{\cdot}03$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.
2′·55	·0234	4′·05	0·255	6′·42	·0274	2′·07	·0292
2·72	·0234	4·52	0·255	7·85	·0272	2·20	·0293
(2) 5·97	·0236	5·15	0·252	10·07	·0270		
3·37	·0233						
3·65	·0234						
Mean rate .	·0234	Mean rate .	·0254	Mean rate .	·0272	Mean rate .	·0292

$t = 22{\cdot}27$, $y_0 = 14{\cdot}03$.		$t = 23{\cdot}2$, $y_0 = 11{\cdot}03$.		$t = 24{\cdot}14$, $y_0 = 8{\cdot}03$.		$t = 26{\cdot}13$, $y_0 = 5{\cdot}03$.		$t = 27{\cdot}1$, $y_0 = 2{\cdot}03$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.	Intervals.	α.
2′·33	0·318	2′·85	·0333	3′·73	·0356	5′·45	·0407	15′·43	·0439
2·58	0·309	3·15	·0333	4·37	·0352	7·05	·0405	67·13	·0437
Mean rate ·0313		Mean rate ·0333		Mean rate ·0354		Mean rate ·0406		Mean rate 0·438	

TABLE XLII.

$(191 - 3{\cdot}64z)\,HCl, \quad 3{\cdot}64\,\{(1-z)\,KI + zHI\}, \quad 3{\cdot}64z\,KCl.$ Temperature t° C.

$t = 30{\cdot}05$, $y_0 = 17$.		$t = 35{\cdot}05$, $y_0 = 10$.		$t = 40{\cdot}05$, $y_0 = 6$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.
3′·85	·0157	4′·73	·0222	5′·85	·0313
4·12	·0157	5·33	·0221	7·17	·0311
4·42	·0157	6·00	·0222	9·32	·0310
4·68	·0158			13·15	·0309
5·03	·0159			22·02	·0316
5·57	·0156				
Mean rate . .	·0157	Mean rate . .	·0222	Mean rate . .	·0312

TABLE XLIII.

$(546 - 5{\cdot}21z)\,HCl, \quad 5{\cdot}21\,\{(1-z)\,KI + zHI\}, \quad 5{\cdot}21z\,KCl.$ Temperature t° C.

$t = 0$, $y_0 = 25{\cdot}25$.		$t = 2{\cdot}98$, $y_0 = 21{\cdot}25$.		$t = 5{\cdot}95$, $y_0 = 17{\cdot}25$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.
6′·80	·00594	6′·60	·00729	6′·43	·00929
7·18	·00586	6·92	·00731	6·88	·00923
7·47	·00589	7·18	·00743	7·40	·00918
Mean rate .	·00590	Mean rate .	·00734	Mean rate .	·00923

$t = 8{\cdot}94$, $y_0 = 13{\cdot}25$.		$t = 11{\cdot}96$, $y_0 = 9{\cdot}25$.		$t = 15{\cdot}03$, $y_0 = 5{\cdot}25$.	
Intervals.	α.	Intervals.	α.	Intervals.	α.
6′·85	·0115	7′·93	·0144	11′·75	·0180
15·47	·0115	8·92	·0145	14·95	·0179
		10·27	·0144	20·40	·0180
Mean rate . .	·0115	Mean rate . .	·0144	Mean rate . .	·0180

TABLE XLIV.

$(273 - 5{\cdot}21z)\,HCl$, $5{\cdot}21\,\{(1 - z)\,KI + z\,HI\}$, $5{\cdot}21\,zKCl$. Temperature t° C.

$t = 15{\cdot}08$, $y_0 = 24{\cdot}13$.	
Intervals.	α.
4′·17	·01014
4·53	·00975
4·65	·00996
4·85	·01000
Mean rate .	·00996

$t = 16{\cdot}07$, $y_0 = 19{\cdot}13$.	
Intervals.	α.
4′·98	·0108
5·27	·0108
Mean rate .	·0108

$t = 17{\cdot}16$, $y_0 = 16{\cdot}13$.	
Intervals.	α.
5′·50	·0116
5·87	·0116
Mean rate .	·0116

$t = 18{\cdot}2$, $y_0 = 13{\cdot}13$.	
Intervals.	α.
6′·27	·0126
6·77	·0127
Mean rate .	·0126

$t = 19{\cdot}28$, $y_0 = 10{\cdot}13$.	
Intervals.	α.
7′·63	·0136
8·48	·0137
Mean rate . . .	·0136

$t = 20{\cdot}34$, $y_0 = 7{\cdot}13$.	
Intervals.	α.
10′·25	·0148
11·97	·0149
Mean rate . . .	·0148

$t = 21{\cdot}32$, $y_0 = 4{\cdot}13$.	
Intervals.	α.
17′·37	·0160
24·28	·0158
39·82	·0159
Mean rate . . .	·0159

The experimental numbers recorded in the preceding tables were corrected for expansion, and values of α calculated by interpolation to correspond to whole numbers of temperature. The value of m in the formula $\alpha/\alpha_0 = (T/T_0)^m$ which most nearly satisfies the four sets of experimental values of α, is 21·17.

The following tables, each of which corresponds to one of the preceding tables give the experimental and calculated values of log α and α.

TABLE XLV.

$(381{\cdot}5 - 7{\cdot}29z)\,HCl$, $7{\cdot}29\,\{(1-z)\,KI + zHI\}$, $7{\cdot}29\,zKCl$. Temperature t° C.

t.	Log. α.		α.	
	Found.	Calculated.	Found.	Calculated.
0		$\bar{3}$·774		·00594
10	$\bar{2}$·114	$\bar{2}$·112	·0130	·0129
15	$\bar{2}$·272	$\bar{2}$·266	·0187	·0185
18	$\bar{2}$·363	$\bar{2}$·361	·0231	·0230
19	$\bar{2}$·397	$\bar{2}$·393	·0249	·0247
20	$\bar{2}$·425	$\bar{2}$·424	·0266	·0265
21	$\bar{2}$·456	$\bar{2}$·455	·0286	·0285
22	$\bar{2}$·489	$\bar{2}$·486	·0308	·0306
23	$\bar{2}$·516	$\bar{2}$·518	·0328	·0330
24	$\bar{2}$·547	$\bar{2}$·549	·0352	·0354
25	$\bar{2}$·573	$\bar{2}$·580	·0374	·0380
26	$\bar{2}$·606	$\bar{2}$·610	·0404	·0407
27	$\bar{2}$·640	$\bar{2}$·641	·0437	·0438

TABLE XLVI.

$(191{\cdot}3 - 3{\cdot}64z)\,HCl$, $3{\cdot}64\,\{(1-z)\,KI + zHI\}$, $3{\cdot}64\,zKCl$. Temperature t° C.

t.	Log. α.		α.	
	Found.	Calculated.	Found.	Calculated.
0		$\bar{3}$·240		·00174
30	$\bar{2}$·198	$\bar{2}$·198	·0158	·0158
35	$\bar{2}$·349	$\bar{2}$·349	·0223	·0223
40	$\bar{2}$·497	$\bar{2}$·497	·0314	·0314

TABLE XLVII.

$(546{\cdot}8 - 5{\cdot}22z)\ HCl,\quad 5{\cdot}22\ \{(1 - z)\ KI + zHI\},\quad 5{\cdot}22\ zKCl.$ Temperature t° C.

t.	Log. α.		α.	
	Found.	Calculated.	Found.	Calculated.
0	$\bar{3}$·770	$\bar{3}$·765	·00589	·00582
3	$\bar{3}$·866	$\bar{3}$·865	·00734	·00733
6	$\bar{3}$·966	$\bar{3}$·965	·00925	·00923
9	$\bar{2}$·062	$\bar{2}$·062	·0115	·0115
12	$\bar{2}$·159	$\bar{2}$·160	·0144	·0144
15	$\bar{2}$·254	$\bar{2}$·257	·0179	·0181

TABLE XLVIII.

$(273{\cdot}4 - 5{\cdot}22z)\ HCl,\quad 5{\cdot}22\ \{(1 - z)\ KI + zHI\},\quad 5{\cdot}22\ zKCl.$ Temperature t° C.

t.	Log. α.		α.	
	Found.	Calculated.	Found.	Calculated.
0		$\bar{3}$·507		·00321
15	$\bar{3}$·995	$\bar{3}$·999	·00989	·00998
16	$\bar{2}$·030	$\bar{2}$·031	·0107	·0107
17	$\bar{2}$·061	$\bar{2}$·062	·0115	·0115
18	$\bar{2}$·096	$\bar{2}$·094	·0125	·0124
19	$\bar{2}$·127	$\bar{2}$·126	·0134	·0134
20	$\bar{2}$·160	$\bar{2}$·157	·0144	·0143
21	$\bar{2}$·191	$\bar{2}$·188	·0152	·0154

In the following table the experimental values of $\log(\alpha/\alpha_0)$ and α/α_0 are compared with the values calculated from the expressions $21{\cdot}17 \log. (T/T_0)$ and $(T/T_0)^{21{\cdot}17}$ respectively.

TABLE XLIX.

t.	log. $\frac{T}{T_0}$.	log. $\frac{\alpha}{\alpha_0}$.					$\frac{\alpha}{\alpha_0}$.				
		XLV.*	XLVI.	XLVII.	XLVIII.	Calculated.	XLV.	XLVI.	XLVII.	XLVIII.	Calculated.
0	·0000	..	..	·005	..	·000	..	..	1·01	..	1·00
3	·0047	..	..	·101	..	·100	..	..	1·26	..	1·26
6	·0094	..	..	·201	..	·200	..	..	1·59	..	1·58
9	·0141	..	..	·297	..	·298	..	..	1·98	..	1·99
10	·0156	·340	..	..	..	·338	2·19	..	..	..	2·18
12	0187	..	..	·394	..	·395	..	..	2·48	..	2·48
15	·0232	·498	..	·489	·488	·492	3·15	..	3·08	3·08	3·10
16	·0247	..		..	·523	·524	..	..	..	3·33	3·34
17	·0262	..	..	..	·554	·555	..	..	..	3·58	3·59
18	·0277	·589	..	..	·589	·587	3·88	..	..	3·88	3·86
19	·0292	·623	..	..	·620	·619	4·20	..	..	4·17	4·16
20	·0307	·651	..	..	·653	·650	4·48	..	..	4·50	4·47
21	·0322	·682	..	..	·684	·681	4·81	..	..	4·83	4·80
22	·0337	·715	..	..	..	·712	5·19	..	..	..	5·15
23	·0351	·742	..	..	..	·744	5·52	..	..	..	5·55
24	·0366	·773	..	..	..	·775	5·93	..	..	..	5·96
25	·0380	·799	..	..	..	·806	6·30	..	..	..	6·40
26	·0395	832	..	..	..	·836	6·80	..	..	..	6·85
27	·0410	·866	..	..	..	·867	7·35	..	..	..	7·36
30	·0453	..	·958	..	..	·958	..	9·08	..	..	9·08
35	·0524	..	1·109	..	..	1·109	..	12·85	..	..	12·85
40	·0594	..	1·257	..	..	1·257	..	18·07	..	..	18·07

These numbers, which agree with each other and with the calculated values within the limits of experimental error, are represented graphically, in the two ways previously described, on Diagram 4, fig. 2, p. 864, and Diagram 5, fig. 2, p. 870.

The following table gives the results of a set of experiments made in a solution containing only hydrogen iodide.

TABLE L.

21·8 *HI.* Temperature t° C.

t = 20·03, y_0 = 11·72.		t = 24·99, y_0 = 7·72.		t = 30, y_0 = 3·72.	
Intervals.	α.	Intervals.	α.	Intervals.	α.
5·62	·0158	5·92	·0234	8·90	·0352
6·20	·0158	6·83	·0236	12·98	·0353
6·85	·0158	8·07	·0238	24·65	·0353
Mean rate .	·0158	Mean rate . .	·0236	Mean rate . .	·0353

The values of α were corrected for expansion and values obtained by interpolation corresponding to whole numbers of temperature. The value of m in the formula $\alpha/\alpha_0 = (T/T_0)^m$ is 24·1, and of log α_0, 3·458. The following table exhibits the agreement of the found and calculated values of log α and α.

TABLE LI.

t.	Log α.		α.		Log (α/α_0).		α/α_0.	
	Found.	Calculated.	Found.	Calculated.	Found.	Calculated.	Found.	Calculated.
0		$\bar{3}$·458		·0029		·000		1·00
20	$\bar{2}$·199	$\bar{2}$·198	·0158	·0158	·740	·740	5·50	5·50
25	$\bar{2}$·374	$\bar{2}$·375	·0237	·0237	·916	·917	8·24	8·26
30	$\bar{2}$·550	$\bar{2}$·549	·0355	·0354	1·092	1·091	12·36	12·33

The numbers are represented graphically on Diagram 4, fig. 3, in the manner described on p. 863.

A comparison of this table with those on pp. 869 and 876 shows that the increase of the rates with temperature is much more rapid than in the cases in which the main ingredients were hydrogen sulphate and hydrogen chloride. At a temperature of 30° the ratio of the chemical change to that at zero is, in the case of hydrogen sulphate, 8·38, in the case of hydrogen chloride 9·08, and in the case of hydrogen iodide 12·33. The value of m in the formula $\alpha/\alpha_0 = (T/T_0)^m$ depends upon the nature of the medium in which the chemical change takes place. This conclusion is confirmed by the results of a set of experiments made with a solution, into which were brought 273 H^2SO^4 and 190 Na^2SO^4. The observations are recorded in the following table.

Reserving the question of the probable interaction between hydrogen sulphate, potassium iodide, and sodium sulphate, the actual ingredients of the solution were, per cub. centim., 273 H^2SO^4, 5·21 KI, and 190 Na^2SO^4.

TABLE LII.—Temperature $t°$ C.

$t = 23$, $y_0 = 11·67$.		$t = 33$, $y_0 = 5·67$.	
Intervals.	α.	Intervals.	α.
6·12	·0147	7·33	·0264
6·80	·0144	9·03	·0267
7·58	·0144	12·07	·0263
8·37	·0146		
9·58	·0146		
Mean rate . .	·0145	Mean rate . .	·0265

The value of m deduced from the rates in this table is 18·1. The ratio of the rate at a temperature of 30° to that at zero is 6·58, which is much less than 8·38, the corresponding ratio in a medium containing only hydrogen sulphate. This variation may probably be due to a change in the actual substances in the solution. At 23° the solution contained

$$(273 - 190y)\ H^2SO^4 + 380y NaHSO^4 + 190\,(1 - y)\ Na^2SO^4.$$

The value of y, that is the equilibrium between the hydrogen sulphate, the sodium hydrogen sulphate, and the sodium sulphate in the solution, is likely to change with rise of temperature, the amount of sodium hydrogen sulphate probably increasing and that of hydrogen sulphate and of sodium sulphate decreasing correspondingly.

The value of m in the formula $\alpha/\alpha_0 = (T/T_0)^m$ is less for solutions containing sodium hydrogen carbonate than any of the values hitherto found. An approximation to the value can be obtained from the results of the experiments recorded on p. 848, with 57·2 $NaHCO^3$ and 8·07 NaI, at a temperature of 17°. The rate with unit of iodide is 2670×10^{-6}, and the rate with the same amounts of $NaHCO^3$ and NaI at a temperature of 15° deduced from the formula (24), p. 847, is 2480×10^{-6}. The value of m deduced from the equation, $\log(2670/2480) = m \log\{(273 + 17)/(273 + 15)\}$, is 10·7. Another approximation can be obtained from the value of the increment of the rate per unit of $NaHCO^3$ found on p. 851. The value of this increment at a temperature of 19° is $45·5 \times 10^{-6}$, but at a temperature of 15° (p. 847) it is 40×10^{-6}. The value of m, deduced from the equation,

$$\log(45·5/40) = m \log\{(273 + 19)/(273 + 15)\} \quad . \ . \ . \ . \quad (48),$$

is 9·3. The mean of these two values, viz., 10, may be taken as an approximation to the value of m in solutions whose main ingredient is sodium hydrogen carbonate.

The following table gives a comparison of the effect of temperature in solutions containing mainly $NaHCO^3$, H^2SO^4, HCl, and HI. The rate at zero centigrade is in each case taken to be unity.

TABLE LIII.

t.	$NaHCO^3$. $m = 10$.	H^2SO^4. $m = 20{\cdot}4$.	HCl. $m = 21{\cdot}2$.	HI. $m = 24{\cdot}1$.
		α.		
0	1·00	1·00	1·00	1·00
10	1·43	2·08	2·18	2·38
20	2·03	4·23	4·47	5·50
30	2·84	8·38	9·08	12·33
40	3·93	16·22	18·07	27·04

A further confirmation of the law of connexion between chemical change and temperature is obtained from experiments on the rate of change of hydrogen chlorate and potassium iodide, by W. H. **PENDLEBURY** and M. **SEWARD**, in vol. 45 of the 'Proceedings of the Royal Society.' On p. 420 are given the rates at different temperatures. In the following table are given these rates and their values calculated from the formula

$$\log (\alpha/\alpha_0) = 40{\cdot}5 \log (T/T_0) \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (49).$$

TABLE LIV.

t.	α.		α/α_0.	
	Found.	Calculated.	Found.	Calculated.
4	·00136	·00139	1·76	1·80
7	·00213	·00215	2·74	2·79
10	·00330	·00324	4·27	4·20
13	·00509	·00498	6·59	6·44
15	·00215	·00212	8·83	8·73
20	·00427	·00427	17·60	17·50
20	·00143	·00142	17·60	17·50
21	·00164	·00163	20·30	20·20
22	·00186	·00187	23·00	23·10
23	·00213	·00214	26·30	26·50
25	·00838	·00840	34·50	34·50
30	·01641	·01660	67·50	68·20

The values of α found agree on the whole with the calculated values within the limits of experimental error. The value of m in the case of the decomposition of hydrogen chlorate at different temperatures is twice as great as in the case of the reaction between hydrogen dioxide and hydrogen iodide in the presence of hydrogen sulphate. Hence, as in the latter case the rate is approximately doubled by a rise in temperature of 10°, it is approximately doubled in the former case by a rise of 5°.

In the discussion of their results the authors adopted a view of the relation between chemical change and temperature, which was originally taken by us, but which we were led to reject for reasons given on p. 859 of this paper.

Discussion of the Preceding Results.

It has been shown, p. 842, that in a medium consisting of water and hydrogen iodide the amount of which in a cubic centimeter varies between 14·5 HI and 43·5 HI, the rate of change α is given by the formula $\alpha = i\,\{a + b\,(i - 1)\}$, i being the number of HI in a cubic centimeter, a the rate when one HI is present and b the increment of a per unit substitution for water of HI. Both a and b remain constant in a set of experiments made at the same temperature but vary with the temperature according to the law $a_1 = a_0\,(\mathrm{T}_1/\mathrm{T}_0)^{24\cdot1}$, $b_1 = b_0\,(\mathrm{T}_1/\mathrm{T}_0)^{24\cdot1}$, a_1, b_1 being the values of a and b at an absolute temperature T_1, and a_0, b_0 their values at a temperature T_0; see p. 877. The most probable values of a and b at a temperature of 30° C. or 303° absolute are 1210×10^{-6} and $19{\cdot}4 \times 10^{-6}$ respectively, so that the formula for α at a temperature 30° C. with a given number i of HI is

$$\alpha = i\,\{1210 + 19{\cdot}4\,(i - 1)\}\,10^{-6} \quad . \; . \; . \; . \; . \; . \; . \quad (50).$$

On pp. 830 and 837 the results of three sets of experiments with 14·22 HI are given in which α has the values ·0210, ·0208, ·0214; the formula gives the value ·0209, which agrees with the mean of the experimental values within the usual limit of error of 1 per cent. On p. 841 the rate with 14·5 HI is ·0214, the formula gives ·0213; on p. 876 the rate with 21·8 HI is ·0353, the formula gives ·0351.

At a temperature of 20° C. or 293° absolute, the value of a is $1210 \times (\tfrac{293}{303})^{24\cdot1} = 540$, and of b, $19{\cdot}4 \times (\tfrac{293}{303})^{24\cdot1} = 8{\cdot}65$, so that the formula for the rate with $i\,HI$ at 20° C. is

$$\alpha = i\,\{540 + 8{\cdot}65\,(i - 1)\}\,10^{-6} \quad . \; . \; . \; . \; . \; . \; . \quad (51).$$

On p. 833 the rate with 22·82 HI at 20° C. is ·0167, the formula gives ·0166; on p. 876 the rate with 21·8 HI at 20° C. is ·0158, the formula gives ·0157; these values agree within the limit of experimental error. It will be observed that in all these comparisons the calculated results fall slightly below the observed values; the reason for this is that no account is taken of the small additions of hydrogen sulphate introduced into the solution with the hydrogen dioxide.

These comparisons of calculated and observed values of the rate of the chemical changes here considered justify the conclusion that the rate at an absolute temperature T_1 with i *HI* is given by the formula $i\{a_1 + b_1(i-1)\}$ and the rate at an absolute temperature T_2 by the formula $i\{a_1 + b_1(i-1)\}(T_2/T_1)^m$ where the value of m is 24·1, and that at a temperature of 30° C. or 303° absolute, the values of a and b are 1210×10^{-6} and $19{\cdot}4 \times 10^{-6}$ respectively. The formula at zero or 273° absolute, may be deduced from formula (50) by means of the equation $\alpha_1 = \alpha_0(T_1/T_0)^{24\cdot1}$. This formula is

$$i\{98 + 1{\cdot}57(i-1)\}\,10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52),$$

and the formula at any temperature t is

$$i\{98 + 1{\cdot}57(i-1)\}\,10^{-6} \times \left(\frac{273+t}{273}\right)^{24\cdot1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (53).$$

We have seen that, at the three different temperatures of 16°, 20°, and 30° C., the law of variation of the rate with hydrogen sulphate, in a medium consisting mainly of water and hydrogen sulphate at a given temperature, is given by the formula

$$\alpha = i\{c_1 + d_1(s - s_1)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54),$$

where s varies between the limits s_1 and s_2, and by the formula

$$\alpha = i\{c_2 + d_2(s - s_2)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55),$$

where s varies between the limits s_2 and s_3; and the explanation given of the different values of d_1 and d_2 is that between the limits s_1 and s_2 two hydrates are present. The hydrate corresponding to s_1 continually diminishes, and the hydrate corresponding to s_2 continually increases, as s increases between these limits, until the limit s_2 is reached, at which point the medium consists entirely of the hydrate corresponding to s_2. A further increase in s diminishes the hydrate corresponding to s_2, and increases the hydrate corresponding to s_3, and so on. On this hypothesis, suppose d_0 to be the increase of rate per unit of the first hydrate formed, d_1 the increase of rate per unit of decrease of the first hydrate and increase of the second, and so on. The formula for the rate from $s = 0$ to $s = s_1$ is of the form,

$$\alpha = i\{c_0 + d_0 s\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56),$$

and this formula will be valid until $s = s_1$, at which point the rate is

$$\alpha_1 = i(c_0 + d_0 s_1) = ic_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57).$$

From $s = s_1$ to $s = s_2$ the formula will become

$$\alpha = i\{c_1 + d_1(s - s_1)\} \quad \ldots\ldots\ldots \quad (58),$$

which will hold good until $s = s_2$, when the rate is

$$\alpha_2 = i\{c_1 + d_1(s_2 - s_1)\} = ic_2 \quad \ldots\ldots\ldots \quad (59).$$

Proceeding in the same manner, the formula from $s = s_{r-1}$ to $s = s_r$ is

$$\alpha = i\{c_{r-1} + d_{r-1}(s - s_{r-1})\} \quad \ldots\ldots\ldots \quad (60)$$

where

$$c_{r-1} = c_{r-2} + d_{r-2}(s_{r-1} - s_{r-2}) \quad \ldots\ldots\ldots \quad (61);$$

hence the final formula which is valid between the values s_{r-1} and s_r is

$$\alpha = i\{c_0 + d_0 s_1 + d_1(s_2 - s_1) + d_2(s_3 - s_2) + \ldots + d_{r-1}(s - s_{r-1})\} \ . \quad (62).$$

This formula may be also written in the form

$$\begin{aligned}\alpha = i\{c_0 + (d_0 - d_1)s_1 + (d_1 - d_2)s_2 + \ldots + (d_{r-2} - d_{r-1})s_{r-1} + d_{r-1}s\} \\ = i(c'_{r-1} + d_{r-1}s) \quad \ldots\ldots\ldots \quad (63),\end{aligned}$$

which shows that if $d_0, d_1 \ldots d_{r-1}$ are in ascending order of magnitude, the quantity c'_{r-1} may become negative; but if they are in descending order of magnitude c'_{r-1} is always positive. There is reason pp. 831, 836, to believe that at first they are in descending order, and finally in ascending order, so that at some value of s_r, c'_{r-1} may become nearly equal to c_0.

If a definite hydrate $n_1 H^2O,H^2SO^4$ is formed when $s=s_1$, and another $n_2 H^2O,H^2SO^4$ when $s = s_2$, and if h_1, h_2 are the quantities of H^2O present when these hydrates are formed, these quantities being with sufficient accuracy assumed to be $h - \mu s_1$, $h - \mu s_2$, the amounts x and y of the first and second hydrate are given by the equation

$$\left.\begin{aligned} n_1 x + n_2 y &= h - \mu s \\ x + y &= s \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (64),$$

hence

$$x = (h - \mu s - n_2 s)/(n_1 - n_2) \quad \ldots\ldots\ldots \quad (65),$$

$$y = (n_1 s - h + \mu s)/(n_1 - n_2) \quad \ldots\ldots\ldots \quad (66),$$

and if e_1, e_2 are the increments of unit rate per unit of the hydrates, the rate between $s = s_1$, and $s = s_2$ is given by the formula

$$\alpha = i\,\{f_1 + e_1 x + e_2 y\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67),$$

$$= i\,[f_1 - (e_2 - e_1)\,h/(n_1 - n_2) + \{\mu\,(e_2 - e_1) + n_1 e_2 - n_2 e_1\}s/(n_1 - n_2)] \quad . \quad (68).$$

Comparing this with the formula (54) we obtain

$$d_1 = \{\mu\,(e_2 - e_1) + n_1 e_2 - n_2 e_1\}/(n_1 - n_2) \quad . \quad . \quad . \quad . \quad . \quad (69),$$

$$c_1 = f_1 - (e_2 - e_1)\,h/(n_1 - n_2) + d_1 s_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (70),$$

from which, if the hydrates n_1, n_2 were known, and their increments e_1, e_2 of unit rate, the increment d_1 could be calculated.

Until the first hydrate ascertained corresponding to $s_1\,H^2SO^4$ is reached, the formula for α with $s\,H^2SO^4$, and $i\,HI$ is at a temperature t_1,

$$\alpha = i\{a + b\,(i - 1) + d_1 s\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71);$$

when $s = s_1$, this becomes

$$\alpha_1 = i\{a + b\,(i - 1) + d_1 s_1\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72).$$

For values of s between s_1 and s_2, the value of s corresponding to the second hydrate ascertained,

$$\alpha = i\{a + b\,(i - 1) + d_1 s_1 + d_2\,(s - s_1)\} \quad . \quad . \quad . \quad . \quad . \quad (73);$$

when $s = s_2$, this becomes

$$\alpha_2 = i\{a + b\,(i - 1) + d_1 s_1 + d_2\,(s_2 - s_1)\} \quad . \quad . \quad . \quad . \quad . \quad (74).$$

For values of s beyond this second hydrate,

$$\alpha = i\{a + b\,(i - 1) + d_1 s_1 + d_2\,(s_2 - s_1) + d_3\,(s - s_2)\} \quad . \quad . \quad (75).$$

The results of the experiments on the variation of rate with temperature show that for any other temperature t the rates corresponding to these are obtained by multiplying them by the factor $\{(273 + t)/(273 + t_1)\}^m$.

The values assumed in the following discussion for the constants in these formulæ are at a temperature of 30°

$b = 19{\cdot}4 \times 10^{-6}$, previously ascertained (p. 880),

$d_1 = 18{\cdot}1 \times 10^{-6}$, obtained as a mean of the values of d, at the temperatures 16°, 20°, 30°, previously employed in the discussion of the experiments on pp. 822 to 836.

$d_2 = 22{\cdot}4 \times 10^{-6}$, similarly obtained,
$d_3 = 26{\cdot}5 \times 10^{-6}$, also similarly obtained,
$s_1 = 515$, corresponding to the hydrate H^2SO^4, $106H^2O$,
$s_2 = 762$, ,, ,, ,, H^2SO^4, $71H^2O$,

values already determined in the same sets of experiments,

$m = 20{\cdot}38$, determined from the experiments on temperature.

The value of a has now to be found. Substituting the values of the constants assumed above in the whole series of experiments in which hydrogen sulphate is a main ingredient of the solution, we obtain the following table; the first column gives the amount and nature of the iodide used, the second the number n of experiments which give the value of $a \times 10^6$, given in the third column. The fourth column is the product $na \times 10^6$. The true value of $a \times 10^6$ is finally obtained by dividing the sum of the numbers $na \times 10^6$ by the sum of the numbers n. An examination of the values of a shows that the value of a corresponding to $i = 22{\cdot}82$ falls considerably below the mean value of the rest. This value is excluded from the computation. In the set of experiments from which this value is found, the ratio of the number s of H^2SO^4 to the number i of HI is at first 2 and finally 12, and it appears hereafter that in a medium of water, hydrogen sulphate, and hydrogen iodide, in which the ratio $s : i$ falls below 20, the constant b has a smaller value than it has in media in which the hydrogen sulphate predominates, and that the value of the temperature-constant m has some value intermediate between 24·1 and 20·38 which are its values when the solution consists mainly of hydrogen iodide and hydrogen sulphate respectively.

TABLE LV.

i.	n.	$a \times 10^6$.	$na \times 10^6$.
KI.			
3·64	8	1220	9760
3·64	5	1230	6150
3·64	3	1240	3720
4·72	1	1190	1190
7·29	1	1180	1180
3·64	1	1290	1290
3·64	1	1220	1220
7·29	1	1220	1220
4·72	1	1220	1220
HI			
12·2	6	1210	7260
12·2	3	1175	3525
12·2	5	1250	6250
11·41	7	1265	8855
11·41	2	1250	2500
	45	1230	55340

Assuming the value of $a \times 10^6$ at a temperature 30° to be 1230, its value at zero is 147; and the values of b, d_1, d_2, d_3 at zero are, when multiplied by the factor 10^6, 2·32, 2·16, 2·68, 3·16 respectively, so that the formulæ for α at zero, corresponding to the general formulæ 71, 73, 75, are

$$\alpha = i\{147 + 2{\cdot}32(i-1) + 2{\cdot}16s\}10^{-6} \qquad (76),$$

$$\alpha = i\{147 + 2{\cdot}32(i-1) + 2{\cdot}16 \times 515 + 2{\cdot}68(s-515)\}10^{-6} \qquad (77),$$

$$\alpha = i\{147 + 2{\cdot}32(i-1) + 2{\cdot}16 \times 515 + 2{\cdot}68(762-515) + 3{\cdot}16(s-762)\}10^{-6} \qquad (78),$$

and the formulæ corresponding to these at a temperature t are obtained by multiplying these by the factor $\{(273+t)/273\}^{20{\cdot}38}$.

At a temperature of 16° the three formulæ are

$$\alpha = i\{469 + 7{\cdot}4(i-1) + 6{\cdot}89s\}10^{-6} \qquad (79),$$

$$\alpha = i\{469 + 7{\cdot}4(i-1) + 6{\cdot}89 \times 515 + 8{\cdot}55(s-515)\}10^{-6} \qquad (80),$$

$$\alpha = i\{469 + 7{\cdot}4(i-1) + 6{\cdot}89 \times 515 + 8{\cdot}55(762-515) + 10{\cdot}09(s-762)\}10^{-6} \qquad (81).$$

At a temperature of 20° they are

$$\alpha = i\{621 + 9{\cdot}8(i-1) + 9{\cdot}13s\}10^{-6} \qquad (82),$$

$$\alpha = i\{621 + 9{\cdot}8(i-1) + 9{\cdot}13 \times 515 + 11{\cdot}32(s-515)\}10^{-6} \qquad (83),$$

$$\alpha = i\{621 + 9{\cdot}8(i-1) + 9{\cdot}13 \times 515 + 11{\cdot}32(762-515) + 13{\cdot}37(s-762)\}10^{-6} \qquad (84).$$

At a temperature of 30° they are

$$\alpha = i\{1230 + 19{\cdot}4(i-1) + 18{\cdot}1s\}10^{-6} \qquad (85),$$

$$\alpha = i\{1230 + 19{\cdot}4(i-1) + 18{\cdot}1 \times 515 + 22{\cdot}4(s-515)\}10^{-6} \qquad (86),$$

$$\alpha = i\{1230 + 19{\cdot}4(i-1) + 18{\cdot}1 \times 515 + 22{\cdot}4(762-515) + 26{\cdot}5(s-762)\}10^{-6} \qquad (87).$$

These formulæ give values of α which agree within the limits of experimental error with all the values of α recorded in the preceding pages, with the exception of (1) those in which $i = 11{\cdot}41$, and s has values ranging from 44 to 219; (2) those in which $i = 12{\cdot}2$, and s has values ranging from 54 to 218; (3) those in which

$i = 22{\cdot}82$, and s has values ranging from 44 to 263. In all these cases the ratio of s to i falls below 20. The values of α are given within the limits of experimental error if the values of b, the increment per unit of iodide, at different temperatures, are assumed to follow the law $b_1 = b_0 (T_1/T_0)^{24{\cdot}1}$, instead of the law $b_1 = b_0 (T/T_0)^{20{\cdot}38}$. In the absence of experiments which would show the law of variation of m in the formula $\alpha_1 = \alpha_0 (T/T_0)^m$, with the amounts of water, hydrogen iodide, and hydrogen sulphate present in the medium, this assumption gives a sufficient approximation to the actual state of the case.

The values of b at 16° and 20° on this assumption are $19{\cdot}4\ (289/303)^{24{\cdot}1} = 6{\cdot}21$, and $19{\cdot}4\ (293/303)^{24{\cdot}1} = 8{\cdot}65$.

At 16° the formula is

$$\alpha = i \{469 + 6{\cdot}21 (i - 1) + 6{\cdot}89 s\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (88).$$

At 20° the formula is

$$\alpha = i \{621 + 8{\cdot}65 (i - 1) + 9{\cdot}13 s\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (89).$$

These formulæ are approximately true for values of i between 11 and 23, and for values of s between 44 and $20i$.

In a medium consisting of water, hydrogen iodide and hydrogen chloride, the formula for the rate at a temperature t_1 is of the form

$$\alpha = i \{a + b (i - 1) + ec\}$$

i and c being the number of HI and HCl present in the solution. The rate at another temperature t is obtained by multiplying by the factor $\{(273 + t)/(273 + t_1)\}^{21{\cdot}17}$.

When the iodide is in the form iKI, and the assumption is made that iHI is produced at the expense of $iHCl$, the formula becomes

$$\alpha = i \{a + b (i - 1) + e (c - i)\}.$$

Assuming further that the value of b at a temperature 30° is $19{\cdot}4 \times 10^{-6}$ as in the case of media consisting of water and hydrogen iodide or of water, hydrogen iodide and hydrogen sulphate, the value of e, the increment per unit of hydrogen chloride per unit of hydrogen iodide, which satisfies the values of α obtained in the experiments recorded on p. 840, and in those made at various temperatures recorded on pp. 874, 875, is $16{\cdot}2 \times 10^{-6}$; and the mean value of a determined in the same manner as in the case of hydrogen sulphate is 1230×10^{-6}. It is seen that this value of a is the same for solutions containing hydrogen chloride as it is for solutions containing hydrogen sulphate.

In all the experiments from which these values of a and e are determined, the values of i range from $3{\cdot}64$ to $14{\cdot}56$, and the values of c range from $191{\cdot}3$ to $381{\cdot}3$.

The ratio $c : i$ exceeds in all cases the value 25. Within these limits the formula for α at 30° is

$$\alpha = i\{1230 + 19{\cdot}4\,(i - 1) + 16{\cdot}2c\}\,10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad (90).$$

The corresponding formula at zero is

$$\alpha = i\{135 + 2{\cdot}14\,(i - 1) + 1{\cdot}78c\}\,10^{-6} \quad . \quad . \quad . \quad . \quad . \quad . \quad (91),$$

and the corresponding formula at a temperature t is obtained by multiplying by the factor $\{(273 + t)/273\}^{21{\cdot}17}$.

In the experiments recorded on p. 837 with 14·22 HI, and with numbers of HCl, ranging from 71·1 to 355·5, the value of e is $16{\cdot}8 \times 10^{-6}$, a value higher than $16{\cdot}2 \times 10^{-6}$, the value which satisfies the experiments discussed above, and the value of a is also larger, being 1345×10^{-6} instead of 1230×10^{-6}. It is also seen that the value of e for the range of numbers of HCl from 0 to 71·1 is 18×10^{-6} instead of $16{\cdot}8 \times 10^{-6}$. An examination of Table XIV. shows that the value of e between 284·4 HCl and 355·5 HCl falls below the values of e for the range 71·1 HCl to 284·4 HCl. It seems probable that in media consisting of water, hydrogen iodide, and hydrogen chloride, in which the ratio $c : i$ falls below 20, the values of a and e are greater than in similar media in which the ratio of $c : i$ is greater than 20. The same kind of difference in the values of a and d has already been observed in the case of media consisting of water, hydrogen iodide, and hydrogen sulphate in which the ratio $s : i$ is less than 20. Further experiments are needed to reveal the cause of this variation in the values of these constants.

An examination of these cases of chemical action in media, the constituents of which are varied, leads to the conclusion that the rate of action at a given temperature t_1, in a medium consisting of water, hydrogen iodide, and another substance X, is given by the equation

$$\alpha_1 = i\{a_1 + b_1(i - 1) + c_1x\},$$

where i is the number of HI, x the number of X in an unit volume, and a_1, b_1, c_1 are constants, provided that the ratio $x : i$ is sufficiently great to make the medium homogeneous throughout the range of the values of x. The rate of action at another temperature t_2 is, under the same conditions,

$$\alpha_2 = i\{a_2 + b_2(i - 1) + c_2x\},$$

a_2, b_2, c_2 being constants connected with a_1, b_1, c_1 by the equations,

$$a_2/a_1 = b_2/b_1 = c_2/c_1 = (T_2/T_1)^m,$$

m being a constant depending only upon the nature of the medium.

APPENDIX.

On Chemical Equilibrium.

The case of reverse action discussed in pp. 854, 855 is one which is probably of frequent occurrence, and deserves a special investigation. The general problem may be thus stated.

In a homogeneous medium two substances AB, CD interact to form the two AC, BD, which also interact to re-form the first pair; the amount of AC formed per unit of time varies as the amount of AB when CD is constant, and the same law governs the action of the other substances; having given the amount of each substance present at a given time, to determine the amount present at any subsequent time.

Let a, a', b, b' be the initial amounts of AB, CD, AC, BD respectively, and x, x', y, y' the corresponding amounts at a subsequent time t; these quantities are connected by the equations $x + y = a + b$, $x + y' = a + b'$, $x' + y = a' + b$, $x' + y' = a' + b'$, so that $\dot{x} = \dot{x}' = -\dot{y} = -\dot{y}'$. If, when an unit of each of the substances AB, CD is constantly present, α of each of them undergoes change in an unit of time, and β is a similar quantity for AC, BD, the law of chemical change above given leads to the equation,

$$\begin{aligned}\dot{x} &= \beta yy' - \alpha xx' \\ &= \beta (a + b - x)(a + b' - x) - \alpha x (a' - a + x), \\ &= \beta \{bb' - (b + b')(x - a) + (x - a)^2\} \\ &\quad - \alpha \{aa' + (a + a')(x - a) + (x - a)^2\}.\end{aligned}$$

Putting

$$\lambda = 4 (\beta - \alpha)(\beta bb' - \alpha aa'),$$

$$\mu = \alpha (a + a') + \beta (b + b'),$$

and

$$\nu^2 = \mu^2 - \lambda = \{(a - a')\alpha + (b - b')\beta\}^2 + 4\alpha\beta (a + b)(a' + b'),$$

the equation becomes

$$4 (\beta - \alpha)(x - a)^{\cdot} = 2 \{(\beta - \alpha)(x - a) - \mu\}^2 - \nu^2;$$

the solution of which is

$$\log \left[\left\{ \frac{2 (\beta - \alpha)(x - a) - \mu - \nu}{2 (\beta - \alpha)(a - a) - \mu + \nu} \right\} \frac{\mu - \nu}{\mu + \nu} \right] = \nu t,$$

or

$$2 (\beta - a)(x - a) = (\mu + \nu) \frac{1 - e^{\nu t}}{1 - \dfrac{\mu + \nu}{\mu - \nu} e^{\nu t}};$$

putting

$$(\mu+\nu)/2(\beta-\alpha)=c,$$
$$(\mu+\nu)/(\mu-\nu)=k,$$
$$x=a+c(1-e^{\nu t})/(1-ke^{\nu t}).$$

It follows that if $u=c(1-e^{\nu t})/(1-ke^{\nu t})$ the quantities of AB, CD, AC, BD present at a time t, are respectively $a+u$, $a'+u$, $b-u$, $b'-u$; u being a positive or negative quantity.

When t is infinite,

$$u=c/k=(\mu-\nu)/2(\beta-\alpha),$$

and

$$\begin{aligned}4(\beta-\alpha)(\beta yy'-\alpha xx')&=4(\beta-\alpha)\{\beta(b-u)(b'-u)-\alpha(a+u)(a'+u)\}\\&=\lambda-4(\beta-\alpha)\mu u+4(\beta-\alpha)^2u^2\\&=\lambda-2\mu(\mu-\nu)+(\mu-\nu)^2\\&=\lambda-\mu^2+\nu^2=0.\end{aligned}$$

So that theoretically after an infinite time chemical equilibrium is established, the quantities of AB, CD, AC, BD constantly present being respectively $a+c/k$, $a'+c/k$, $b-c/k$, $b'-c/k$.

We may, without loss of generality, consider β to be greater than α, so that c is positive, and the sign of k depends on that of $\mu-\nu$ or λ. Now λ is positive or negative according as $\beta bb'-\alpha aa'$ is positive or negative, *i.e.*, according as the amount of AB is initially increasing or diminishing. If k is negative, say $-k'$, the equation for the amount of AB at a time t is

$$x=a-c\frac{e^{\nu t}-1}{k'e^{\nu t}+1},$$

and when equilibrium is established the amounts of AB, CD, AC, BD present are respectively $a-c/k'$, $a'-c/k'$, $b+c/k'$, $b'+c/k'$.

In the particular case when $\alpha=\beta$

$$\begin{aligned}x&=a+\frac{bb'-aa'}{a+a'+b+b'}\{1-e^{-(a+a'+b+b')\alpha t}\}\\&=\frac{(a+b)(a+b')}{a+a'+b+b'}-\frac{bb'-aa'}{a+a'+b+b'}e^{-(a+a'+b+b')\alpha t},\end{aligned}$$

and the quantity of AB present when equilibrium is established is

$$(a+b)(a+b')/(a+a'+b+b').$$

In the case in which AB, CD are present initially in equivalent quantities, each equal to a, and neither AC nor BD are initially present,

$$x=a\left\{1-m\frac{e^{\nu t}-1}{ne^{\nu t}+1}\right\}.$$

Where $m = \alpha^{\frac{1}{2}}/(\beta^{\frac{1}{2}} - \alpha^{\frac{1}{2}})$, $n = (\beta^{\frac{1}{2}} + \alpha^{\frac{1}{2}})/(\beta^{\frac{1}{2}} - \alpha^{\frac{1}{2}})$, and when equilibrium is established the amount of AB or CD present is $a\beta^{\frac{1}{2}}/(\beta^{\frac{1}{2}} + \alpha^{\frac{1}{2}})$, and the amount of AC or BD present is $a\alpha^{\frac{1}{2}}/(\beta^{\frac{1}{2}} + \alpha^{\frac{1}{2}})$.

In the case in which there are originally present a units of AB, ra units of CD, and, at any subsequent time t, $a - y$ units of AB, $ra - y$ units of CD, and y units of AC or BD,

$$\begin{aligned}\dot{y} &= \alpha\,(a - y)\,(ra - y) - \beta y^2\\ &= (\beta - \alpha)\,(p - y)\,(q + y),\end{aligned}$$

p and $-\,q$ being the roots of

$$(\beta - \alpha)\,y^2 + a\alpha\,(r + 1)\,y - \alpha r a = 0,$$

the course of the change is given by the equation

$$\log\,\{p\,(q + y)\,/\,q\,(p - y)\} = (\beta - \alpha)\,(p + q)\,t$$

or

$$y = pq\,(e^{\nu t} - 1)\,/\,(p + qe^{\nu t})$$

where $\nu = (\beta - \alpha)\,(p + q)$, and the equilibrium is established, theoretically after an infinite time, when there are present $a - p$ units of AB, $ra - p$ units of CD, and p units of AC or BD.

The equations obtained in this case are applicable to the experiments contained in a paper by Dr. GLADSTONE published in the Transactions of the Royal Society ('Phil. Trans.,' vol. 145, p. 187). The most probable value of a is 792, and of $(\beta - \alpha)/a\alpha$ is $\cdot 0705$. Therefore $\beta = 57\alpha$, so that the rate of the change

$$\mathrm{AC} + \mathrm{BD} = \mathrm{AB} + \mathrm{CD}$$

is fifty-seven times the rate of the change

$$\mathrm{AB} + \mathrm{CD} = \mathrm{AC} + \mathrm{BD}.$$

The equation of the course of the change when $r = 1$ is

$$\log\left\{\cdot 766\left(\frac{\cdot 153 + y}{\cdot 117 - y}\right)\right\} = 15{\cdot}1\alpha t\,;$$

when $r = 15{\cdot}4$ it is

$$\log\left\{\cdot 576\left(\frac{\cdot 691 + y}{\cdot 398 - y}\right)\right\} = 60{\cdot}9\alpha t\,;$$

when $r = 125$ it is

$$\log_e\left\{\cdot 249\left(\frac{2{\cdot}99 + y}{\cdot 745 - y}\right)\right\} = 209\alpha t$$

If in each of these cases the change is allowed to proceed until the value of y is within 1 per cent. of the value p which it attains, after a theoretically infinite time, when equilibrium is established, and t_1, t_2, t_3 are in each case the intervals from the commencement of the change to the instant when y has the value $\cdot 99p$, then $t_1 : t_2 : t_3 = 30 : 7 : 2$ nearly; so that if the change reaches this point in the third case in two minutes, it will not reach it in the first case until the change has proceeded for at least half an hour. This fact probably accounts for the difference between the observed and calculated values of y in the earlier part of the following table; for it is possible that the observations in the experiments in which a small quantity of potassium sulphocyanide was taken, were made before the change had practically reached its limit, in which case the observed values of p would be too small. In the following table r is the number of units of potassium sulphocyanide (the amount of ferric nitrate being unity), p' the amount of red salt formed expressed in the arbitrary units adopted by Dr. GLADSTONE and p the number of units of change when equilibrium is established.

r.	p.	p'.	
	Calculated.	Found.	Calculated.
1·0	·117	88	93
2·0	·164	127	130
3·2	·205	156	162
4·2	·231	176	183
5·4	·259	195	205
6·4	·279	213	221
9·4	·327	266	259
15·4	·398	318	316
21·0	·447	356	354
33·0	·522	419	414
45·0	·575	457	456
63·0	·634	508	502
81·0	·676	539	536
99·0	·710	560	562
125·0	·745	587	590

The relation between p and r is

$$(1 - p)(r - p) = 57p^2$$

and

$$p' = 792p.$$

From $r = 15\cdot 4$ to $r = 125$ the calculated results agree with those found within the limits of experimental error, and the divergence of the earlier results in the table has been explained above.

Another set of observations, p. 199 of the same paper, gives $a = 126$ $(\alpha - \beta) / a\alpha = \cdot 00596$, therefore $\alpha = 4 \cdot 07\beta$. In this case the rate of the change

$$AB + CD = AC + BD$$

is nearly four times the rate of the reverse change

$$AC + BD = AB + CD.$$

In the following table r is the number of units of potassium ferrocyanide (the amount of ferric citrate being unity), p the number of units of change when equilibrium is established, p' the amount of blue salt formed, in the arbitrary unit of Dr. GLADSTONE'S paper.

r.	p.	p'.	
	Calculated.	Found.	Calculated.
1	·668	88	85
2	·847	107	107
3	·906	113	115
5	·942	120	119

The relation between p and r is

$$(1 - p)(r - p) = \cdot 246p^2$$

and

$$p' = 126p.$$

The experiments on p. 193 of the same paper are given in the following table. The ratio of the equivalents of ferric chloride and gallic acid is 1 : 1·26.

r.	p.	p'.	
	Calculated.	Found.	Calculated.
1·26	·624	88	89
2·52	·781	108	111
3·78	·845	120	120
5·04	·880	128	125
7·56	·919	133	131

The relation between p and r is

$$(1 - p)(r - p) = \cdot 63p^2$$

and

$$p' = 143p.$$

The experiments on p. 189, treated in the same way, give the relation

$$(1 - p)(r - p) = 95p^2.$$

In all these cases there occur a direct action and a reverse action, between which equilibrium is ultimately established. The ratio of the rates of the two actions is very different in the four cases here discussed; they are, taken in order, 57 : 1, 1 : 4, 1 : 1·6, 95 : 1. The cases chosen for calculation are those in which the curves representing the observations appear to be regular. The theoretical curve is in each case a hyperbola whose equation is of the form $(1 - y)(x - y) = my^2$.

A case of equilibrium similar to the cases here discussed has been investigated by one of us in a note appended to a paper by Professor DIXON, published in the Transactions of the Royal Society ('Phil. Trans.,' vol. 175, Part II., p. 682). The reactions considered in the note are

$$H^2O + CO = H^2 + CO^2,$$
$$H^2 + CO^2 = H^2O + CO.$$

If α, β are the rates of these reactions, the ratio $\beta : \alpha$ is shown to be independent of the quantities of the substances taking part in the reaction, and to depend upon the conditions of temperature, pressure, &c. The ratio $\beta : \alpha$ varies in the experiments made under different conditions from 2·7 to 7.

ADDENDUM.

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Another kind of chemical equilibrium occurs when a substance C, which is formed by the reaction between the substances A and B, reacts with A to form B. After a time, which is theoretically infinite, the quantities of B and C remain constant, whilst the quantity of A is continually diminishing. The equations of these reactions are

$$A + B = C + D, \quad A + C = B + E.$$

Let a, b be the masses of A, B, respectively, initially present, x, y, z the masses of A, B, C present at a subsequent time t, then

$$\dot{x} = -(\alpha y + \beta z)\,x,$$
$$\dot{z} = (\alpha y - \beta z)\,x,$$

α being the amount of each of A and B which react in an unit of time when an unit of each of them is present, and β the amount of each of A and C which react in an unit of time when an unit of each of them is present. When the masses of B and C become constant, $\dot{z} = 0$ and $\alpha y = \beta z$, so that when equilibrium is established the mass of B is $b\beta/(\alpha + \beta)$, and the mass of C is $b\alpha/(\alpha + \beta)$. The quantity of A remaining afterwards is determined from the equation

$$\dot{x} = -\, 2\alpha\beta b x/(\alpha + \beta),$$

or

$$x = a_1 \text{ exp. } \{-\, 2\alpha\beta b t/(\alpha + \beta)\},$$

a_1 being the mass of A present when equilibrium is established, and t the subsequent time.

The course of the chemical change previous to the establishment of equilibrium is sufficiently indicated by a study of the case when $\beta = \alpha$. The equations of the change become in this case

$$\dot{x} = -\, \alpha\,(y + z)\, x = -\, \alpha b x,$$
$$\dot{z} = \alpha\,(y - z)\, x = \alpha\,(b - 2z)\, x,$$

or

$$x = a \text{ exp. } (-\, \alpha b t),$$
$$\log \{b/(b - 2z)\} = 2a\, \{1 - \text{exp. } (-\, \alpha b t)\}/b.$$

It is seen from these equations that, after an infinite time, A is completely decomposed and the amounts of B and C present are $\frac{1}{2} b\, \{1 + \text{exp. } (-\, 2a/b)\}$ and $\frac{1}{2} b\, \{1 - \text{exp. } (-\, 2a/b)\}$, respectively. Theoretically, equilibrium between B and C has not been completely established unless a is infinitely great in comparison with b.

An examination of a particular case will show that equilibrium may be very nearly established in a finite time. When $a = 5$, $b = 2$, $\alpha = {\cdot}25$, the following table of values are obtained :—

t.	x.	y.	z.
·5	3·89	1·33	·67
1	3·03	1·14	·86
2	1·84	1·04	·96
3	1·11	1·02	·98
4	0·68	1·01	·99
5	0·41	1·01	·99

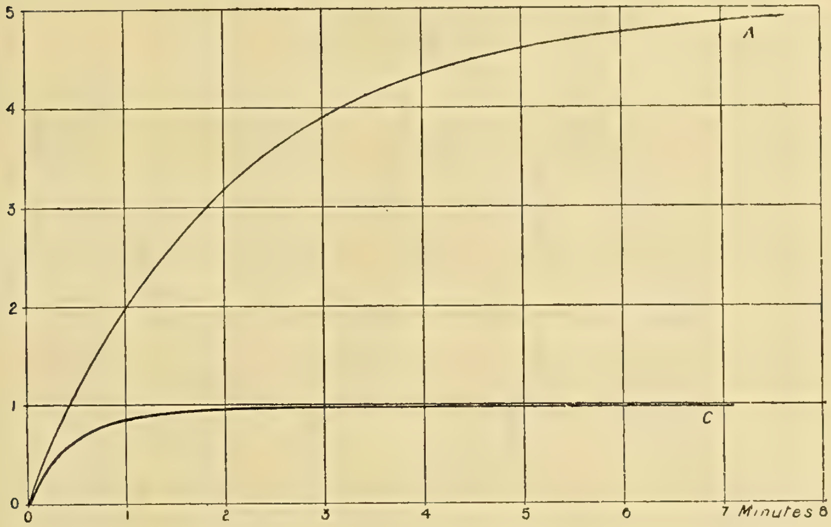
This table shows that, within the usual limit of error in chemical experiments, equilibrium is practically attained after the reaction has proceeded for four minutes, if the rate of change is such that when an unit mass of each of the substances undergoing change is present, one quarter of the total change is effected in one minute.

These results are represented graphically in the accompanying plate, in which the curves A and C represent the course of the change of the substances A and C.

This theory explains the case of equilibrium recorded on p. 854, in which the reactions are

$$H^2O^2 + 2HI = I^2 + 2H^2O,$$
$$H^2O^2 + I^2 = 2HI + O^2.$$

Diagram 6.



XXIII. *Theorems on the Attraction of Ellipsoids for certain Laws of Force other than the Inverse Square.*

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1. To find the potential at an internal point P of a thin ellipsoidal shell bounded by similar concentric ellipsoids, the law of force being the inverse κth power of the distance.

Let (x, y, z) be the coordinates of P; (l, m, n) the direction cosines of a radius vector drawn from P to any point Q on the surface; and let PQ $= r$. Let (a, b, c) be the semi-axes of the ellipsoids, and (α, β, γ) their squared reciprocals. Then from the equation of the ellipsoid

$$(\alpha l^2 + \beta m^2 + \gamma n^2) r^2 + 2 (\alpha l x + \beta m y + \gamma n z) r - (1 - \alpha x^2 - \beta y^2 - \gamma z^2) = 0 \quad . \quad (1).$$

For the sake of brevity we shall put

$$\left.\begin{aligned} 1/D^2 &= \alpha l^2 + \beta m^2 + \gamma n^2 \\ F &= \alpha lx + \beta my + \gamma nz \\ E &= 1 - \alpha x^2 - \beta y^2 - \gamma z^2 \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (2).$$

Thus D is the semi-diameter parallel to the radius vector r, F is a linear function of (x, y, z) such that $F = 0$ represents a plane through the centre O diametral to PQ, and $E = 0$ is the equation of the ellipsoid.

The quadratic may therefore be written in either of the forms

$$\left.\begin{aligned} r^2 + 2FD^2 r - ED^2 = 0 \\ u^2 - \frac{2F}{E} u - \frac{1}{ED^2} = 0 \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (3)$$

where u is the reciprocal of r. The roots r_1, r_2 have opposite signs and

$$r_1 + r_2 = -2FD^2, \qquad r_1 r_2 = -ED^2 \quad \ldots \ldots \quad (4).$$

Taking together the two elements of the shell enclosed by an elementary cone whose vertex is the attracted point P and whose solid angle is $d\omega$, the potential of the whole shell is evidently

$$V = \frac{1}{\kappa - 1}\iint\left\{\frac{\rho_1 r_1^2\, d\omega\, dr_1}{r_1^{\kappa - 1}} + \frac{\rho_2 r_2^2\, d\omega\,(-dr_2)}{(-r_2)^{\kappa - 1}}\right\} \quad \ldots \ldots \quad (5),$$

where r_1, r_2 have their proper signs as given by the standard quadratic (3), r_1 being the positive root and r_2 the negative root. Also ρ_1, ρ_2 are the densities of the shell at the extremities of the chord through P.

It is evident that if we integrate all round the point P every element of the surface is taken twice over. *We must therefore halve the result given by* (5).

2. The values of dr_1, dr_2 depend on the kind of shell under consideration. Supposing that the shell is a thin homœoid, we differentiate the quadratic (1) on the supposition that b/a and c/a are constant. We thus find

$$dr = \frac{da}{a} \cdot \frac{D^2}{r + FD^2}.$$

Using the relations (2) and (4), it immediately follows that

$$dr_1 = \frac{da}{a} \frac{2D^2}{r_1 - r_2}, \qquad dr_2 = \frac{da}{a} \frac{2D^2}{r_2 - r_1} \quad \ldots \ldots \quad (6).$$

These values of dr_1, dr_2 may easily be verified by a short geometrical proof.

The potential at an internal point, P, of a thin homœoid is therefore

$$V = \frac{1}{\kappa - 1} \frac{da}{a} \iint \frac{\rho_1 r_1^{3-\kappa} + \rho_2 (-r_2)^{3-\kappa}}{r_1 - r_2} D^2 \, d\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad (7),$$

where the integration extends all round the point P. Writing $da/a = \mu$, this formula also represents the potential of a thin stratum placed on the ellipsoid whose surface-density at any point Q is $\mu\rho p$, where p is the perpendicular from the centre on the tangent plane at Q.

3. *If the law of force is any even power of the distance,* κ is even, *and if the density is uniform and equal to unity the expression* (7) takes the form

$$V = \frac{1}{\kappa - 1} \frac{da}{a} \iint \frac{r_1^{3-\kappa} - r_2^{3-\kappa}}{r_1 - r_2} D^2 \, d\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8),$$

in which the subject of integration is a symmetrical function of the roots of the fundamental quadratic (3). Since such a function can be expressed as an integral rational function of the roots, the integration is much facilitated.

If κ be greater than 3, it will be more convenient to write $r = 1/u$. Supposing κ to be an even integer the formula (7) then becomes

$$V = \frac{1}{\kappa - 1} \frac{da}{a} \frac{1}{E} \iint \frac{\rho_1 u_1^{\kappa-3} - \rho_2 u_2^{\kappa-3}}{u_1 - u_2} \, d\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

If the law of force is any odd power of the distance and the density is uniform, the potential at an internal point becomes

$$V = \frac{1}{\kappa - 1} \frac{da}{a} \iint \frac{r_1^{3-\kappa} + r_2^{3-\kappa}}{r_1 - r_2} D^2 \, d\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10).$$

This is not a symmetrical function of the roots of the quadratic. It appears, therefore, that the odd powers of the force lead to a more complicated analysis than the even ones.

4. *We may apply a similar method to solid ellipsoids.* Not to complicate matters at the beginning of the investigation, let us suppose that the ellipsoid is homogeneous and of unit density. The potential of the elementary cone at its vertex is found by integrating from $r = 0$ to r. The potential of the whole solid at an internal point P is therefore

$$V = \tfrac{1}{2} \frac{-1}{(\kappa - 1)(\kappa - 4)} \iint (r_1^{4-\kappa} + (-r_2)^{4-\kappa}) \, d\omega + C \quad . \quad . \quad . \quad . \quad (11),$$

where the result has been halved for the same reason as before. Writing $d\omega = \sin\theta \, d\theta \, d\phi$ the integration extends from $\phi = 0$ to 2π, $\theta = 0$ to π.

When κ is negative or positive and less than 4, the constant C is zero. When κ is positive and greater than 4 the constant is infinite. Since the potential of a small sphere of fixed radius ϵ and unit density at its centre is an absolute constant, we may take the lower limit in the integration for r to be ϵ, and omit the potential of the small sphere. The value of C will then be finite and depend on the magnitude of ϵ. As it will disappear when V is differentiated to find the forces, we shall omit it here also.

5. Before discussing the general values of the expressions (7) and (8), it will be useful to notice a few particular cases.

Suppose that the ellipsoidal shell is homogeneous and that the force varies as the inverse fourth power of the distance. We then have $\kappa = 4$, and the potential at any internal point P is

$$\mathrm{V} = \frac{\mu}{3}\iint \frac{-\mathrm{D}^2}{r_1 r_2}\, d\omega = \frac{4\pi\mu}{3}\frac{1}{\mathrm{E}}$$

by using the standard relations (4). Here, as before,

$$\mathrm{E} = 1 - \alpha x^2 - \beta y^2 - \gamma z^2.$$

Thus *the level surfaces inside the homœoid are quadrics similar and similarly situated to the bounding surface.* This agrees with the result given by TOWNSEND, in the 'Quarterly Journal,' vol. 12, 1873, "on the attraction of the ellipsoid for the law of inverse fourth power of the distance."

The potential V′ *at an external point* $(x'y'z')$ follows immediately from that at an internal point. We have

$$\mathrm{V}' = \frac{4\pi\mu}{3}\,\frac{abc}{a'b'c'}\cdot\frac{1}{\mathrm{E}'}$$

where E′ may be written on either of the forms

$$\mathrm{E}' = 1 - \frac{a^2x'^2}{a'^4} - \frac{b^2y'^2}{b'^4} - \frac{c'z'^2}{c'^4} = \epsilon^2 \cdot \left(\frac{x'^2}{a'^4} + \frac{y'^2}{b'^4} + \frac{z'^2}{c'^4}\right)$$

and

$$\frac{x'^2}{a^2+\epsilon^2} + \frac{y'^2}{b^2+\epsilon^2} + \frac{z'^2}{c^2+\epsilon^2} = 1.$$

Here a', b', c' are the semi-axes of the confocal through the attracted point.

The potential of a solid ellipsoid at an internal point may also be easily found for the same law of force. The integration for r, in equation (11), takes a logarithmic form when $\kappa = 4$, and we have

$$\mathrm{V} = \tfrac{1}{2}\,\tfrac{1}{3}\int d\omega\,\{\log r_1 + \log(-r_2)\} = \tfrac{1}{6}\int d\omega \log(\mathrm{ED}^2)$$

by using the standard forms (4). Integrating and omitting the constant, viz.: $\int d\omega \log D^2$, we have

$$V = \frac{2\pi}{3} \log E.$$

From this we easily find, by differentiating with regard to x, that the force at *an internal point* resolved parallel to x is

$$X = -\frac{4\pi}{3}\frac{1}{E}\frac{x}{a^2}.$$

By IVORY'S theorem it immediately follows that the force at *an external point* (x', y', z') is

$$X' = -\frac{4\pi}{3}\frac{abc}{a'b'c'}\cdot\frac{x'}{a'^2}\frac{1}{\epsilon^2}\left(\frac{x'^2}{a'^4}+\frac{y'^2}{b'^4}+\frac{z'^2}{c'^4}\right)^{-1}.$$

These values of X, X′ agree with those given by TOWNSEND in the memoir already referred to.

We may deduce from the values of X′, Y′, Z′ the potential at any external point. Since

$$\frac{x'^2}{a^2+\epsilon^2}+\frac{y'^2}{b^2+\epsilon^2}+\frac{z'^2}{c^2+\epsilon^2}=1,$$

we have

$$\frac{2x'}{a^2+\epsilon^2}=\left\{\frac{x'^2}{(a^2+\epsilon^2)^2}+\frac{y'^2}{(b^2+\epsilon^2)^2}+\frac{z'^2}{(c^2+\epsilon^2)^2}\right\}\frac{d\epsilon^2}{dx'}.$$

Hence

$$X' = -\frac{2\pi}{3}\frac{abc}{a'b'c'}\frac{1}{\epsilon^2}\frac{d\epsilon^2}{dx'}.$$

It follows by an easy integration that *the potential of a homogeneous solid ellipsoid at an external point for the law of the inverse fourth is*

$$V' = -\frac{2\pi}{3}\int_{\infty}^{\epsilon^2}\frac{abc}{a'b'c'}\frac{d\lambda}{\lambda}.$$

We notice that the potential is constant if ϵ^2 is constant; thus the *external level surfaces are confocals.*

Let p' be the perpendicular from the centre on a tangent plane to a confocal, then $2p'\,dp' = d\lambda$. The force normal to the confocal is therefore

$$F = \frac{dV'}{dp'} = \frac{dV'}{d\lambda}\frac{d\lambda}{dp'} = -\frac{4\pi}{3}\frac{abc}{a'b'c'}\frac{p'}{\epsilon^2},$$

where a', b', c' are the semi-axes of the confocal and $\epsilon^2 = a'^2 - a^2$. *For points on the* *same confocal the normal force is therefore proportional to the perpendicular on the tangent plane.*

The law of the inverse fourth power derives some importance from a theorem of JELLETT'S. JELLETT showed that, if V_κ be the potential when the law of force is the inverse κ^{th} power of the distance,

$$V_{\kappa+2} = \frac{1}{(\kappa+1)(\kappa-2)} \left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2} \right) V_\kappa .$$

By using this theorem, we may deduce the potential for any inverse integral power greater than the square, when the potential for the two laws of the inverse distance and the inverse fourth power are completely known. This theorem was communicated to the British Association at its meeting in Dublin in 1857, and briefly alluded to, rather than explained, in the report at page 3. But TOWNSEND, in the 'Quarterly Journal,' vol. 16, 1879, has called attention to the theorem and used it to find the potentials of circular rings and plates, &c., for various laws of force.

In this way the potential, when the force varies as the inverse sixth power, may be derived from the potential for the inverse fourth, but as some of these potentials have the function we have called E in the denominator, while others have the still more complicated function $a'b'c'E'$ in the same position, the labour will be found to become very great as we proceed to higher laws of force.

We notice that the potentials of a homogeneous thin homœoid, at both external and internal points, have been expressed as algebraic functions of the coordinates of the attracted point. It follows from JELLETT'S theorem that these potentials for all even higher laws of force can also be expressed free from all signs of integration. A similar argument will obviously apply to several other cases.

TOWNSEND also observes that JELLETT'S theorem remains true when the number of variables is extended beyond three. Thus, if

$$V_\kappa = \Sigma \frac{m}{r^\kappa}, \quad r^2 = (x_1 - a_1)^2 + (x_2 - a_2)^2 + \ldots + (x_n - a_n)^2$$

we find by simple differentiation that

$$V_{\kappa+2} = \frac{1}{(\kappa+1)(\kappa+1-n)} \left(\frac{d^2}{dx_1^2} + \ldots + \frac{d^2}{dx_n^2} \right) V_\kappa .$$

6. *Suppose the ellipsoidal shell to be homogeneous and the force to vary as the inverse sixth power of the distance.* We then have $\kappa = 6$, and the potential at any internal point P is by equation (8)

$$V = \frac{\mu}{5} \iint (r_1^2 + r_1 r_2 + r_2^2) \frac{D^2 \, d\omega}{- r_1^3 r_2^3} .$$

Using the formulæ (4) this becomes

$$V = \frac{\mu}{5}\iint(4F^2D^4 + ED^2)\frac{d\omega}{D^4E^3}$$
$$= \frac{\mu}{5}\left[\frac{4}{E^3}\iint(\alpha lx + \beta my + \gamma nz)^2\,d\omega + \frac{1}{E^2}\iint(\alpha l^2 + \beta m^2 + \gamma n^2)\,d\omega\right].$$

Writing $d\omega = \sin\theta\,d\theta\,d\phi$, as usual, and putting $\cos^2\theta$ for l^2, m^2, n^2 in turn, these integrals follow at once; the terms with the products lm, mn, nl being obviously zero. We therefore have

$$V = \frac{\mu}{5}\,\frac{4\pi}{3}\left[\frac{4(\alpha^2x^2 + \beta^2y^2 + \gamma^2z^2)}{(1 - \alpha x^2 - \beta y^2 - \gamma z^2)^3} + \frac{\alpha + \beta + \gamma}{(1 - \alpha x^2 - \beta y^2 - \gamma z^2)^2}\right].$$

The potential at an external point x', y', z' is therefore

$$V' = \frac{4\pi\mu}{15}\,\frac{abc}{a'b'c'}\left(\frac{p'}{\epsilon}\right)^4\left[\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} + \frac{4}{\epsilon^2} - 4p'^2\left(\frac{x'^2}{a'^6} + \frac{y'^2}{b'^6} + \frac{z'^2}{c'^6}\right)\right],$$

where a', b', c' are the semi-axes of the confocal through the attracted point, $\epsilon^2 = a'^2 - a^2$, and p' is the perpendicular on the tangent plane at the attracted point.

The potential of a solid homogeneous ellipsoid at an internal point for the inverse sixth power is easily seen to be

$$V = -\frac{2\pi}{15}\left\{2\cdot\frac{\alpha^2x^2 + \beta^2y^2 + \gamma^2z^2}{E^2} + \frac{\alpha + \beta + \gamma}{E}\right\}.$$

The potential at an external point is given later.

7. Another interesting case is when the index κ of the law of force is zero or negative.

Taking $\kappa = 0$ so that the force is independent of the distance, we see from equation (8) that the potential of a homogeneous thin homœoid at an internal point is

$$V = \frac{\mu}{\kappa - 1}\iint(r_1^2 + r_1r_2 + r_2^2)\,D^2\,d\omega.$$

Substituting (exactly as in the last article) from the standard forms (4) we see at once that the potential takes the form

$$V = Ax^2 + By^2 + Cz^2 + D$$

where A, B, C, D are constants which depend on the axes of the ellipsoid.

In the same way when $\kappa = -2$, or the force varies as the square of the distance, we find that V is a quartic function of the coordinates of the attracted point containing only even powers.

8. *To find the potential of a thin homogeneous homœoid at an internal point when the force varies as any even inverse power of the distance greater than the square.*

Putting $u = 1/r$ and using the formulæ (4), the potential (8) becomes

$$\mathrm{V} = \frac{\mu}{\kappa - 1} \frac{1}{\mathrm{E}} \iint \frac{u_1^{\kappa-3} - u_2^{\kappa-3}}{u_1 - u_2} d\omega,$$

where u_1, u_2 are the roots of the quadratic

$$u^2 - \frac{2\mathrm{F}}{\mathrm{E}} u - \frac{1}{\mathrm{ED}^2} = 0.$$

Writing the quadratic in the form $u^2 - pu - q = 0$, we have by a known theorem

$$u_1^n + u_2^n = p^n + np^{n-2} q + \ldots + n \frac{\mathrm{L}(n - f - 1)}{\mathrm{L}(f)\,\mathrm{L}(n - 2f)} p^{n-2f} q^f + \ldots,$$

where $\mathrm{L}(f) = 1 . 2 . 3 \ldots f$. This series stops at the first negative power of p, so that the number of terms is $\frac{1}{2}n + 1$ or $\frac{1}{2}(n + 1)$ according as n is even or odd.

We find by simply differentiating that

$$\frac{u_1^n - u_2^n}{u_1 - u_2} = \frac{1}{n} \frac{d}{dp} (u_1^n + u_2^n),^*$$

the differentiation being performed on the supposition that $u_1 u_2 = -q$ is constant.

We thus find that

$$\frac{u_1^n - u_2^n}{u_1 - u_2} = p^{n-1} + (n - 2) p^{n-3} q + \ldots + \frac{\mathrm{L}(n - f - 1}{\mathrm{L}(f) . \mathrm{L}(n - 2f - 1)} p^{n-2f-1} q^f + \ldots$$

In order to find V, we require the value of the integral

$$\iint p^{\kappa-4-2f} q^f \, d\omega = \iint \left(\frac{2\mathrm{F}}{\mathrm{E}}\right)^{\kappa-4-2f} \left(\frac{1}{\mathrm{ED}^2}\right)^f d\omega.$$

---

* One case of this equation is given by SERRET, who uses it to find the expansion of $x^n - 1/x^n$ in powers of $x + 1/x$, when that of $x^n + 1/x^n$ is known. Probably the equation for this case was suggested by differentiating $\cos n\theta$. See 'Cours d'Algèbre Supérieure,' fourteenth lesson.

We know that

$$\iint (\lambda l + \mu m + \nu n)^{2s}\, d\omega = (\lambda^2 + \mu^2 + \nu^2)^s \frac{4\pi}{2s+1}.$$

Hence in our case

$$\iint \left(\alpha l x + \beta m y + \gamma n z\right)^{2s} d\omega = (\alpha^2 x^2 + \beta^2 y^2 + \gamma^2 z^2)^s \frac{4\pi}{2s+1}.$$

Operating with

$$\nabla = \frac{1}{\alpha}\frac{d^2}{dx^2} + \frac{1}{\beta}\frac{d^2}{dy^2} + \frac{1}{\gamma}\frac{d^2}{dz^2}$$

on both sides, we have

$$\iint (\alpha l x + \beta m y + \gamma n z)^{2s-2} (\alpha l^2 + \beta m^2 + \gamma n^2)\, d\omega = \frac{4\pi}{(2s+1)\, 2s\, (2s-1)} \nabla \mathrm{P}.$$

Repeating this process, and writing $2s = \kappa - 4$,

$$\iint \mathrm{F}^{\kappa-4-2f} \left(\frac{1}{\mathrm{D}^2}\right)^f d\omega = \frac{4\pi\, \mathrm{L}(\kappa - 4 - 2f)}{\mathrm{L}(\kappa - 3)} \nabla^f \mathrm{P},$$

where

$$\mathrm{P} = (\alpha^2 x^2 + \beta^2 y^2 + \gamma^2 z^2)^{\frac{1}{2}(\kappa-4)}.$$

The potential of the thin homœoid when the force varies as the inverse κth power of the distance is therefore

$$\mathrm{V} = \frac{2\pi\mu}{(\kappa-1)(\kappa-3)} \left(\frac{2}{\mathrm{E}}\right)^{\kappa-3} \left\{1 + \frac{1}{2^2}\frac{\mathrm{E}}{\kappa-4} + \frac{1}{2^4}\frac{\mathrm{E}^2\nabla^2}{1.2.(\kappa-4)(\kappa-5)} + \ldots\right\}\mathrm{P}.$$

The general term within the bracket is

$$\frac{1}{2^{2f}} \frac{\mathrm{L}(\kappa-4-f)}{\mathrm{L}(f).\mathrm{L}(\kappa-4)} \mathrm{E}^f \nabla^f,$$

and ∇, P have the meanings given above, viz.,

$$\mathrm{E} = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2},$$

$$\mathrm{P} = \left(\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}\right)^{\frac{1}{2}(\kappa-4)},$$

$$\nabla = a^2\frac{d^2}{dx^2} + b^2\frac{d^2}{dy^2} + c^2\frac{d^2}{dz^2}.$$

It follows, therefore, that the potential of the homœoid has been found free from all sign of integration. It has the form

$$\mathrm{V} = \mathrm{A}\frac{\mathrm{P}}{\mathrm{E}^{\kappa-3}} + \mathrm{B}\frac{\nabla\mathrm{P}}{\mathrm{E}^{\kappa-4}} + \mathrm{C}\frac{\nabla^2\mathrm{P}}{\mathrm{E}^{\kappa-5}} + \ldots$$

where A, B, C are numerical factors whose values have just been found. The numerators of the powers of E are homogeneous functions of x^2, y^2, z^2, whose dimensions are continually reduced by the differentiations implied in the operator ∇.

The series has $\frac{1}{2}(\kappa - 2)$ terms. Thus for the law of the inverse fourth it reduces to the one term given in Art. 5 ; for the law of the inverse sixth power it becomes the two terms found in Art. 6, and so on.

The potential at an external point P′, whose coordinates are (x', y', z'), follows from that at an internal point by what is practically a method of inversion. We describe through P′ a confocal homœoid whose axes are a', b', c', and of the same volume as the original homœoid. The potentials of the two homœoids at corresponding points are then equal. Remembering the transformation already made in Art. 5, we find that the potential at an external point is

$$\mathrm{V}' = \frac{2\pi\mu}{(\kappa-1)(\kappa-3)} \frac{abc}{a'b'c'} \left(\frac{2}{\mathrm{E}'}\right)^{\kappa-3} \left\{1 + \frac{1}{2^2} \frac{\mathrm{E}'\nabla'}{\kappa-4} + \frac{1}{2^4} \frac{\mathrm{E}'^2\nabla'^2}{1.2(\kappa-4)(\kappa-5)} + \dots\right\} \mathrm{P}',$$

where E′ and P′ have either of the forms

$$\mathrm{E}' = 1 - \frac{a^2x'^2}{a'^4} - \frac{b^2y'^2}{b'^4} - \frac{c^2z'^2}{c'^4} = \epsilon^2 \left(\frac{x'^2}{a'^4} + \frac{y'^2}{b'^4} + \frac{z'^2}{c'^4}\right),$$

$$\mathrm{P}' = \left(\frac{a^2x'^2}{a'^6} + \frac{b^2y'^2}{b'^6} + \frac{c^2z'^2}{c'^6}\right)^{\frac{1}{2}(\kappa-4)},$$

$$\nabla' = \frac{a'^4}{a^2} \frac{d^2}{dx'^2} + \frac{b'^4}{b^2} \frac{d^2}{dy'^2} + \frac{c'^4}{c^2} \frac{d^2}{dz'^2}.$$

It should be noticed that the differentiations implied in the operator ∇' are to be performed on (x', y', z') on the supposition that a', b', c' are constant.

9. *To find the potential of a thin homogeneous homœoid at an internal point when the force varies as the inverse κth power of the distance and* $\kappa = 2$, *zero, or any even negative number.*

We now use the potential (8) in the form

$$\mathrm{V} = \frac{\mu}{\kappa - 1} \iint \frac{r_1^{\,3-\kappa} - r_2^{\,3-\kappa}}{r_1 - r_2} \mathrm{D}^2 \, d\omega$$

where r_1, r_2 are the roots of the quadratic

$$r^2 + 2\mathrm{FD}^2 r - \mathrm{ED}^2 = 0.$$

Putting $p = -2\mathrm{FD}^2$, $q = \mathrm{ED}^2$, we substitute in the series

$$\frac{r_1^{\,3-\kappa} - r_2^{\,3-\kappa}}{r_1 - r_2} = p^{2-\kappa} + (1-\kappa)\, p^{-\kappa} q + \frac{(-\kappa)(-\kappa-1)}{1.2} p^{-\kappa-2} q^2 + \dots$$

This series has $\frac{1}{2}(4-\kappa)$ terms, and the general term has been written at length in Art. 8.

To determine the potential we must therefore find the value of

$$\iint p^{2-\kappa-2f} q^f \mathrm{D}^2\, d\omega = \mathrm{E}^f \iint (2\mathrm{F})^{2-\kappa-2f} (\mathrm{D}^2)^{3-\kappa-f}\, d\omega$$

where f has any positive value from zero to $\frac{1}{2}(2-\kappa)$. To effect this integration we shall use the following lemma.

10. *Lemma.* By a known theorem we have

$$\iint \frac{d\omega}{\alpha l^2+\beta m^2+\gamma n^2} = 2\pi \int_0^\infty \frac{v^{-\frac{1}{2}}\, dv}{\mathrm{Q}},$$

where

$$\mathrm{Q}^2 = (\alpha+v)(\beta+v)(\gamma+v).$$

Now

$$\left(\frac{d}{d\alpha}+\frac{d}{d\beta}+\frac{d}{d\gamma}\right)\frac{1}{\mathrm{Q}} = \frac{d}{dv}\frac{1}{\mathrm{Q}}.$$

Hence, operating t times on the equation with the left-hand operator, we find

$$\iint \frac{d\omega}{(\alpha l^2+\beta m^2+\gamma n^2)^{t+1}} = \frac{2\pi(-1)^t}{\mathrm{L}(t)} \int_0^\infty v^{-\frac{1}{2}}\, dv \left(\frac{d}{dv}\right)^t \frac{1}{\mathrm{Q}}.$$

Differentiating again f times with regard to α, g times with regard to β, h times with regard to γ, we have

$$\iint \frac{l^{2f} m^{2g} n^{2h}\, d\omega}{(\alpha l^2+\beta m^2+\gamma n^2)^{s+t+1}} = \mathrm{K}\,.\int_0^\infty v^{-\frac{1}{2}}\, dv \left(\frac{d}{dv}\right)^t \frac{1}{(\alpha+v)^f(\beta+v)^g(\gamma+v)^h\mathrm{Q}},$$

where

$$\mathrm{K} = \frac{2\pi(-1)^t}{\mathrm{L}(s+t)}\cdot\frac{\mathrm{L}(2f)}{\mathrm{L}(f)}\cdot\frac{\mathrm{L}(2g)}{\mathrm{L}(g)}\frac{\mathrm{L}(2h)}{\mathrm{L}(h)}\frac{1}{2^{2s}},$$

where $s = f+g+h$ and $\mathrm{L}(f) = 1\,.\,2\,.\,3\,.\,.\,.\,f$.

By an expansion we deduce from this

$$\iint \frac{(\lambda l+\mu m+\nu n)^{2s}\, d\omega}{(\alpha l^2+\beta m^2+\gamma n^2)^{s+t+1}} = \mathrm{K}' \int_0^\infty v^{-\frac{1}{2}}\, dv \left(\frac{d}{dv}\right)^t \left\{\frac{\lambda^2}{\alpha+v}+\frac{\mu^2}{\beta+v}+\frac{\nu^2}{\gamma+v}\right\}^s \frac{1}{\mathrm{Q}},$$

where

$$\mathrm{K}' = \frac{\mathrm{L}(2s)}{\mathrm{L}(s)\,\mathrm{L}(s+t)}\cdot\frac{2\pi(-1)^t}{2^{2s}}.$$

The expansion is not difficult, for the odd powers of l, m, n, on the left hand side give zero on integration, and the general integral for any set of even powers is written down just above.

The subject of integration on the right hand side is infinite when $v = 0$, but it is not difficult to see that the integral from zero to any very small value of v is ultimately zero.

11. To apply the lemma to the integral required in our proposition we refer to Art. 1 for the definitions of F and D^2. We have

$$F = \alpha lx + \beta my + \gamma nz.$$

We therefore put $\lambda = \alpha x$, $\mu = \beta y$, $\nu = \gamma z$, $2s = 2 - \kappa - 2f$, $s + t + 1 = 3 - \kappa - f$. This gives $t = \frac{1}{2}(2 - \kappa)$ so that t is a positive integer when κ is even and not greater than 2. We thus have

$$\begin{aligned} V &= \frac{\mu}{\kappa - 1} \iint \Sigma \frac{L(2 - \kappa - f)}{L(f) \,.\, L(2 - \kappa - 2f)} \,.\, E^f \,.\, (2F)^{2 - \kappa - 2f} (D^2)^{3 - \kappa - f}\, d\omega, \\ &= \frac{2\pi\mu}{\kappa - 1} (-1)^t \int_0^\infty v^{-\frac{1}{2}}\, dv \left(\frac{d}{dv}\right)^t \Sigma \frac{E^f}{L(f)} \frac{1}{L(s)} \left\{\frac{\alpha^2 x^2}{\alpha + v} + \ldots\right\}^s \frac{1}{Q}. \end{aligned}$$

The summation extends from $f = 0$ to $\frac{1}{2}(2 - \kappa)$ and s varies from $\frac{1}{2}(2 - \kappa)$ to 0. The Σ is therefore equivalent to an expansion by the binomial theorem. We thus find for the required potential

$$V = \frac{2\pi\mu}{\kappa - 1} \frac{(-1)^t}{L(t)} \int_0^\infty v^{-\frac{1}{2}}\, dv \left(\frac{d}{dv}\right)^t . \left\{E + \frac{\alpha^2 x^2}{\alpha + v} + \frac{\beta^2 y^2}{\beta + v} + \frac{\gamma^2 z^2}{\gamma + v}\right\}^t \frac{1}{Q}.$$

Substituting for E its value (given in Art. 1), the potential becomes

$$V = \frac{2\pi\mu (-1)^t}{(\kappa - 1) L(t)} \int_0^\infty v^{-\frac{1}{2}}\, dv \left(\frac{d}{dv}\right)^t \left\{1 - \frac{\alpha v x^2}{\alpha + v} - \frac{\beta v y^2}{\beta + v} - \frac{\gamma v z^2}{\gamma + v}\right\}^t \frac{1}{Q},$$

where $t = \frac{1}{2}(2 - \kappa)$.

Writing $v = 1/u$ we find that the potential of a thin homœoid when the index κ of the force is even and not greater than 2 at an internal point, is

$$V = \frac{2\pi\mu}{\kappa - 1} \int_0^\infty u^{-\frac{3}{2}}\, du \left(u^2 \frac{d}{du}\right)^t \frac{abc\, u^{\frac{3}{2}}}{Q_1} \cdot \frac{R^t}{L(t)},$$

where

$$R = 1 - \frac{x^2}{a^2 + u} - \frac{y^2}{b^2 + u} \quad \frac{z^2}{c^2 + u}$$

$$Q_1{}^2 = (a^2 + u)(b^2 + u)(c^2 + u).$$

At an external point $(x'y'z')$ *the potential is*

$$V' = \frac{2\pi\mu}{\kappa - 1}\int_{\epsilon^2}^{\infty}(u - \epsilon^2)^{-\frac{3}{2}}\, du \left\{(u - \epsilon^2)^2 \frac{d}{du}\right\}^t \frac{abc\,(u - \epsilon^2)^{\frac{3}{2}}}{Q_1}\left(\frac{u}{u - \epsilon^2}\right)^t \frac{R'^t}{L\,(t)}$$

where R' differs from R only in having $(x'y'z')$ written for xyz.

Here $t = \frac{1}{2}(2 - \kappa)$, so that when $t = 0$ the formulæ reduce to the well-known results for the law of the inverse square.

It is easy to verify these results in the case of a spherical shell.

12. *To find the potential at an internal point* P *of a thin heterogeneous homœoid when the law of force is an inverse even positive power of the distance, and the density at any point* (ξ, η, ζ), *is* $\phi\,(\xi, \eta, \zeta)$.

The coordinates of the attracted point P being (x, y, z), the coordinates of any point Q on the surface are $\xi = x + lr$, &c. If ρ_1, ρ_2 are the densities at the extremities of a chord through P, we have

$$\rho_1 = \phi\,(x + lr_1, y + mr_1, z + nr_1) = \phi + r_1\,\delta\phi + \frac{r_1^{\,2}}{1\,.\,2}\,\delta^2\phi + \dots$$

where $\delta = l\,d/dx + m\,d/dy + n\,d/dz$. In the same way

$$\rho_2 = \phi + r_2\,\delta\phi + \frac{r_2^{\,2}}{1\,.\,2}\,\delta^2\phi + \dots$$

The potential at P (as in Art. 3) is

$$V = \frac{\mu}{\kappa - 1}\,\frac{1}{E}\iint \frac{\rho_1 u_1^{\,\kappa - 3} - \rho_2 u_2^{\,\kappa - 3}}{u_1 - u_2}\, d\omega.$$

Substituting for ρ_1, ρ_2, this becomes

$$V = \frac{\mu}{\kappa - 1}\,\frac{1}{E}\iint\left\{\phi\,\frac{u_1^{\,\kappa - 3} - u_2^{\,\kappa - 3}}{u_1 - u_2} + \delta\phi\,\frac{u_1^{\,\kappa - 4} - u_2^{\,\kappa - 4}}{u_1 - u_2} + \dots \frac{\delta^{\kappa - 4}\phi}{L\,(\kappa - 4)}\,\frac{u_1 - u_2}{u_1 - u_2}\right.$$
$$\left. + \text{zero} + \frac{\delta^{\kappa - 2}\phi}{L\,(\kappa - 2)}\,\frac{r_1 - r_2}{r_1 - r_2}(-\, r_1 r_2) + \frac{\delta^{\kappa - 1}\phi}{L\,(\kappa - 1)}\,\frac{r_1^{\,2} - r_2^{\,2}}{r_1 - r_2}(-\, r_1 r_2) + \dots\right\} d\omega.$$

The series is divided into two parts, one with u's and the other with r's, so that all the powers used are positive. The two parts will be represented by V_1 and V_2, so that $V = V_1 + V_2$.

13. *Let us examine first the terms which contain* u *only.* These may be written in the form

$$V_1 = \frac{\mu}{\kappa - 1}\,\frac{1}{E}\iint d\omega\left\{\phi S_{\kappa - 3} + \delta\phi S_{\kappa - 4} + \dots + \frac{\delta^{\kappa - 3 - n}\,\phi}{L\,(\kappa - 3 - n)}\,S_n + \dots + \frac{\delta^{\kappa - 4}\,\phi}{L\,(\kappa - 4)}\,S_1\right\},$$

where, as in Art. 8,

$$S_n = p^{n-1} + (n-2)\,p^{n-3}q + \ldots + \frac{L(n-f-1)}{L(f).L(n-2f-1)}\,p^{n-2f-1}q^f + \ldots$$

$$p = \frac{2F}{E} = \frac{2}{E}(\alpha xl + \beta ym + \gamma zn), \qquad q = \frac{1}{ED^2}.$$

Before attempting the general integration it is useful to examine what kind of integral we shall arrive at. We notice that every term of $\delta^{\kappa-3-n}\phi.S_n$ is an integral rational function of the direction cosines (l, m, n) of the chord. All the integrations can, therefore, be completely effected. The terms with odd powers of (l, m, n) give zero on integration, while those with even powers may be found by the theorem

$$\iint l^{2f}m^{2g}n^{2h}\,d\omega = 2\,\frac{\Gamma(f+\frac{1}{2})\,\Gamma(g+\frac{1}{2})\,\Gamma(h+\frac{1}{2})}{\Gamma(f+g+h+\frac{3}{2})}.$$

Since every term of V_1 is of the same dimensions in (l, m, n), viz., $\kappa - 4$ dimensions, we notice that the denominator of this expression, viz., $\Gamma(\kappa - 4 + \frac{3}{2})$, is the same for every term.

Supposing the density $\phi(\xi, \eta, \zeta)$ to be a homogeneous function of (ξ, η, ζ) of i dimensions, we see that V_1 will take the general form

$$V_1 = \frac{H_{\kappa-4+i}}{E^{\kappa-3}} + \frac{H_{\kappa-6+i}}{E^{\kappa-4}} + \frac{H_{\kappa-8+i}}{E^{\kappa-5}} + \ldots$$

where H_s is a homogeneous function of (x, y, z) of s dimensions. If $i > \kappa - 4$, the series ends when the exponent of E is unity, if $i < \kappa - 4$ the series ends when the numerator reduces to a constant.

Referring again to the general expression for the potential ν, we see that where the highest power in $\phi(x, y, z)$ is less than $\kappa - 2$, the only terms in the potential are those with u. It follows that *when the law of force is an even inverse κ^{th} power of the distance ($\kappa > 2$), the potential of a heterogeneous thin homœoid can be found free from signs of integration, provided the law of density at (ξ, η, ζ) is a function of (ξ, η, ζ) of less than $\kappa - 2$ dimensions.*

14. Let us now consider the general integration of the expression for V_1. The following lemma will be required.

Beginning (as in Art. 8) with the equation

$$\iint(\alpha xl + \beta ym + \gamma zn)^{2s}\,d\omega = (\alpha^2x^2 + \beta^2y^2 + \gamma^2z^2)^s\,\frac{4\pi}{2s+1}$$

where the integration extends all round the origin, let

$$\nabla = \frac{1}{\alpha}\frac{d^2}{dx^2} + \frac{1}{\beta}\frac{d^2}{dy^2} + \frac{1}{\gamma}\frac{d^2}{dz^2},$$

$$\nabla' = \frac{A}{\alpha}\frac{d}{dx} + \frac{B}{\beta}\frac{d}{dy} + \frac{C}{\gamma}\frac{d}{dz}$$

where A, B, C are three arbitrary constants.

We then find

$$\iint \frac{(\alpha x l + \beta y m + \gamma z n)^{2s-2f-g}}{\mathrm{L}\,(2s - 2f - g)} (\alpha l^2 + \beta m^2 + \gamma n^2)^f (\mathrm{A} l + \mathrm{B} m + \mathrm{C} n)^g \, d\omega$$
$$= 4\pi \, \nabla^f \, \nabla'^g \, \frac{(\alpha^2 x^2 + \beta^2 y^2 + \gamma^2 z^2)^s}{\mathrm{L}\,(2s + 1)}$$

where, as before $\mathrm{L}\,(s) = 1.2.3 \ldots s$. In this equation all the exponents are to be positive.

To apply this lemma we let A, B, C signify $\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}$, which operate solely on the ϕ and not on the other x, y, z which occur in the equation. The equation may then be written

$$\iint \frac{\mathrm{F}^{2s-2f-g}}{\mathrm{L}\,(2s-2f-g)} \left(\frac{1}{\mathrm{D}^2}\right)^f \delta^g \, d\omega = 4\pi \, \nabla^f \, \nabla'^g \, \frac{(\alpha^2 x^2 + \beta^2 y^2 + \gamma^2 z^2)^s}{\mathrm{L}\,(2s + 1)}$$

We see by what precedes that it will be convenient to arrange the expression for V_1 in a series of inverse powers of E. Let us, therefore, seek the term containing $\left(\frac{1}{\mathrm{E}}\right)^{h+1}$. Putting $n - f - 1 = h$, we notice that $\frac{\mathrm{S}_n \, \delta^{\kappa - 3 - n}}{\mathrm{L}\,(\kappa - 3 - n)}$ contributes the one term

$$\frac{\mathrm{L}\,(h)}{\mathrm{L}\,(n - h - 1)} \, \frac{(2\mathrm{F})^{2h - n + 1}}{\mathrm{L}\,(2h - n + 1)} \left(\frac{1}{\mathrm{D}^2}\right)^{n - h - 1} \frac{\delta^{\kappa - 3 - n}}{\mathrm{L}\,(\kappa - 3 - n)} \, \frac{1}{\mathrm{E}^h},$$

where every exponent is positive. To find all the terms with the given power of E we sum this expression, beginning at $n = h + 1$ and ending at the lesser of the two $n = 2h + 1$ or $n = \kappa - 3$.

Using the lemma, the corresponding term of V_1 becomes

$$\frac{2\pi\mu}{\kappa - 1} \, \mathrm{L}\,(h) \Sigma \, \frac{(\frac{1}{2}\Delta)^{n - h - 1}}{\mathrm{L}\,(n - h - 1)} \, \frac{\Delta'^{\kappa - 3 - n}}{\mathrm{L}\,(\kappa - 3 - n)} \, \frac{(\alpha^2 x^2 + \beta^2 y^2 + \gamma^2 z^2)^{\frac{1}{2}(\kappa - 4)}}{\mathrm{L}\,(\kappa - 3)} \left(\frac{2}{\mathrm{E}}\right)^{h + 1},$$

where Σ implies a summation with regard to n.

If $2h + 1 > \kappa - 3$ the summation extends from $n = h + 1$ to $\kappa - 3$ and the several terms form a complete binomial series. If $2h + 1 < \kappa - 3$ the summation extends from $n = h + 1$ to $2h + 1$. In this case the missing operators required to complete the binomial series are $\Delta^{h + 1} \, \Delta'^{\kappa - 5 - 2h}$, $\Delta^{h + 2} \, \Delta'^{\kappa - 6 - 2h}$, &c.

Since Δ, Δ' are differential operators of the orders d^2/dx^2 and d/dx, while the subject is of the order $x^{\kappa - 4}$, the missing operators reduce the subject to zero. We may, therefore, suppose the binomial series to be complete.

It immediately follows that

$$\mathrm{V}_1 = \frac{2\pi\mu}{(\kappa - 1)\,\mathrm{L}\,(\kappa - 3)} \Sigma \frac{\mathrm{L}\,(h)}{\mathrm{L}\,(\kappa - 4 - h)} \cdot \left(\frac{2}{\mathrm{E}}\right)^{h+1} (\Delta' + \tfrac{1}{2}\Delta)^{\kappa - 4 - h} \, \mathrm{P}\phi,$$

where (as in Art. 8),

$$\mathrm{P} = (\alpha^2 x^2 + \beta^2 y^2 + \gamma^2 z^2)^{\frac{1}{2}(\kappa-4)},$$

and the Σ implies summation from $h = 0$ to $h = \kappa - 4$.

When the ellipsoidal shell is homogeneous we omit the operator Δ', and put the density ϕ equal to unity. The result then agrees with that already obtained in Art. 8.

The differentiations with regard to x, y, z, which are implied by ∇, ∇', operate only on P, while the differentiations, A, B, C, in ∇' operate only on ϕ. When written at length the operator takes the form,

$$(\tfrac{1}{2}\nabla + \nabla')^s \mathrm{P}\phi = \phi(\tfrac{1}{2}\nabla)^s + s\left(\frac{1}{\alpha}\frac{d\phi}{dx}\frac{d}{dx} + \frac{1}{\beta}\frac{d\phi}{dy}\frac{d}{dy} + \dots\right)(\tfrac{1}{2}\nabla)^{s-1}\mathrm{P}$$

$$+ s\frac{s-1}{2}\left(\frac{1}{\alpha^2}\frac{d^2\phi}{dx^2}\frac{d^2}{dx^2} + \dots + \frac{2}{\alpha\beta}\frac{d^2\phi}{dx\,dy}\frac{d^2}{dx\,dy} + \dots\right)(\tfrac{1}{2}\nabla)^{s-2}\mathrm{P} + \dots$$

15. We may write the result in a different form so as to avoid having two kinds of operators. We have

$$\nabla + 2\nabla' = \frac{1}{\alpha}\left(\frac{d}{dx} + \mathrm{A}\right)^2 + \frac{1}{\beta}\left(\frac{d}{dy} + \mathrm{B}\right)^2 + \frac{1}{\gamma}\left(\frac{d}{dz} + \mathrm{C}\right)^2 - \frac{\mathrm{A}^2}{\alpha} - \frac{\mathrm{B}^2}{\beta} - \frac{\mathrm{C}^2}{\gamma}.$$

Let ∇_t represent the total operator ∇ when the differentiations implied act on both P and ϕ. Let ∇_ϕ be the partial operator ∇ when the differentiations act on ϕ only, then

$$\nabla + 2\nabla' = \nabla_t - \nabla_\phi.$$

The potential of the heterogeneous homœoid therefore becomes

$$\mathrm{V}_1 = \frac{2\pi\mu}{(\kappa-1)\,\mathrm{L}(\kappa-3)}\Sigma\frac{\mathrm{L}(h)}{\mathrm{L}(\kappa-4-h)}\left(\frac{2}{\mathrm{E}}\right)^{h+1}(\nabla_t - \nabla_\phi)^{\kappa-4-h}\,\mathrm{P}\phi\,(\tfrac{1}{2})^{\kappa-4-h}.$$

16. As an example, let us find the potential of a thin homœoid at an internal point (x, y, z) when the force varies as the inverse fourth power and the density at (ξ, η, ζ) is $m\xi$.

The series has only one term given by $h = 0$, since $\kappa = 4$. The result is seen at a glance to be

$$\mathrm{V} = \frac{2\pi\mu m}{3}\frac{2}{\mathrm{E}}x.$$

Thus the potential is the same as if the homœoid were homogeneous and its density were mx.

This curious result may be verified by independent reasoning. Let P be the given internal point, PQ any radius vector of the shell. Let the direction cosines of PQ be (l, m, n) and its length be r. The density at Q is therefore proportional to $(x + lr)$. The potential is

$$\mathrm{V} = \frac{x}{3}\iint\frac{\mu p d\sigma}{r^3} + \frac{1}{3}\iint\frac{\mu p l d\sigma}{r^2}.$$

The first term is the potential of the homogeneous shell. The second is the x attraction of a homogeneous homœoid where the law of force is the inverse square and is therefore zero.

As another example, let us find the potential of a heterogeneous thin homœoid when the force varies as the inverse sixth power and the density at (ξ, η, ζ) is $m\xi^2$.

The series has three terms given by $h = 0, 1, 2$. Hence, since $\kappa = 6$,

$$\mathrm{V} = \frac{\pi\mu}{15}\left\{\frac{2}{\mathrm{E}}\,\frac{1}{2}\left(\nabla' + \frac{\nabla}{2}\right)^2 + \frac{4}{\mathrm{E}^2}\left(\nabla' + \frac{\nabla}{2}\right) + \frac{8}{\mathrm{E}^3}\,2\right\}\mathrm{P}\phi.$$

Now $\nabla\mathrm{P} = 2(\alpha + \beta + \gamma)$, $\nabla^2\mathrm{P} = 0$, $\nabla\nabla'\mathrm{P} = 0$, $\nabla'\mathrm{P} = 2\alpha\mathrm{A}x + \text{\&c.}$, $\nabla'^2\mathrm{P} = 2\mathrm{A}^2 + \text{\&c.}$

Remembering that $\phi = mx^2$ and $\mathrm{A}\phi = d\phi/dx$, $\mathrm{A}^2\phi = d^2\phi/dx^2$, we find, after an easy reduction,

$$\mathrm{V} = \frac{4\pi\mu m}{15}\left\{\frac{1}{\mathrm{E}} + \frac{(5\alpha + \beta + \gamma)\,x^2}{\mathrm{E}^2} + \frac{4\,(\alpha^2x^2 + \beta^2y^2 + \gamma^2z^2)\,x^2}{\mathrm{E}^3}\right\}.$$

It immediately follows that the potential at an external point x', y', z' for the same law of density is

$$\mathrm{V}' = \frac{4\pi\mu m}{15}\,\frac{abc}{a'b'c'}\,\frac{a^2}{a'^2}\left\{\frac{1}{\mathrm{E}'} + \left(\frac{5}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}\right)\frac{a^2x'^2}{a'^2}\,\frac{1}{\mathrm{E}'^2} + \left(\frac{a^2x'^2}{a'^6} + \frac{b^2y'^2}{b'^6} + \frac{c^2z'^2}{c'^6}\right)\frac{a^2x'^2}{a'^2}\,\frac{4}{\mathrm{E}'^3}\right\},$$

where

$$\mathrm{E}' = \epsilon^2\left(\frac{x'^2}{a'^4} + \frac{y'^2}{b'^4} + \frac{z'^2}{c'^4}\right) = \frac{\epsilon^2}{p'^2},$$

and a', b', c' are the semi-axes of the confocal through the attracted point.

17. As a third example, we may apply the formula to find the potential of a thin homogeneous focaloid when κ is greater than 4.

The thickness of a thin homœoid is μp, while that of a thin focaloid varies inversely as p. A homœoid, whose density is m/p^2, is equivalent to a homogeneous focaloid of unit density whose $ada = m\mu$. Putting then

$$\phi = m/p^2 = m\,(\alpha^2x^2 + \beta^2y^2 + \gamma^2z^2),$$

we have, if $n = \kappa - 4 - h$,

$$\begin{aligned}(\nabla_l - \nabla_\phi)^n \mathrm{P}\phi &= \{\nabla_l^n - n\nabla_l^{n-1}\nabla_\phi\}\,\mathrm{P}\phi \\ &= \nabla^n(\mathrm{P}\phi) - 2n(\alpha+\beta+\gamma)\,m\nabla^{n-1}\mathrm{P} \\ &= m\nabla^n\left(\frac{1}{p^2}\right)^{\frac{1}{2}(\kappa-2)} - 2n(\alpha+\beta+\gamma)\,m\nabla^{n-1}\left(\frac{1}{p^2}\right)^{\frac{1}{2}(\kappa-4)}.\end{aligned}$$

Substituting this in the formula for V, already written down, we have the required potential.

Suppose the law of force to be the inverse sixth power, the potential of a homogeneous thin focaloid, at an internal point, is

$$\mathrm{V} = \frac{4\pi\mu m}{15}\left\{\frac{\alpha^2+\beta^2+\gamma^2}{\mathrm{E}} + \frac{1}{\mathrm{E}^2}\left(\frac{\alpha+\beta+\gamma}{p^2} + 4(\alpha^3x^2+\beta^3y^2+\gamma^3z^2)\right) + \frac{4}{\mathrm{E}^3}\frac{1}{p^4}\right\},$$

and at an external point

$$\mathrm{V}' = \frac{4\pi\mu m}{15}\,\frac{abc}{a'b'c'}\,\frac{1}{\epsilon^4}\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{4}{\epsilon^2}\right)p'^2,$$

where a', b', c' are the axes of the confocal through the external point, and $\epsilon^2 = a'^2 - a^2$. It appears that the potential varies as the square of the perpendicular p' on the tangent plane for all points on the same confocal.

18. When the density $\phi(\xi, \eta, \zeta)$ of the thin homœoid is a function of the coordinates of a degree higher than $\kappa - 2$, the potential includes some of the terms of Art. 12, which contain r_1 and r_2. We shall now examine these terms. In this way we shall arrive at an expression for *the potential of a thin heterogeneous homœoid at an internal point where the index κ of the force is a positive even integer greater than the square.*

These additional terms are given by

$$\mathrm{V}_2 = \frac{\mu}{\kappa-1}\iint\left[\frac{\delta^{\kappa-2}\phi}{\mathrm{L}(\kappa-2)}\mathrm{S}_1 + \frac{\delta^{\kappa-1}\phi}{\mathrm{L}(\kappa-1)}\mathrm{S}_2 + \ldots + \frac{\delta^{\kappa+n-3}\phi}{\mathrm{L}(\kappa+n-3)}\mathrm{S}_n + \ldots\right]\left(\frac{-r_1r_2}{\mathrm{E}}\right)d\omega,$$

$$\mathrm{S}_n = \frac{r_1^n - r_2^n}{r_1 - r_2} = p^{n-1} + (n-2)p^{n-3}q + \ldots + \frac{\mathrm{L}(n-f-1)}{\mathrm{L}(f)\,.\,\mathrm{L}(n-2f-1)}p^{n-2f-1}q^f + \ldots,$$

$$p = -2\mathrm{D}^2\mathrm{F}, \qquad q = -r_1r_2 = \mathrm{E}\mathrm{D}^2,$$

and D, E, F are expressed as functions of the direction cosines (l, m, n) in Art. 1.

Before attempting the general integration, it is convenient to examine the kind of integral we shall arrive at.

Consider first the dimensions of the several terms of V_2 as regards the coordinates (x, y, z) of the attracted point. Let $\phi(x, y, z)$ be homogeneous and of i dimensions, then remembering that F is a linear function of (x, y, z) we see that, if the terms are arranged in powers of q or E, V_2 takes the form

$$\mathrm{V}_2 = \mathrm{H}_{i-\kappa+2} + \mathrm{E}\mathrm{H}_{i-\kappa} + \mathrm{E}^2\mathrm{H}_{i-\kappa-2} + \ldots$$

where H_s is a homogeneous function of (x, y, z) of s dimensions. The series terminates when H_s reduces to a constant.

To calculate the values of H_s we must consider the dimensions of V_2 with regard to (l, m, n). Since positive powers of D now occur in the series, the integrations cannot be completely effected. The coefficients of the powers of (x, y, z) in H_s will be definite integrals of the same kind as those which occur when the force varies as the inverse square.

The terms of V_2 which contain any odd exponents of (l, m, n) give zero after integration. Omitting these it will be seen that every term of V_2 is of the form $l^{2f}m^{2g}n^{2h}\,D^{2(f+g+h)-(\kappa-4)}$, where f, g, h here stand for any positive integers. Now putting $t = \frac{1}{2}(\kappa - 2)$, the integral of any one of these terms may be found by the equation

$$\iint \frac{l^{2f}m^{2g}n^{2h}\,d\omega}{(\alpha l^2 + \beta m^2 + \gamma n^2)^{s+1-t}} = \mathrm{N}\left(\frac{d}{d\alpha}\right)^f\left(\frac{d}{d\beta}\right)^g\left(\frac{d}{d\gamma}\right)^h \int_0^\infty \frac{v^{t-\frac{1}{2}}\,dv}{\mathrm{Q}},$$

where

$$\mathrm{N} = \frac{2\pi \mathrm{L}(t)}{\mathrm{L}(2t)}\,2^{2t}\,\frac{(-1)^s}{\mathrm{L}(s-t)}, \qquad s = f + g + h,$$

$$\mathrm{Q}^2 = (\alpha + v)(\beta + v)(\gamma + v).$$

In any particular example, therefore, the integrations may be effected (as far as they can be done) by using this equation.

19. To obtain the general integral of the series for V_2, we require a lemma.

We deduce from the integral given in the last article that

$$\iint \frac{(\lambda l + \mu m + \nu n)^{2s}\,d\omega}{(\alpha l^2 + \beta m^2 + \gamma n^2)^{s+1-t}} = \mathrm{K}\int_0^\infty \left(\frac{\lambda^2}{\alpha + v} + \frac{\mu^2}{\beta + v} + \frac{\nu^2}{\gamma + v}\right)^s \frac{v^{t-\frac{1}{2}}\,dv}{\mathrm{Q}},$$

where

$$\mathrm{K} = \frac{\mathrm{L}(2s)}{\mathrm{L}(s)}\,\frac{1}{\mathrm{L}(s-t)}\,\frac{2\pi}{2^{2s-2t}}\,\frac{\mathrm{L}(t)}{\mathrm{L}(2t)},$$

$$\mathrm{Q}^2 = (\alpha + v)(\beta + v)(\gamma + v),$$

and t is a positive integer less than s (see the author's 'Statics,' 1892, vol. 2, Arts. 219 and 220 for this integral and the one used in the last article).

Let

$$\Delta = \mathrm{A}\frac{d}{d\lambda} + \mathrm{B}\frac{d}{d\mu} + \mathrm{C}\frac{d}{d\nu},$$

where A, B, C are any constants, we find

$$\iint \frac{(\lambda l + \mu m + \nu n)^{2s-g}\,(\mathrm{A}l + \mathrm{B}m + \mathrm{C}n)^g\,d\omega}{(\alpha l^2 + \beta m^2 + \gamma n^2)^{s+1-t}}$$

$$= \frac{\mathrm{L}(2s-g)}{\mathrm{L}(2s)}\,\mathrm{K}\,\Delta^g \int_0^\infty \left\{\frac{\lambda^2}{\alpha + v} + \&\mathrm{c.}\right\}^s \frac{v^{t-\frac{1}{2}}\,dv}{\mathrm{Q}}.$$

To apply this lemma we suppose, as before, that A, B, C represent differentiations with regard to x, y, z, but to operate only on ϕ. Hence supposing λ, μ, ν to stand for αx, βy, γz, and using the abbreviations given in Art. 1, we write the lemma in the form

$$\iint F^{2s-g}\,(D^2)^{s+1-t}\,\delta^g\,d\omega = \frac{L\,(2s-g)}{L\,(2s)}\,K\,\Delta^g\int_0^\infty \left\{\frac{\lambda^2}{\alpha+v} + \&c.\right\}^s \frac{v^{t-\frac{1}{2}}\,dv}{Q}.$$

20. By substituting for S_n its value we find

$$\iint \frac{\delta^{\kappa+n-3}\,\phi}{L\,(\kappa+n-3)}\,S_n\,D^2\,d\omega = \iint \frac{\delta^{\kappa+n-3}\,\phi}{L\,(\kappa+n-3)}\,\Sigma\,\frac{L\,(n-f-1)}{Lf\,.\,L\,(n-2f-1)}\,(-\,2F)^{n-2f-1}\,D^{2(n-f)}\,E^f.$$

To use the lemma we put $g = \kappa + n - 3$, $2s - g = n - 2f - 1$, $s + 1 - t = n - f$. These give $t = \frac{1}{2}\,(\kappa - 2)$ so that κ must not be less than 2. They also make $s - t$ positive. The double integral therefore becomes

$$\frac{(-\,1)^{n-1}}{L\,(n+2t-1)}\,\frac{2\pi}{2^{n-1}}\,\frac{L\,(t)}{L\,(2t)}\,\Delta^{n+2t-1}\int_0^\infty \Sigma\,\frac{E^f}{L\,(f)\,L\,(s)}\left(\frac{\lambda^2}{\alpha+v} + \ldots\right)^s \frac{v^{t-\frac{1}{2}}\,dv}{Q},$$

where $2\,(t+1)$ has been written for κ and $s = n - f + t - 1$.

Since Σ here implies summation from $f = 0$ to $f = \frac{1}{2}\,n - 1$ or $\frac{1}{2}\,(n-1)$, the terms which follow the Σ do not form a complete binomial expansion. But, since the operator Δ^{n+2t-1} contains differential coefficients of the order $(d/d\lambda)^{n+2t-1}$, while the subject is of the order $\lambda^{2(n-f+t-1)}$, the missing terms are zero. We may therefore sum the series from $f = 0$ to $n + t - 1$. We therefore have

$$V_2 = \frac{2\pi\mu}{\kappa-1}\cdot\frac{L\,(t)}{L\,(2t)}\cdot\Sigma\int_0^\infty \frac{\Delta^{n+2t-1}}{L\,(n+2t-1)}\,\frac{R^{n+t-1}}{L\,(n+t-1)}\,(-\,\tfrac{1}{2})^{n-1}\,\phi\,\frac{v^{t-\frac{1}{2}}\,dv}{Q},$$

where

$$R = E + \frac{\lambda^2}{\alpha+v} + \frac{\mu^2}{\beta+v} + \frac{\nu^2}{\gamma+v}$$

and Σ implies summation for integer values of n, beginning at $n = 1$ onwards until the terms vanish by the operation of the differential coefficients A, B, C, contained in Δ on the subject $\phi\,(x, y, z)$.

21. We may simplify this series by performing the operation $\Delta^{n+2t-1}\,R^{n+t-1}$ once for all. For the sake of brevity let

$$S = 2\left(\frac{A\lambda}{\alpha+v} + \frac{B\mu}{\beta+v} + \frac{C\nu}{\gamma+v}\right) \qquad T = \frac{A^2}{\alpha+v} + \frac{B^2}{\beta+v} + \frac{C^2}{\gamma+v}.$$

We then find, when $g < h$,

$$\frac{\Delta^g R^h}{L(g)\,.\,L(h)} = \frac{R^{h-g}S^g}{L(h-g)\,.\,L(g)} + \frac{R^{h-g+1}S^{g-2}T}{L\,(h-g+1)\,.\,L\,(g-2)} + \frac{R^{h-g+2}S^{g-4}T^2}{L\,(h-g+2)\,.\,L\,(g-4)\,.\,L(2)} + \&c.,$$

the series stopping when the exponent of S becomes negative. The form of the expansion having been found, the simplest proof is that supplied by the method of induction. Performing Δ on both sides of this equation and noticing that

$$\Delta R^m S^n = mR^{m-1}S^{n+1} + nR^m S^{n-1} \,.\, 2T,$$

we may see that since the theorem is true for $g = 0$, it is true for all positive integer values of g. It is unnecessary to reproduce the work.

When $g > h$, some of the indices of R at the beginning of the series are negative. These terms are to be omitted and we then have

$$\frac{\Delta^g R^h}{L(g)L(h)} = \frac{S^{2h-g}T^{g-h}}{L(2h-g).L(g-h)} + \frac{RS^{2h-g-2}T^{g-h+1}}{L(2h-g-2)L(g-h+1)} + \frac{R^2 S^{2h-g-4}T^{g-h+2}}{L(2)L(2h-g-4)L(g-h+2)} + \&c.$$

We thus have

$$\frac{\Delta^{n+2t-1} R^{n+t-1}}{L(n+2t-1)\,L(n+t-1)}\left(-\tfrac{1}{2}\right)^{n-1} = \frac{(-\frac{1}{2}S)^{n-1}T^t}{L(n-1)\,L(t)} + \frac{R(-\frac{1}{2}S)^{n-3}T^{t+1}}{L(n-3)\,L(t+1)}\,\frac{1}{2^2} + \ldots$$

In this equality S and T contain A, B, C, which represent differential coefficients of the function $\phi\,(x, y, z)$. Also to find V_2 we have to sum both sides of this equality for all values of n from $n = 1$ onwards, until the terms become zero by the continued operation of S and T on ϕ. Since the series stops at the first negative power of S, all negative exponents are to be omitted. We therefore write the summation in the form

$$\Sigma \frac{\Delta^{n+2t-1}R^{n+t-1}}{L(n+2t-1)\,L(n+t-1)}(-\tfrac{1}{2})^{n-1}\phi = \frac{T^t}{L(t)}\,\Sigma\frac{(-\frac{1}{2}S)^{n-1}\phi}{L(n-1)} + \frac{RT^{t+1}}{L(t+1)}\,\frac{1}{2^2}\,\Sigma\frac{(-\frac{1}{2}S)^{n-3}\phi}{L(n-3)} + \ldots$$

We notice that on the right-hand side the first Σ implies summation from $n = 1$ to ∞, the second from $n = 3$ to ∞, and so on. Now by TAYLOR's theorem

$$e^{-\frac{1}{2}S}\phi = \Sigma\frac{(-\frac{1}{2}S)^h\phi}{L(h)} = \phi\left(x - \frac{\lambda}{\alpha + v},\quad y - \frac{\mu}{\beta + v},\quad z - \frac{\nu}{\gamma + v}\right)$$

where the Σ implies summation from $h = 0$ to ∞. We therefore have

$$V_2 = \frac{2\pi\mu}{\kappa - 1}\,\frac{L(t)}{L(2t)}\int_0^\infty \frac{v^{t-\frac{1}{2}}\,dv}{Q}\left[\frac{T^t}{L(t)} + \frac{RT^{t+1}}{L(t+1)}\,\frac{1}{2^2} + \ldots\right]\phi\left(x - \frac{\lambda}{\alpha + v}, \&c.\right).$$

Here T is an operator on (x, y, z). The quantities λ, μ, ν are constants for which we write αx, βy, γz after the operations indicated by T have been performed. We may reverse the order of these operations and write

$$x - \frac{\lambda}{\alpha + v} = \frac{vx}{\alpha + v}, \quad y - \frac{\mu}{\beta + v} = \frac{vy}{\beta + v}, \text{ \&c.,}$$

before the operation T has been performed, provided we replace the d/dx, d/dy, d/dz included in T by $\frac{\alpha + v}{v}\frac{d}{dx}$, $\frac{\beta + v}{v}\frac{d}{dy}$, $\frac{\gamma + v}{v}\frac{d}{dz}$ respectively.

The operator T then takes the form

$$\mathrm{T} = \frac{1}{v^2}\left\{(\alpha + v)\frac{d^2}{dx^2} + (\beta + v)\frac{d^2}{dy^2} + (\gamma + v)\frac{d^2}{dz^2}\right\};$$

also by the same substitution

$$\mathrm{R} = 1 - \frac{\alpha v x^2}{\alpha + v} - \frac{\beta v y^2}{\beta + v} - \frac{\gamma v z^2}{\gamma + v}$$

which must now be placed on the left-hand side of the operator to indicate that it is not included in the subject.

We thus find for the partial potential of a thin heterogeneous homœoid, whose density at the point (ξ, η, ζ) is $\phi\,(\xi, \eta, \zeta)$,

$$\mathrm{V}_2 = \frac{2\pi\mu}{\kappa - 1}\frac{\mathrm{L}(t)}{\mathrm{L}(2t)}\int_0^\infty \frac{v^{t-\frac{1}{2}}dv}{\mathrm{Q}}\left\{\frac{\mathrm{T}^t}{\mathrm{L}(t)} + \frac{\mathrm{RT}^{t+1}}{\mathrm{L}(t+1)}\frac{1}{2^2} + \ldots\right\}\phi\left(\frac{vx}{\alpha + v}, \ldots\right).$$

To this we add the partial potential V_1, already found in Art. 14, for the complete potential.

22. It is sometimes more convenient to use the axes a, b, c of the ellipsoid instead of the squared reciprocals α, β, γ. Let us also suppose that the density at any point (ξ, η, ζ) is given by $\psi\,(\xi/a, \eta/b, \zeta/c)$, so that we may the more easily deduce the potential at an external point. Also writing $v = 1/u$, we have

$$\mathrm{R} = 1 - \frac{x^2}{a^2 + u} - \frac{y^2}{b^2 + u} - \frac{z^2}{c^2 + u},$$

$$\mathrm{D} = \frac{\mathrm{T}}{u} = \frac{a^2 + u}{a^2}\frac{d^2}{dx^2} + \frac{b^2 + u}{u^2}\frac{d^2}{dy^2} + \frac{c^2 + u}{c^2}\frac{d^2}{dz^2},$$

$$\mathrm{Q}_1^2 = (a^2 + u)(b^2 + u)(c^2 + u).$$

The partial potential of a thin heterogeneous homœoid at an internal point is then

$$\mathrm{V}_2 = \frac{2\pi\mu}{\kappa - 1}\frac{\mathrm{L}(t)}{\mathrm{L}(2t)}\int_0^\infty \frac{abc\,du}{\mathrm{Q}_1}\mathrm{M}\psi\left(\frac{ax}{a^2 + u}, \frac{by}{b^2 + u}, \frac{cz}{c^2 + u}\right),$$

where the operator M *is*

$$\mathrm{M} = \frac{\mathrm{D}^t}{\mathrm{L}\,(t)} + \frac{u\mathrm{R}\mathrm{D}^{t+1}}{\mathrm{L}\,(t+1)}\,\frac{1}{2^2} + \ldots + \frac{u^n\mathrm{R}^n\mathrm{D}^{t+n}}{\mathrm{L}\,(n)\,.\,\mathrm{L}\,(t+n)}\,\frac{1}{2^{2n}} + \ldots,$$

and $t = \frac{1}{2}(\kappa - 2)$ *where* κ *is the index of the law of force.*

Having found the law of the potential of a homœoid for the inverse κ^{th} power, the potential for the inverse $(\kappa + 2)^{\text{th}}$ power may be obtained by Jellett's theorem, already mentioned in Art. 5. We might thus obtain a new proof of the formula by the method of induction. The theorem has been verified in this way, but the demonstration is long and rather tedious.

23. To deduce the potential of the heterogeneous homœoid at an external point (x', y', z'), we follow the same process as before. Let a', b', c' be the axes of the confocal through the attracted point. Let $a'^2 - a^2 = \epsilon^2$, so that ϵ^2 is determined by

$$\frac{x'^2}{a^2 + \epsilon^2} + \frac{y'^2}{b^2 + \epsilon^2} + \frac{z'^2}{c^2 + \epsilon^2} = 1.$$

We now write a', b', c' and μ' for a, b, c and μ, where $\mu' \; a'b'c' = \mu \; abc$, since the volumes of the two shells are to be equal. Lastly, putting $u + \epsilon^2 = w$, $x/a = x'/a'$, &c., we find after some easy reductions that the *partial potential of the heterogeneous homœoid at an external point is*

$$\mathrm{V}'_2 = \frac{2\pi\mu}{\kappa - 1}\,\frac{\mathrm{L}\,(t)}{\mathrm{L}\,(2t)}\int_{\epsilon^2}^{\infty}\frac{abc\,dw}{\mathrm{Q}_1}\,\mathrm{M}\psi\left(\frac{ax'}{a^2 + w}, \frac{by'}{b^2 + w}, \frac{cz'}{c^2 + w}\right),$$

$$u\mathrm{R} = w\mathrm{R}' = w\left\{1 - \frac{x'^2}{a^2 + w} - \frac{y'^2}{b^2 + w} - \frac{z'^2}{c^2 + w}\right\},$$

$$\mathrm{D}' = \frac{a^2 + w}{a^2}\,\frac{d^2}{dx'^2} + \frac{b^2 + w}{b^2}\,\frac{d^2}{dy'^2} + \frac{c^2 + w}{c^2}\,\frac{d^2}{dz'^2},$$

$$\mathrm{M} = \frac{\mathrm{D}'^t}{\mathrm{L}\,(t)} + \frac{w\mathrm{R}'\mathrm{D}'^{t+1}}{\mathrm{L}\,(t+1)}\,\frac{1}{2^2} + \ldots + \frac{w^n\mathrm{R}'^n\mathrm{D}'^{t+n}}{\mathrm{L}\,(n)\,\mathrm{L}\,(t+n)}\,\frac{1}{2^{2n}} + \ldots$$

24. As a first example, *let the law of force be the inverse square.* We then have $\kappa = 2$ and therefore $t = 0$. The partial potential V_1 is then zero and the complete potentials, inside and outside, are given by the expressions for V_2 and V_2' just written down. These agree with those given by Mr. Dyson in the 'Quarterly Journal,' vol. 25, 1891.

As a second example, let us find the potential of a thin homœoid when the *law of force is the inverse fourth power and the density* at (ξ, η, ζ) *is* $m\;(\xi/a^2)$. We then find by Arts. (14) and (22),

$$\mathrm{V}_1 = \frac{2\pi\mu}{3}\,\frac{2}{\mathrm{E}}\,m\left(\frac{x}{a}\right)^2, \qquad \mathrm{V}_2 = \frac{2\pi\mu}{3}\int_0^{\infty}\frac{abc\,du}{\mathrm{Q}_1}\,\frac{m}{a^2 + u}.$$

The potential at an internal point is the sum of these two.

25. *To find the potential of a thin heterogeneous homœoid when the force varies as the inverse κ^{th} power of the distance, and $\kappa = 2$, zero, or any even negative number.*

As the investigation resembles that of the last proposition, except as regards the lemma, it is sufficient merely to sketch the argument.

To avoid confusion as to the sign of κ, let us, in the first instance, put $\kappa = -\kappa_1$ We then have, by Art. 2,

$$V = -\frac{\mu}{\kappa_1+1}\iint\frac{\rho_1 r_1^{\kappa_1+3} - \rho_2 r_2^{\kappa_1+3}}{r_1 - r_2} D^2 d\omega\,;$$

writing as before (Art. 12),

$$\rho = \phi + r\delta\phi + r^2\frac{\delta^2\phi}{1\,.\,2} + \ldots$$

$$V = -\frac{\mu}{\kappa_1+1}\iint\left\{\phi S_{\kappa_1+3} + \delta\phi S_{\kappa_1+4} + \ldots\right\}D^2 d\omega$$

$$= -\frac{\mu}{\kappa_1+1}\Sigma\iint\frac{\delta^{n-\kappa_1-3}\phi}{L(n-\kappa_1-3)}S_n D^2 d\omega,$$

where Σ implies summation from $\kappa_1 + 3$ to ∞. We now substitute, as before,

$$S_n = \frac{r_1^n - r_2^n}{r_1 - r_2} = \Sigma\frac{L(n-f-1)}{L(f)L(n-2f-1)}p^{n-2f-1}q^f,$$

where $p = -2D^2F$, $q = ED^2$.

The lemma by which the integration of each term is effected is as follows. By Art. 10 we have

$$\iint\frac{(\lambda l + \mu m + \nu n)^{2s}\,d\omega}{(\alpha l^2 + \beta m^2 + \gamma n^2)^{s+t+1}} = K'\,.\int_0^\infty v^{-\frac{1}{2}}dv\left(\frac{d}{dv}\right)^t\left\{\frac{\lambda^2}{\alpha+v} + \frac{\mu^2}{\beta+v} + \frac{\nu^2}{\gamma+v}\right\}^s\frac{1}{Q}\,.$$

Following the same notation as before, we let

$$\Delta = A\,d/d\lambda + B\,d/d\mu + C\,d/d\nu.$$

We then find, as in Art. 19,

$$\iint F^{2s-g}(D^2)^{s+t+1}\delta^g\,d\omega = \frac{L(2s-g)}{L(s)\,L(s+t)}\,\frac{2\pi(-1)^t}{2^{2s}}\,\Delta^g\int_0^\infty v^{-\frac{1}{2}}dv\left(\frac{d}{dv}\right)^t\{\&c.\}^s\frac{1}{Q}\,,$$

where g is positive and less than $2s$.

To apply the lemma we put $g = n - \kappa_1 - 3$, $2s - g = n - 2f - 1$, $s + t = n - f - 1$. These give $t = \frac{1}{2}(\kappa_1 + 2) = \frac{1}{2}(2 - \kappa)$ and this requires that the index κ of the force should be less than 2.

After completing and summing the same binomial series as before, we have

$$V = -\frac{2\pi\mu}{\kappa_1 + 1}\Sigma(-1)^{n-t-1}\frac{(\frac{1}{2}\Delta)^{n-2t-1}\phi}{L(n-2t-1)}\int_0^\infty v^{-\frac{1}{2}}\,dv\left(\frac{d}{dv}\right)^t\frac{R^{n-t-1}}{L(n-t-1)}\frac{1}{Q},$$

where, as in Art. 20,

$$R = E + \frac{\lambda^2}{\alpha + v} + \frac{\mu^2}{\beta + v} + \frac{\nu^2}{\gamma + v}.$$

By using the theorem of Art. 21, we can perform the operations indicated by Δ. Noticing that the index of the operator (viz., $n - 2t - 1$) is here less than that of the subject (viz., $n - t - 1$), we find

$$V = -\frac{2\pi\mu}{\kappa_1 + 1}(-1)^t\int_0^\infty v^{-\frac{1}{2}}\,dv\left(\frac{d}{dv}\right)^t M\frac{1}{Q}\phi\left(x - \frac{\lambda}{\alpha + v},\ y - \frac{\mu}{\beta + v},\ \&c.\right),$$

$$M = \frac{R^t}{L(t)} + \frac{R^{t+1}T}{L(t+1)}\frac{1}{2^2} + \ldots\frac{R^{t+n}T^n}{L(t+n)L(n)}\frac{1}{2^{2n}} + \ldots$$

We now write $\lambda = \alpha x$, &c., and make the necessary changes in the meanings of R and T. We thus have

$$T = \frac{1}{v^2}\left\{(\alpha + v)\frac{d^2}{dx^2} + (\beta + v)\frac{d^2}{dy^2} + (\gamma + v)\frac{d^2}{dz^2}\right\},$$

$$R = 1 - \frac{\alpha v x^2}{\alpha + v} - \frac{\beta v y^2}{\beta + v} - \frac{\gamma v z^2}{\gamma + v}.$$

26. We may conveniently express the result in terms of the axes of a, b, c of the ellipsoid instead of the squared reciprocals. We find that, when the index κ of the force is an even integer < 2 or negative, the potential of a thin heterogeneous homœoid, the density at (ξ, η, ζ) being $\psi\,(\xi/a, \eta/b, \zeta/c)$ at an internal point, is

$$V = \frac{2\pi\mu}{\kappa - 1}\int_0^\infty u^{-\frac{3}{2}}\,du\left(u^2\frac{d}{du}\right)^t\frac{abc\,u^{\frac{3}{2}}}{Q_1}M\psi\left(\frac{ax}{a^2 + u},\ \frac{by}{b^2 + u},\ \frac{cz}{c^2 + u}\right),$$

where

$$M = \frac{R^t}{L(t)} + \frac{R^{t+1}}{L(t+1)}uD.\frac{1}{2^2} + \ldots + \frac{R^{t+n}}{L(t+n)}\frac{(u\,D)^n}{L(n)}\frac{1}{2^{2n}} + \ldots,$$

and R, D, and Q_1 have the meanings given to them in Art. 22. Also $t = \frac{1}{2}(2 - \kappa)$.

At an external point the potential is

$$V' = \frac{2\pi\mu}{\kappa - 1}\int_{\epsilon^2}^\infty (u - \epsilon^2)^{-\frac{3}{2}}\,du\left\{(u - \epsilon^2)^2\frac{d}{du}\right\}^t\frac{abc\,u^t}{Q_1(u - \epsilon^2)^{t-\frac{3}{2}}}M\psi\left(\frac{ax}{a^2 + u}, \&c.\right),$$

where (x, y, z) are now the coordinates of the external attracted point, so that ϵ^2 is defined by

$$\frac{x^2}{a^2 + \epsilon^2} + \frac{y^2}{b^2 + \epsilon^2} + \frac{z^2}{c^2 + \epsilon^2} = 1.$$

27. *To find the potential of a solid homogeneous ellipsoid at an internal point when the law of force is the inverse κ^{th} power and κ is even and* > 4.

Referring to Art. 4, we see that when $\kappa > 4$,

$$V = \frac{-1}{2(\kappa-1)(\kappa-4)} \iint (u_1^{\kappa-4} + u_2^{\kappa-4})\, d\omega.$$

Proceeding as in Art. 8, we have

$$\frac{u_1^n + u_2^n}{n} = \Sigma \frac{L(n-f-1)}{L(f) L(n-2f)} \left(\frac{2F}{E}\right)^{n-2f} \left(\frac{1}{ED^2}\right)^f,$$

where Σ implies summation from $f = 0$ to $\frac{1}{2}n$. Writing $n = \kappa - 4$, and remembering the integral given in Art. 8, we have

$$V = \frac{-2\pi}{(\kappa-1) L(\kappa-3)} \left(\frac{2}{E}\right)^{\kappa-4} \Sigma \frac{L(\kappa-5-f)}{L(f)} \frac{E^f \nabla^f}{2^2} P,$$

where

$$P = \left(\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}\right)^{\frac{1}{2}(\kappa-4)},$$

$$\nabla = a^2 \frac{d^2}{dx^2} + b^2 \frac{d^2}{dy^2} + c^2 \frac{d^2}{dz^2},$$

and Σ implies summation from $f = 0$ to $\frac{1}{2}(\kappa - 4)$. Thus the series has but two terms when the law of force is the inverse sixth, three terms when the inverse eighth, and so on.

28. *To find the potential of a homogeneous solid ellipsoid at an internal point when the law of force is the inverse κ^{th} and κ is even* < 4, *and may be negative.*

Here

$$V = \frac{-1}{2(\kappa-1)(\kappa-4)} \iint (r_1^{4-\kappa} + r_2^{4-\kappa})\, d\omega,$$

$$\frac{r_1^n + r_2^n}{n} = \Sigma \frac{L(n-f-1)}{L(f) \,.\, L(n-2f)} (-2FD^2)^{n-2f} (ED^2)^f.$$

Substituting and using the integral form given in the lemma, Art. 10, we find, after summing the same binomial series,

$$V = \frac{\pi(-1)^t}{(\kappa-1) L(t+1)} \int_0^\infty v^{-\frac{1}{2}} dv \left(\frac{d}{dv}\right)^t \frac{R^{t+1}}{Q},$$

where

$$R = 1 - \frac{v\alpha x^2}{\alpha+v} - \frac{v\beta y^2}{\beta+v} - \frac{v\gamma z^2}{\gamma+v}$$

$$Q^2 = (\alpha+v)(\beta+v)(\gamma+v)$$

and

$$t = \tfrac{1}{2}(2-\kappa).$$

29. Let us next turn our attention to the attraction of a heterogeneous solid ellipsoid at an internal point. The density at any point x, y, z being $\phi(x, y, z)$, the density at a neighbouring point $x + \xi, y + \eta, z + \zeta$ is

$$\phi + \phi_x\xi + \phi_y\eta + \phi_z\zeta + \dots$$

Let us describe a sphere of small radius r, whose centre is the point x, y, z. The x attraction at the centre of the matter filling this sphere, supposing the law of attraction to be the inverse κ^{th} and the density to be $m\xi$, is seen by an easy integration to be $\frac{4\pi}{3} m \left[\frac{r^{4-\kappa}}{4-\kappa}\right]_0^r$ except when $\kappa = 4$. If κ is greater than 4, this attraction is infinite. If $\kappa = 4$ the integral is $\frac{4\pi}{3}[\log r]_0^r$ and is again infinite. The attraction at any internal point of a heterogeneous ellipsoid is therefore in general infinite whenever the law of force is such that the index κ is greater than 3. We shall therefore confine our attention to those laws of force which make κ less than 4. When the ellipsoid is homogeneous the x attraction of the matter filling the elementary sphere is zero at the centre, and there is no restriction on the value of κ.

30. *To find the potential at an internal point* P *of a heterogeneous ellipsoid whose density is* $\phi(x, y, z)$, *when the law of force is the inverse* κ^{th}, *and* κ *is even and less than* 4.

Referring to Art. 4 we see that the potential at P of the conical element whose axis is in the positive direction of the radius vector is $\frac{1}{\kappa - 1}\int \frac{r^2\,d\omega\,dr\rho}{r^{\kappa-1}}$, the limits being $r = 0$ to r. As in Art. 12, we write

$$\rho = \phi + r\,\delta\phi + \frac{r^2}{1\,.\,2}\delta^2\phi + \dots + \frac{r^m}{\mathrm{L}(m)}\delta^m\phi + \dots$$

The potential of the positive conical element is therefore $\frac{d\omega}{\kappa-1}\Sigma\frac{r^{4-\kappa+}}{4-\kappa+m}\frac{\delta^m\phi}{\mathrm{L}(m)}$. To find the potential of the negative cone, we let R be the distance from P of one of its elements Q′, taken positively. The density at Q′ is found by writing $-$ R for r in the above series for ρ. The potential of the negative cone is, therefore, $\frac{d\omega}{\kappa-1}\Sigma\frac{\mathrm{R}^{4-\kappa+m}}{4-\kappa+m}(-1)^m\frac{\delta^m\phi}{\mathrm{L}(m)}$, and here we must write $-r_2$ for R.

Taking the two conical elements together, and writing n for $4 - \kappa + m$, the potential of the whole ellipsoid at P becomes

$$\mathrm{V} = \frac{1}{2(\kappa-1)}\iint d\omega\,\Sigma\frac{r_1^n + r_2^n}{n}\frac{\delta^{n+\kappa-4}\phi}{\mathrm{L}(n+\kappa-4)}.$$

The integral is to be taken all round the attracted point. The summation represented by Σ extends from $n = 4 - \kappa$ to infinity.

The investigation now proceeds on the same general lines as in Art. 25. By Art. 8 we have

$$\frac{r_1^n + r_2^n}{n} = \Sigma \frac{\mathrm{L}(n-f-1)}{\mathrm{L}f \,.\, \mathrm{L}(n-2f)} (-2\mathrm{FD}^2)^{n-2f} (\mathrm{ED}^2)^f,$$

where the sum extends from $f = 0$ to $\frac{1}{2}n$ or $\frac{1}{2}(n-1)$. Substituting and effecting an integration by the use of the lemma of Art. 25, we find, after putting $t = \frac{1}{2}(2-\kappa)$,

$$\iint \frac{r_1^n + r_2^n}{n} \frac{\delta^{n+\kappa-4}}{\mathrm{L}(n+\kappa-4)} d\omega = \frac{(-1)^{n+t} 2\pi}{\mathrm{L}(n-2t-2)} \left(\frac{\Delta}{2}\right)^{n-2t-2} \int_0^\infty v^{-\frac{1}{2}} dv \left(\frac{d}{dv}\right)^t \cdot \Sigma \frac{\mathrm{E}^f(\mathrm{R}-\mathrm{E})^s}{\mathrm{L}(f) \,.\, \mathrm{L}(s)} \cdot \frac{1}{\mathrm{Q}},$$

where

$$\mathrm{R} = \mathrm{E} + \frac{\lambda^2}{\alpha+v} + \frac{\mu^2}{\beta+v} + \frac{\nu^2}{\gamma+v},$$

$$\Delta = \mathrm{A}\frac{d}{d\lambda} + \mathrm{B}\frac{d}{d\mu} + \mathrm{C}\frac{d}{d\nu} \cdot$$

The summation implied by the Σ does not make a complete binomial series. But since the operator contains differential coefficients of the order $n - 2t - 2$ while the subject is of the order $2s$ where $s = n - f - t - 1$, the terms given by any value of f greater than $\frac{1}{2}n$ are zero. The missing terms of the binomial series are therefore zero. Summing the series we find

$$\mathrm{V} = \Sigma \frac{\pi(-1)^{n+t}}{\kappa-1} \int_0^\infty v^{-\frac{1}{2}} dv \left(\frac{d}{dv}\right)^t \frac{(\frac{1}{2}\Delta)^{n-2t-2}\mathrm{R}^{n-t-1}}{\mathrm{L}(n-2t-2) \,.\, \mathrm{L}(n-t-1)} \frac{\phi}{\mathrm{Q}},$$

where Σ here implies summation from $n = 2t + 2$ to ∞.

To effect this summation we take the series for $\Delta^g \mathrm{R}^h$ given in Art. 21, where $g < h$. Putting $g = n - 2t - 2$, $h = n - t - 1$, we find

$$\frac{(-\frac{1}{2}\Delta)^g \mathrm{R}^h}{\mathrm{L}g \,.\, \mathrm{L}h} = \frac{\mathrm{R}^{t+1}(-\frac{1}{2}\mathrm{S})^g}{\mathrm{L}(t+1)\,\mathrm{L}(g)} + \frac{\mathrm{R}^{t+2}(-\frac{1}{2}\mathrm{S})^{g-2}\mathrm{T}}{\mathrm{L}(t+2)\,\mathrm{L}(g-2)\,\mathrm{L}(1)} \frac{1}{2^2} + \ldots$$

Using TAYLOR's theorem in the manner described in Art. 21, the result of performing this operation on ϕ is

$$\left\{\frac{\mathrm{R}^{t+1}}{\mathrm{L}(t+1)} + \frac{\mathrm{R}^{t+2}}{\mathrm{L}(t+2)} \frac{\mathrm{T}}{1} \cdot \frac{1}{2^2} + \frac{\mathrm{R}^{t+3}}{\mathrm{L}(t+3)} \frac{\mathrm{T}^2}{\mathrm{L}(2)} \frac{1}{2^4} + \ldots\right\} \phi\left(x - \frac{\lambda}{\alpha+v}, \text{\&c.}\right).$$

We now make the changes in the operators described in the article just referred to, and the expression for the potential becomes

$$\mathrm{V} = \frac{\pi(-1)^t}{\kappa-1} \int_0^\infty v^{-\frac{1}{2}} dv \left(\frac{d}{dv}\right)^t \frac{1}{\mathrm{Q}} \mathrm{M}\phi\left(\frac{vx}{\alpha+v}, \frac{vy}{\beta+v}, \frac{vz}{\gamma+v}\right),$$

where

$$\mathrm{M} = \frac{\mathrm{R}^{t+1}}{\mathrm{L}(t+1)} + \frac{\mathrm{R}^{t+2}}{\mathrm{L}(t+2)}\frac{\mathrm{T}}{1}\frac{1}{2^2} + \ldots\ldots + \frac{\mathrm{R}^{t+n}}{\mathrm{L}(t+n)}\frac{\mathrm{T}^n}{\mathrm{L}(n)}\frac{1}{2^{2n}} + \ldots,$$

$$\mathrm{R} = 1 - \frac{\alpha v x^2}{\alpha + v} - \frac{\beta v y^2}{\beta + v} - \frac{\gamma v z^2}{\gamma + v},$$

$$\mathrm{T} = \frac{1}{v^2}\left\{(\alpha + v)\frac{d^2}{dx^2} + (\beta + v)\frac{d^2}{dy^2} + (\gamma + v)\frac{d^2}{dz^2}\right\}.$$

It is often more convenient to use the axes a, b, c of the ellipsoid instead of the squared reciprocals α, β, γ. Let us also suppose that the density at the point x, y, z, is $\psi(x/a, y/b, z/c)$. Putting $v = 1/u$, we have

$$\mathrm{R} = 1 - \frac{x^2}{a^2 + u} - \frac{y^2}{b^2 + u} - \frac{z^2}{c^2 + u},$$

$$\mathrm{D} = \frac{\mathrm{T}}{u} = \frac{a^2 + u}{a^2}\frac{d^2}{dx^2} + \frac{b^2 + u}{b^2}\frac{d^2}{dy^2} + \frac{c^2 + u}{c^2}\frac{d^2}{dz^2},$$

$$\mathrm{Q}_1^2 = (a^2 + u)(b^2 + u)(c^2 + u).$$

The potential at an internal point of the solid heterogeneous ellipsoid is then

$$\mathrm{V} = \frac{\pi abc}{\kappa - 1}\int_0^\infty u^{-\frac{3}{2}}\,du\left(u^2\frac{d}{du}\right)^t \frac{u^{\frac{3}{2}}}{\mathrm{Q}_1}\mathrm{M}\psi\left(\frac{ax}{a^2 + u}, \frac{by}{b^2 + u}, \frac{cz}{c^2 + u}\right),$$

$$\mathrm{M} = \frac{\mathrm{R}^{t+1}}{\mathrm{L}(t+1)} + \frac{\mathrm{R}^{t+2}}{\mathrm{L}(t+2)}\frac{u\mathrm{D}}{1}\frac{1}{2^2} + \ldots + \frac{\mathrm{R}^{t+n+1}}{\mathrm{L}(t+n+1)}\frac{u^n\mathrm{D}^n}{\mathrm{L}(n)}\frac{1}{2^{2n}} + \ldots,$$

and $t = \frac{1}{2}(2 - \kappa)$ where κ is the index of the law of force.

As an example, let the law be the inverse square, we then have $\kappa = 2$ and $t = 0$. The expression then becomes

$$\mathrm{V} = \pi abc\int_0^\infty \frac{du}{\mathrm{Q}_1}\mathrm{M}\psi\left(\frac{ax}{a^2 + u}, \text{\&c.}\right)$$

$$\mathrm{M} = \mathrm{R} + \frac{\mathrm{R}^2}{\mathrm{L}(2)}\frac{u\mathrm{D}}{1}\frac{1}{2^2} + \ldots + \frac{\mathrm{R}^{n+1}}{\mathrm{L}(n+1)}\frac{u^n\mathrm{D}^n}{\mathrm{L}(n)}\frac{1}{2^{2n}} + \ldots,$$

which agrees with that given by Mr. DYSON in the 'Quarterly Journal,' 1891.

31. In CAYLEY'S memoir on Prepotentials we find a simple and elegant formula for the potential of an ellipsoid in the form of a definite integral (see 'Phil. Trans.,' 1875, p. 745). Taking the notation of this paper, let V′ be the potential of a solid ellipsoid when the force varies as the inverse κ^{th} power of the distance at an *external* point (x, y, z). When the density ρ at an internal point is given by

$$\rho = \left(1 - \frac{\xi^2}{a^2} - \frac{\eta^2}{b^2} - \frac{\zeta^2}{c^2}\right)^{n+t},$$

where a, b, c are the semi-axes of the bounding surface, it is shown that

$$(\kappa - 1)\,\mathrm{V}' = \frac{\{\Gamma(\frac{1}{2})\}^3\,\Gamma(1 + t + n)}{\Gamma(\frac{3}{2} + t)\,\Gamma(1 + n)}\,abc\int_{\epsilon^2}^{\infty}\frac{du}{u^{t+1}\mathrm{Q}_1}\cdot\left(1 - \frac{x^2}{a^2 + u} - \frac{y^2}{b^2 + u} - \frac{z^2}{c^2 + u}\right)^n,$$

where $t = \frac{1}{2}(\kappa - 4)$ and Q_1, ϵ^2 have the meaning given in Art. 31. It is necessary, also, that each of the three quantities $\frac{3}{2} + t$, $1 + n$, $1 + t + n$ should be positive.

When the ellipsoid is homogeneous $n + t = 0$, and the restrictions just mentioned require that the index κ of the force should lie between $-\frac{3}{2}$ and 4. When the index κ is greater than 4, the index $n + t$ of the density ρ cannot be less than $\frac{1}{2}(\kappa - 6)$.

LEJEUNE-DIRICHLET (LIOUVILLE'S 'Journal,' 1839, p. 165) gives a similar formula (also mentioned by CAYLEY) for the attraction of a solid homogeneous ellipsoid, but the index κ of the law of force is restricted to lie between 2 and 3.

In the 'Cambridge and Dublin Mathematical Journal,' vol. 3, 1843, CAYLEY has given the value of a multiple integral which, when the number of variables is reduced to three, is equivalent to the potential of a homogeneous ellipsoid. Taking as before the notation of this paper, and supposing the force to vary as the inverse κ^{th} power of the distance, the potential at an external point x, y, z becomes

$$\mathrm{V}' = \frac{\pi^{\frac{3}{2}}abc}{(\kappa - 1)\,2^{2t-2}\,\mathrm{L}\,(t)\,\Gamma\,(\frac{3}{2} + t)}\,\Delta^{t-1}\,\frac{1}{\lambda^2\mathrm{Q}\left\{\dfrac{x^2}{(\lambda + a^2)^2} + \dfrac{y^2}{(\lambda + b^2)^2} + \dfrac{z^2}{(\lambda + c^2)^2}\right\}},$$

where $t = \frac{1}{2}(\kappa - 4)$, $\quad \mathrm{Q}^2 = (\lambda + a^2)(\lambda + b^2)(\lambda + c^2)$,

$$\Delta = d^2/dx^2 + d^2/dy^2 + d^2/dz^2,$$

$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} + \frac{z^2}{c^2 + \lambda} = 1.$$

It is obvious that t must be an integer, so that κ must be an even integer and must be greater than 4.

As the subject of operation is not an integral rational function of x, y, z, this formula has the disadvantage that the algebraical work of using the operator is very long.

An elementary proof of this result will be given in Art. 37, by which it will appear that it is really a case of JELLET'S theorem.

Another method of finding the potential of a heterogeneous ellipsoid has been given by Dr. FERRERS in the 'Quarterly Journal,' 1875. The method requires that the potential V of an ellipsoid, whose density is $\rho = (1 - \alpha\xi^2 - \beta\eta^2 - \gamma\zeta^2)^n$, should be known for all integer values of n. To deduce the potential V′, when the density at (ξ, η, ζ) is $\rho' = \xi^f\eta^g\zeta^h$ he finds a general expression for ρ' in a series of differential coefficients of ρ with descending powers of n. The value of V′ is then expressed in a corresponding series of differential coefficients of V. The law of force is supposed to be the inverse square, but the method applies for other laws of force, provided the fundamental expression for V is known.

32. When the attracted point is external, we may for all laws of force deduce the resolved attractions of a homogeneous ellipsoid by the use of IVORY'S theorem, the attractions at an internal point having been found in Arts. 27, 28. But when we know the potential of a thin homœoid at an external point, we may also express as an integral the potential of a solid ellipsoid in which the density ρ at any point (ξ, η, ζ) is given by

$$\rho = f\,(\xi^2/a^2 + \eta^2/b^2 + \zeta^2/c^2).$$

When the force varies as the inverse square, this problem has in effect been solved by POISSON. The potential at an external point (x', y', z') is in that case

$$\mathrm{V}' = \pi abc \int_0^\infty \{f_1\,(1) - f_1\,(m^2)\}\,\frac{du}{\mathrm{Q}_1}$$

where $f_1\,(m)$ is the integral of $f\,(m)$, and

$$m^2 = \frac{x'^2}{a^2+u} + \frac{y'^2}{b^2+u} + \frac{z'^2}{c^2+u}, \quad \mathrm{Q}_1{}^2 = (a^2+u)\,(b^2+u)\,(c^2+u).$$

Let us turn our attention to the corresponding integral when the law of force is the inverse κ^{th} power, where κ is greater than the square.

The potential of a thin homogeneous homœoid bounded by ellipsoids, whose major axes are a and $a\,(1+\mu)$ at an external point, has been found in a finite series in Art. 8. Let us use this to find the potential of a thin shell bounded by ellipsoids, whose major axes are ma and $(m + dm)\,a$. The density of the shell is $f(m^2)$, and the integral of its potential between the limits $m = 0$ to 1 is the potential of the heterogeneous solid ellipsoid.

Referring then to the formula in Art. 8, we unite dm/m for μ; ma, mb, mc, for a, b, c; and $m^2a^2 + \lambda$, $m^2b^2 + \lambda$, $m^2c^2 + \lambda$ for a'^2, b'^2, c'^2. We also put $\lambda = m^2u$ to simplify our results.

Since the attracted point lies on the confocal a', b', c', we have

$$\frac{x'^2}{a^2+u} + \frac{y'^2}{b^2+u} + \frac{z'^2}{c^2+u} = m^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

The potential of the solid ellipsoid is therefore

$$V' = \frac{2\pi}{(\kappa-1)(\kappa-3)}\int_0^1 \frac{abc}{Q_1}\left(\frac{2}{Hu}\right)^{\kappa-3} f(m^2)\, m\, dm \left\{1 + \frac{1}{2^2}\frac{HuD}{\kappa-4} + \ldots\right\} K^{\frac{1}{2}(\kappa-4)},$$

where

$$Q_1^{\,2} = (a^2+u)(b^2+u)(c^2+u),$$

$$H = \frac{x'^2}{(a^2+u)^2} + \frac{y'^2}{(b^2+u)^2} + \frac{z'^2}{(c^2+u)^2},$$

$$K = \frac{a^2x'^2}{(a^2+u)^3} + \frac{b^2y'^2}{(b^2+u)^3} + \frac{c^2z'^2}{(c^2+u)^3},$$

$$D = \frac{(a^2+u)^2}{a^2}\frac{d^2}{dx'^2} + \frac{(b^2+u)^2}{b^2}\frac{d^2}{dy'^2} + \frac{(c^2+u)^2}{c^2}\frac{d^2}{dz'^2},$$

the quantities represented by H, K, D being derived from E′, P′, Δ′ respectively by the changes described above.

Since by differentiating (1) we have

$$-2m\frac{dm}{du} = \frac{x'^2}{(a^2+u)^2} + \frac{y'^2}{(b^2+u)^2} + \frac{z'^2}{(c^2+u)^2} = H \quad . \quad . \quad . \quad . \quad . \quad (2),$$

we may conveniently change the independent variable from m to u, the limits being $u = \infty$ to $u = \epsilon^2$ where $a^2+\epsilon^2$, $b^2+\epsilon^2$, $c^2+\epsilon^2$ are the squared semi-axes of the ellipsoid passing through the attracted point, and confocal with the external surface of the given ellipsoidal body. We thus have

$$V' = \frac{2\pi}{(\kappa-1)(\kappa-3)} \cdot \int_\infty^{\epsilon^2} \frac{-abc}{Q_1}\frac{f(m^2)\,du}{u}\left(\frac{2}{Hu}\right)^{\kappa-4}\left\{1 + \frac{1}{2^2}\frac{HuD}{\kappa-4} + \ldots\right\} K^{\frac{1}{2}(\kappa-4)},$$

where m^2 has the value given by (1).

This formula expresses the potential of a solid ellipsoid, whose strata of equal density are ellipsoids similar to the bounding surface, the law of force being the inverse κ^{th} power of the distance where κ is not less than the fourth. The series has $\frac{1}{2}(\kappa-2)$ terms.

We shall now apply this equation to the special cases in which the force varies as the inverse fourth, sixth, eighth and tenth powers of the distance.

33. *Suppose the law of force to be the inverse fourth,* then $\kappa = 4$, and the series reduces to its first term. The potential of the ellipsoid at an external point is therefore

$$V' = \frac{2\pi}{3}\int_\infty^{\epsilon^2} \frac{-abc}{(a^2+u)^{\frac{1}{2}}(b^2+u)^{\frac{1}{2}}(c^2+u)^{\frac{1}{2}}}\frac{f(m^2)\,du}{u}.$$

If the density is uniform, $f(m^2) = 1$, and we fall back on the potential already obtained in Art. 5.

Let us, however, consider what forms of $f(m^2)$ will enable us to effect the integrations. Solving (1), Art. 32, we have

$$m^2 = \frac{\mathrm{A}u^2 + 2\mathrm{B}u + \mathrm{C}}{(a^2+u)(b^2+u)(c^2+u)}$$

where

$$\begin{aligned} \mathrm{A} &= x'^2 + y'^2 + z'^2 \\ 2\mathrm{B} &= (x'^2 + y'^2 + z'^2)(a^2 + b^2 + c^2) - a^2x'^2 - b^2y'^2 - c^2z'^2 \\ \mathrm{C} &= a^2b^2c^2(x'^2/a^2 + y'^2/b^2 + z'^2/c^2). \end{aligned}$$

If we put $f(m^2) = \mathrm{M}/m$, where M is some constant, the potential reduces to the form

$$\mathrm{V}' = \frac{2\pi}{3}\int_{\infty}^{\epsilon^2} \frac{-abc\mathrm{M}}{(\mathrm{A}u^2 + 2\mathrm{B}u + \mathrm{C})^{\frac{1}{2}}}\,\frac{du}{u}.$$

Writing $u = 1/v$ this takes the simple form

$$\mathrm{V}' = \frac{2\pi}{3}\,abc\int \frac{\mathrm{M}\,dv}{(\mathrm{A} + 2\mathrm{B}v + \mathrm{C}v^2)^{\frac{1}{2}}} \quad v = 0 \text{ to } 1/\epsilon^2$$

where ϵ^2 is given by

$$\frac{x'^2}{a^2+\epsilon^2} + \frac{y'^2}{b^2+\epsilon^2} + \frac{z'^2}{c^2+\epsilon^2} = 1.$$

This integral can be easily evaluated, but as the result is rather long it seems unnecessary to reproduce it here.

If the radius vector of the ellipsoid through any internal point R, whose coordinates are x, y, z, meet the surface in Q, we know that

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = \left(\frac{\mathrm{OR}}{\mathrm{OQ}}\right)^2$$

where O is the centre. It follows that when the law of force is the inverse fourth power and the density at any point R varies as OQ/OR, the potential at any external point can be found in finite terms free from all signs of integration.

This theorem can be generalized. For if we take the density to be

$$\rho = \mathrm{F}(m^2)/m$$

where F is a rational fraction, we arrive at an integral which can be evaluated by the use of partial fractions. The potential of the ellipsoid can therefore be found in this case also.

There is an analogous case where the force varies as the inverse square of the distance, and the density of the ellipsoid follows the law $\rho = \mathrm{F}(m^2)/m$. This is discussed by POISSON, in the '*Connaissance des Tems*' for 1837. After alluding to a statement of JACOBI, that the attraction of certain heterogeneous ellipsoids can be expressed in finite terms, he shows how this result follows from the formulæ for the attraction of an ellipsoid which he had previously given in *volume* 13 of the '*Mémoires de l'Académie des Sciences.*' POISSON remarks that the simple case is not the homogeneous ellipsoid, but that in which the density varies inversely as m.

34. Another interesting case is that in which

$$\rho = f(m^2) = \frac{\mathrm{M}}{(1 - m^2)^{\frac{1}{2}}},$$

where, as before, M is any constant. Substituting, we have

$$\mathrm{V}' = \frac{2\pi}{3}\, abc \int_\infty^{\epsilon^2} \frac{-\mathrm{M}\,du}{u\mathrm{Q}_1}\left(1 - \frac{x^2}{a^2+u} - \frac{y^2}{b^2+u} - \frac{z^2}{c^2+u}\right)^{-\frac{1}{2}},$$

where

$$\mathrm{Q}_1{}^2 = (a^2+u)(b^2+u)(c^2+u),$$

and ϵ^2 is the positive root of

$$\frac{x'^2}{a^2+\mu} + \frac{y'^2}{b^2+\mu} + \frac{z'^2}{c^2+\mu} = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{A}).$$

If μ, μ', μ'' are the three roots of equation (A), we may obviously write the potential in the form

$$\mathrm{V}' = \frac{2\pi}{3}\, abc \int_\infty^{\mu} \frac{du}{u} \frac{-\mathrm{M}}{(u-\mu)^{\frac{1}{2}}(u-\mu')^{\frac{1}{2}}(u-\mu'')^{\frac{1}{2}}},$$

where $\mu = \epsilon^2$ has been chosen to be the positive root.

We may generalize this result by taking

$$\rho = \mathrm{M}(1 - m^2)^{n+\frac{1}{2}},$$

the potential of the ellipsoid is then

$$\mathrm{V}' = \frac{2\pi}{3}\, abc. \int_\infty^{\epsilon^2} \frac{-\mathrm{M}\,du}{u\mathrm{Q}_1}\left(1 - \frac{x^2}{a^2+u} - \frac{y^2}{b^2+u} - \frac{z^2}{c^2+u}\right)^{n+\frac{1}{2}}.$$

The first of these two cases has some importance, for when the least axis (say c) is put zero, the ellipsoid reduces to a homogeneous elliptic disc. The surface density is then $\sigma = \mathrm{M}c\pi$. The expression for the potential, the law of force being the inverse fourth, is then

$$\mathrm{V}' = \tfrac{2}{3} ab\sigma \int_\infty^{\mu} \frac{-du}{u\mathrm{Q}_1}\left\{1 - \frac{x^2}{a^2+u} - \frac{y^2}{b^2+u} - \frac{z^2}{u}\right\}^{-\frac{1}{2}}.$$

We may verify this result by comparing it with that given by CAYLEY in the 'Proceedings of the Mathematical Society,' 1875, for the potential of a homogeneous elliptic disc where the force varies as the inverse square of the distance. His result is

$$\mathrm{V} = 2ab\sigma \int_{\infty}^{\mu} \frac{-du}{\mathrm{Q}} \left\{1 - \frac{x^2}{a^2+u} - \frac{y^2}{b^2+u} - \frac{z^2}{u}\right\}^{\frac{1}{2}}.$$

If V_{κ} be the potential of a disc when the law of force is the inverse κ^{th} power, we have

$$\mathrm{V}_{\kappa+2} = -\frac{1}{\kappa+1} \frac{d\mathrm{V}_{\kappa}}{z dz}.$$

This theorem is proved by ROBERTS in the 'Quarterly Journal,' 1881. Effecting the differentiation indicated, the two values of the potential are seen to agree, though the modes of demonstration are totally different. Dr. FERRERS has given an extension of CAYLEY's theorem on the potential of a disc, which, when differentiated, also agrees with the results given above, see the 'Quarterly Journal,' 1875.

35. Let us next consider the potential at an external point when *the force varies as the inverse sixth power.*

Here $\kappa = 6$ and the series in Art 32 is reduced to the two first terms. We have

$$\mathrm{V}' = \frac{2\pi}{5.3} \int_{\infty}^{\epsilon^2} \frac{-abc\, f(m^2) du}{\mathrm{Q}_1 \quad u} \left(\frac{2}{\mathrm{H}u}\right)^2 \left\{\mathrm{K} + \frac{\mathrm{H}u}{4.2}\mathrm{DK}\right\},$$

where H, K, D have the meanings given in the article referred to. We notice that, if p' be the perpendicular from the centre on the tangent plane to the confocal at the attracted point,

$$\mathrm{H} = \frac{1}{p'^2}, \qquad \mathrm{K} = \frac{1}{2u}\frac{d}{du}(\mathrm{H}u^2)$$

$$\mathrm{DK} = 2\left\{\frac{1}{a^2+u} + \frac{1}{b^2+u} + \frac{1}{c^2+u}\right\} = \frac{4}{\mathrm{Q}_1}\frac{d\mathrm{Q}_1}{du}$$

$$\mathrm{Q}_1^{\,2} = (a^2+u)(b^2+u)(c^2+u).$$

Substituting in the expression for V′ we have

$$\mathrm{V}' = \frac{4\pi}{5.3}\int_{\infty}^{\epsilon^2} abc \left\{-\frac{2p'^2}{u^3\mathrm{Q}_1} + \frac{2p'}{u^2\mathrm{Q}_1} - \frac{p'^2}{u^2\mathrm{Q}_1^{\,2}}\frac{d\mathrm{Q}_1}{du}\right\} du \,.\, f(m^2)$$

$$= \frac{4\pi}{5.3}\, abc \int_{\infty}^{\epsilon^2} f(m^2) \frac{d}{du}\left(\frac{p'^2}{u^2\mathrm{Q}_1}\right) du,$$

where ϵ^2 is the positive root of

$$\frac{x'^2}{a^2+\epsilon^2} + \frac{y'^2}{b^2+\epsilon^2} + \frac{z'^2}{c^2+\epsilon^2} = 1.$$

If the ellipsoid is homogeneous, we have $f(m^2) = 1$, and the potential takes the simple form

$$\mathrm{V}' = \frac{4\pi}{5 \, . \, 3} \frac{abc \, p'^2}{\epsilon^4 (a^2 + \epsilon^2)^{\frac{1}{2}} (b^2 + \epsilon^2)^{\frac{1}{2}} (c^2 + \epsilon^2)^{\frac{1}{2}}} \, .$$

It appears that for all points on the same confocal the potential varies as the square on the tangent plane.

36. When the density is not uniform, we have by an integration by parts

$$\mathrm{V}' = \frac{4\pi}{5 \, . \, 3} \, abc \left\{ \frac{f(1) \, p'^2}{\epsilon^4 \, a'b'c'} + \int_{\infty}^{\epsilon^2} \frac{f'(m^2) \, du}{u^2 \mathrm{Q}_1} \right\} ,$$

since $m^2 = 1$ when $u = \epsilon^2$, and by equation (2) of Art. 32 $dm^2/du = -\mathrm{H} = -1/p'^2$. We have also written a', b', c' for the semi-axes of the confocal which passes through the attracted point.

When the density $f(m^2) = \mathrm{M}m$ where M is any constant, we have $f'(m^2) = \mathrm{M}/2m$. We then find (see Art. 33)

$$\mathrm{V}' = \frac{4\pi}{5 \, . \, 3} \, abc \, \frac{\mathrm{M}}{2} \left\{ \frac{p'^2}{\epsilon^4 \, a'b'c'} + \int_{\infty}^{\epsilon^2} \frac{du}{u^2 (\mathrm{A}u^2 + 2\mathrm{B}u + \mathrm{C})^{\frac{1}{2}}} \right. .$$

The integration can be effected by writing $u = 1/v$.

37. The expression for the potential of a homogeneous ellipsoid, when the law of force is the inverse sixth power, having been found freed from all signs of integration, it follows by Jellett's theorem that the potential for any even power greater than 6 can also be expressed in an integrated form (Art. 5). In fact, if V_κ be the potential of the ellipsoid when the force varies as the inverse κ^{th} power, we have when $t = \frac{1}{2}(\kappa - 4)$

$$\mathrm{V} = \frac{8\pi \, abc}{(\kappa - 1) \, \mathrm{L} (\kappa - 3)} \Delta^{t-i} \frac{p'^2}{\epsilon^4 (a^2 + \epsilon^2)^{\frac{1}{2}} (b^2 + \epsilon)^{\frac{1}{2}} (c^2 + \epsilon^2)^{\frac{1}{2}}} \, .$$

Writing λ for ϵ^2 and remembering the expression for the perpendicular p' on the tangent plane to the confocal, this is easily seen to agree with Cayley's formulæ quoted in Art. 31.

38. *When the force varies as the inverse eighth power*, $\kappa = 8$, and the series for the potential is reduced to three terms. We thus have (Art. 32)

$$\mathrm{V}' = \frac{2\pi}{7 \, . \, 5} \, abc \int_{\infty}^{\epsilon^2} \frac{-f(m^2) \, du}{\mathrm{Q}_1 u} \left(\frac{2}{\mathrm{H}u} \right)^4 \left\{ \mathrm{K}^2 + \frac{\mathrm{H}u}{4 \, . \, 4} \mathrm{DK}^2 + \frac{\mathrm{H}^2 u^2}{2^4 . \, 8 \, . \, 3} \mathrm{D}^2\mathrm{K}^2 \right\} .$$

Referring to the values of H, K already written down (Art. 32), we find

$$\mathrm{DK}^2 = 8 \left\{ \frac{a^2 x'^2}{(a^2 + u)^4} + \dots \right\} + 4\mathrm{K} \left\{ \frac{1}{a^2 + u} + \dots \right\}$$

which may be written shortly in the form

$$\mathrm{DK}^2 = -\frac{8}{3}\frac{d\mathrm{K}}{du} - 8\,\mathrm{KQ}_1\frac{d}{du}\frac{1}{\mathrm{Q}_1},$$

where the differentiations with regard to u are partial, the co-ordinates x', y', z' of the attracted point not being varied. Similarly we have

$$\mathrm{D}^2\mathrm{K}^2 = 32\,\mathrm{Q}_1\frac{d^2}{du^2}\frac{1}{\mathrm{Q}_1}.$$

Substituting these in the value of V′, we find, after some reductions,

$$\mathrm{V}' = -\frac{8\pi}{7\,.\,5\,.\,3}\,abc\int_\infty^{\epsilon^2}(fm^2)\,du\,\frac{d}{du}\left\{p'^2\frac{d}{du}\left(\frac{p'^2}{u^3\mathrm{Q}_1}\right)\right\}.$$

If the ellipsoid is homogeneous, the potential at an external point, when the law of force is the inverse eighth, is

$$\mathrm{V}' = -\frac{8\pi}{7\,.\,5\,.\,3}\,abc\,.\,p'^2\frac{d}{du}\left(\frac{p'^2}{u^3\mathrm{Q}_1}\right),$$

where $u = \epsilon^2$.

When written at length this becomes

$$\mathrm{V}' = \frac{4\pi}{7\,.\,5\,.\,3}\,abc\frac{p'^4}{u^3\mathrm{Q}_1}\left[\frac{6}{u} + \frac{1}{a^2+u} + \frac{1}{b^2+u} + \frac{1}{c^2+u} - 4p^2\left\{\frac{x^2}{(a^2+u)^3} + \frac{y^2}{(b^2+u)^3} + \frac{z^2}{(c^2+u)^3}\right\}\right].$$

When the density is not uniform we have, after twice integrating by parts,

$$\mathrm{V}' = -\frac{8\pi}{7\,.\,5\,.\,3}\,abc\left[f(1)\,p'^2\frac{d}{du}\left(\frac{p'^2}{u^3\mathrm{Q}_1}\right) + f'(1)\frac{p'^2}{u^3\mathrm{Q}_1} + \int_\infty^{\epsilon^2}\frac{f''(m)}{u^3\mathrm{Q}_1}\,du\right],$$

where $u = \epsilon^2$ in the integrated parts.

This result takes a very simple form when the density is given by $f(m^2) = m^2$. The unintegrated term is then zero, and the potential reduces to the two first terms.

39. When the law of force is the inverse tenth power of the distance the expression becomes more complicated. Putting

$$\mathrm{U} = \frac{p'^2}{u^2}\left[\frac{u^2}{\mathrm{Q}_1}\frac{d^2}{du^2}\left(\frac{p'}{u}\right)^4 + \left(3u^2\frac{d}{du}\frac{1}{\mathrm{Q}_1} - \frac{4u}{\mathrm{Q}_1}\right)\frac{d}{du}\left(\frac{p'}{u}\right)^4 + \left(2u^2\frac{d^2}{du^2}\frac{1}{\mathrm{Q}_1} - 4u\frac{d}{du}\frac{1}{\mathrm{Q}_1} + \frac{4}{\mathrm{Q}_1}\right)\left(\frac{p'}{u}\right)^4\right]$$

where the differentiations with regard to u are partial, we find, after integrating by parts, that the potential of a homogeneous ellipsoid at an external point is

$$\mathrm{V}' = \frac{8\pi}{9\,.\,7\,.\,5\;\;3}\,abc\,\mathrm{U}.$$

When the density is not uniform the potential is

$$V' = \frac{8\pi}{9.7.5.3} abc \left[f(1) U + \int_{\infty}^{\epsilon^2} \frac{U f'(m^2)}{p'^2} du \right].$$

40. When the index of the law of force is less than 4, the formula in Art. 32 becomes inconvenient because the index of K is negative. To find the potential of a solid ellipsoid at an external point, we must, in this case, integrate the expression given in Art. 26 for the potential of an elementary shell. The integration, however, presents great difficulties except in the case in which the law of force is the inverse square. The method of proceeding is then as follows:

To find the potential of a thin homœoid whose density is $x^f y^g z^h$ and axes (ma, mb, mc) and $((m + dm)\, a,$ &c.) we use the formula of Art. 26. Writing dm/m for μ, ma, mb, mc for a, b, c, $m^2\omega$ for u, and $m^2\lambda$ for ϵ^2, we find that the potential of the shell is

$$\pi abc\, dm^2 \int_{\lambda}^{\infty} \Sigma \frac{(m^2 - 1 + R)^n \omega^n D^n}{L(n)\, L(n)\, 2^{2n}} \left(\frac{a^2 x}{a^2 + \omega}\right)^f \left(\frac{b^2 y}{b^2 + \omega}\right)^g \left(\frac{c^2 z}{c^2 + \omega}\right)^h \frac{d\omega}{Q_1},$$

where R, D have the meaning given to them in Art. 22, except that ω replaces u, and Σ implies summation from $n = 0$ onwards.

To deduce the potential of the heterogeneous ellipsoid we integrate this from $m^2 = 0$ to 1.

We notice that

$$m^2 = \frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} + \frac{z^2}{c^2 + \lambda},$$

let μ be the value of λ when $m = 1$, so that $a^2 + \mu$, $b^2 + \mu$, $c^2 + \mu$ are the squared semi-axes of that confocal to the boundary of the solid ellipsoid which passes through the attracted point, viz., x, y, z. Then, as m^2 passes from 0 to 1, λ passes from ∞ to μ.

Since $R = 1 - m^2$ when $\omega = \lambda$, we have

$$\frac{d}{dm^2} \int_{\lambda}^{\infty} \frac{(m^2 - 1 + R)^{n+1}}{n + 1} f(\omega)\, d\omega = 0 + \int_{\lambda}^{\infty} \frac{d}{dm^2} \frac{(m^2 - 1 + R)^{n+1}}{n + 1} f(\omega)\, d\omega$$
$$= \int_{\lambda}^{\infty} (m^2 - 1 + R)^n f(\omega)\, d\omega.$$

We shall now integrate this equation from $m^2 = 0$ to 1, *i.e.*, $\lambda = \infty$ to μ. The left side becomes

$$\int_{\mu}^{\infty} \frac{R^{n+1}}{n + 1} f(\omega)\, d\omega - \int_{\infty}^{\infty} \frac{(R - 1)^{n+1}}{n + 1} f(\omega)\, d\omega.$$

But R and $f(\omega)$ are both zero for all infinite values of ω, hence

$$\int_{\mu}^{\infty} \frac{R^{n+1}}{n + 1} f(\omega)\, d\omega = \int^{1} dm^2 \int_{\lambda}^{\infty} (m^2 - 1 + R)^n f(\omega)\, d\omega.$$

The potential of a solid ellipsoid whose density is $x^f y^g z^h$ at an external point is therefore

$$V' = \pi abc \int_{\mu}^{\infty} \Sigma \frac{R^{n+1}\omega^n D^n}{L\,(n+1)\,L\,(n)\,2^{2n}} \left(\frac{a^2 x}{a^2+\omega}\right)^f \left(\frac{b^2 y}{b^2+\omega}\right)^g \left(\frac{c^2 z}{c^2+\omega}\right)^h \frac{d\omega}{Q_1}.$$

This agrees with Mr. DYSON's result, 'Quarterly Journal,' 1891.

41. When the force varies as an odd power of the distance, the analysis is different from that given above. There are two cases, according as the inverse index κ is negative or positive and greater than unity.

The former case is the easier, and ultimately depends on integrals resembling those used in finding moments of inertia, but of a higher power. It will be convenient to take this case first.

The latter case is the more interesting as we cannot suppose that the force can vary as a direct power of the distance except within a limited space. The integrals which present themselves in this case are similar to those used in the ordinary theory when the force varies as the inverse square. On account of their complexity, it will be useful to begin this part of the discussion with the simpler case, in which the force varies as the inverse cube, and then to proceed to the more general law of force.

Though we have here distinguished the cases according as the index κ is negative or positive, this is only to obtain finite results. The expression obtained for the potential when κ is an odd negative integer, may be applied to the case in which κ is even or odd, positive or negative, but the expression then takes the form of an infinite series.

42. Before proceeding to this formula, it will be useful to notice two other methods of arriving at the same potential.

Mr. W. D. NIVEN has given in the 'Phil. Trans.' for 1879, a useful formula resembling the one used in Art. 43. Let U be any function of the coordinates (ξ, η, ζ) of a point on an ellipsoid. Taking the integrations over the whole surface, we have

$$\iint Up\,dS = 4\pi abc\left(1 + \frac{\nabla}{L\,(3)} + \frac{\nabla^2}{L\,(5)} + \ldots\right) U$$

where

$$\nabla = a^2\,d^2/d\xi^2 + b^2\,d^2/d\eta^2 + c^2\,d^2/d\zeta^2,$$

and (ξ, η, ζ) are to be put zero after the differentiations are performed. There is a similar expression when the integration extends throughout the volume of the ellipsoid, and the strata of equal density are similar ellipsoids. By giving U different forms, he obtains the values of some useful integrals.

To apply this to find the potential of an ellipsoidal shell at a point (x, y, z) for the law of the inverse square, he puts

$$U = \{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2\}^{\frac{1}{2}}.$$

He notices that the differentiations with regard to (ξ, η, ζ) can be changed into differentiations with regard to (x, y, z), and that, therefore, ξ, η, ζ may be put equal to zero before instead of after differentiation.

Dr. HOBSON has given another method in the 'Proceedings of the London Mathematical Society,' vol. 24, 1893. He finds the expansions in series of the potentials of ellipsoidal shells, solid ellipsoids and discs of variable density, the law of force being any given function of the distance. Taking the case of a homœoid, he first expands the surface density ρ in a series of spherical harmonics, say

$$\rho/p = \Sigma \mathrm{Y}_n (x/a,\ y/b,\ z/c)$$

where p is the perpendicular on the tangent plane. The potential due to a surface distribution on the ellipsoid represented by any one of these terms, when the law of force is $-\phi'(\mathrm{R})$, is then given by the equation

$$\iint \mathrm{Y}_n \phi(\mathrm{R})\, p\, d\sigma$$
$$= 4\pi abc(-1)^n \frac{2^n \mathrm{L}(n)}{\mathrm{L}(2n+1)} \left\{1 + \frac{\Delta}{2(2n+3)} + \&\mathrm{c.}\right\} \mathrm{Y}_n\left(a\frac{d}{dx}, b\frac{d}{dy}, c\frac{d}{dz}\right)\phi(r)$$

where $r^2 = x^2 + y^2 + z^2$, R is the distance of the attracted point from the element $p\, d\sigma$, and

$$\Delta = a^2\, d^2/dx^2 + b^2\, d^2/dy^2 + c^2\, d^2/dz^2.$$

This series terminates when $\phi(\mathrm{R})$ is an integral rational function of R, and in other cases the potential is given by an infinite series.

43. *To find the potential of a thin homogeneous homœoid at an internal or external point* P *when the force varies as the inverse* κ^{th} *power, where* κ *is an odd negative integer.*

Let x, y, z be the coordinates of the attracted point P, ξ, η, ζ those of any elementary mass $\mu p\, d\sigma$ of the homœoid. Writing $1 - \kappa = 2t$ to avoid negative quantities, we see that the potential of the ellipsoidal shell is

$$\mathrm{V} = \frac{\mu}{-2t}\iint p\, d\sigma\, [(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2]^t.$$

Let A, B, C, as before, be d/dx, d/dy, d/dz; then the potential is

$$\mathrm{V} = \frac{\mu}{-2t}\iint p\, d\sigma e^{-\mathrm{A}\xi - \mathrm{B}\eta - \mathrm{C}\zeta}(x^2 + y^2 + z^2)^t.$$

We shall now expand the exponential and integrate the several terms over the whole surface of the ellipsoid, x, y, z being constant. The terms of the expansion with odd exponents need not be considered, as after integration they are clearly zero. The integral for any even power is best obtained from the corresponding integral for a sphere by the method of projections. We have

$$\iint p\, d\sigma \frac{(A\xi + B\eta + C\zeta)^{2s}}{L(2s)} = 4\pi abc \frac{(a^2A^2 + b^2B^2 + c^2C^2)^s}{L(2s+1)},$$

where s is a positive integer.

Let $\Delta = a^2\, d^2/dx^2 + b^2\, d^2/dy^2 + c^2\, d^2/dz^2$.

Let the mass $4\pi\mu abc = M$. The potential then takes the simple form

$$V = -\frac{M}{2t}\left\{1 + \frac{\Delta}{L(3)} + \frac{\Delta^2}{L(5)} + \ldots + \frac{\Delta^s}{L(25+1)} + \ldots\right\}(x^2 + y^2 + z^2)^t.$$

Here $2t = 1 - \kappa$, so that the *series is finite* if the index of the force is an odd negative integer. The expansion is, however, clearly true whether t is integral or not, provided only the series is convergent.

44. *To find the potential of a heterogeneous homœoid when κ is an odd negative integer.*

Let the law of density be $\rho = \xi^f\eta^g\zeta^h$. Following the same reasoning as before, we have

$$V = -\frac{\mu}{2t}\iint p\, d\sigma e^{-A\xi - B\eta - C\zeta}(x^2 + y^2 + z^2)^t\, \xi^f\eta^g\zeta^h.$$

This is the same as

$$V = -\frac{\mu}{2t}\iint p\, d\sigma \left(-\frac{d}{dA}\right)^f\left(-\frac{d}{dB}\right)^g\left(-\frac{d}{dC}\right)^h e^{-A\xi - B\eta - C\zeta}(x^2 + y^2 + z^2)^t.$$

Making the same expansion, we have

$$V = -\frac{\mu}{2t}\left(-\frac{d}{dA}\right)^f\left(-\frac{d}{dB}\right)^g\left(-\frac{d}{dC}\right)^h 4\pi abc\, \Sigma \frac{(A^2a^2 + B^2b^2 + C^2c^2)^s}{L(2s+1)} r^{2t}.$$

When the law of density is $\rho = \phi(\xi, \eta, \zeta)$, we have

$$V = -\frac{4\pi\mu abc}{2t}\phi\left(-\frac{d}{dA}, -\frac{d}{dB}, -\frac{d}{dC}\right)\left\{1 + \frac{A^2a^2 + B^2b^2 + C^2c^2}{L(3)} + \ldots\right\} r^{2t},$$

where $r^2 = x^2 + y^2 + z^2$, A, B, C stand for d/dx, d/dy, d/dz, and $2t = 1 - \kappa$. The series is finite when κ is an odd negative integer.

We may also deduce this expression from the series given in NIVEN'S memoir.

Putting

$$U = \xi^f \eta^g \zeta^h \psi\{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2\},$$

the general term of his expansion is

$$\frac{4\pi abc}{L(2i+1)}\left(a^2\frac{d^2}{d\xi^2} + b^2\frac{d^2}{d\eta^2} + c^2\frac{d^2}{d\zeta^2}\right)^i U.$$

Let $\delta = d/d\xi$ and $A = d/dx$. Using the generalized theorem of LEIBNITZ for the differential coefficient of a product, and putting $\xi = 0$, after differentiation we have

$$\phi(\delta)\,\xi^f\psi\{(\xi-x)^2 + \ldots\} = \left(\frac{d}{d\delta}\right)^f \phi(\delta)\,.\,\psi\{(\xi-x)^2 + \ldots\} = \left(-\frac{d}{dA}\right)^f \phi(-A)\,\psi(x^2 + \ldots),$$

the last term being obtained by changing the subject of differentiation from ξ to x. Treating η^g and ζ^h in the same way, we have

$$\phi(\nabla)\,U = (-d/dA)^f(-d/dB)^g(-d/dC)^h\,\phi(A^2a^2 + B^2b^2 + C^2c^2)\,\psi(x^2 + y^2 + z^2).$$

When we replace $\phi(\nabla)$ by ∇^i and ψ by a t^{th} power, this becomes virtually identical with the general term of the expansion found above for V.

45. As an example, let the law of force be the direct cube of the distance, and let the density be $\rho = \xi$. We have

$$V = \frac{4\pi\mu abc}{4}\frac{d}{dA}\left\{r^4 + \frac{A^2a^2 + \ldots}{L(3)}r^4 + \frac{(A^2a^2 + \ldots)^2}{L(5)}r^4\right\}$$

$$= \pi\mu abc\left\{\frac{2a^2}{L(3)}Ar^4 + \frac{4a^2}{L(5)}(A^2a^2 + B^2b^2 + C^2c^2)\,Ar^4\right\}.$$

Remembering the meaning of A, B, C, this is clearly

$$V = \frac{4\pi\mu}{3\,.\,5}a^3bc\,.\,x\,(5r^2 + 3a^2 + b^2 + c^2).$$

The potential for a homogeneous homœoid for the same law of force is

$$V = -\frac{\pi\mu}{3}abc\,\{3r^4 + 4(a^2x^2 + b^2y^2 + c^2z^2) + 2(a^2 + b^2 + c^2)\,r^2\},$$

a constant term being omitted.

46. We may use the same method *to find the potential of a solid ellipsoid when* κ *is any odd negative integer.*

Proceeding as before, we have

$$V = -\frac{1}{2t}\int dv\,\{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2\}^t = -\frac{1}{2t}\int dv\, e^{-A\xi - B\eta - C\zeta}(x^2 + y^2 + z^2)^t$$

where *dv* is an element of volume. Either by DIRICHLET's theorem, or by projections from a sphere, we have

$$\int dv\,\frac{(A\xi + B\eta + C\zeta)^{2s}}{L\,(2s)} = \frac{4\pi abc}{2s+3}\,\frac{(A^2a^2 + B^2b^2 + C^2c^2)^s}{L\,(2s+1)}.$$

It immediately follows that for a *homogeneous solid ellipsoid* the potential is

$$V = -\frac{4\pi abc}{2t}\cdot\left\{\tfrac{1}{3} + \frac{A^2a^2 + \ldots}{L\,(3)\,.\,5} + \frac{(A^2a^2 + \ldots)^2}{L\,(5)\,.\,7} + \ldots\right\}(x^2 + y^2 + z^2)^t$$

where A, B, C stand for d/dx, d/dy, d/dz as before.

For a *heterogeneous ellipsoid* whose density is $\rho = \phi\,(\xi, \eta, \zeta)$, we have

$$V = -\frac{4\pi abc}{2t}\,\phi\left(-\frac{d}{dA}, -\frac{d}{dB}, -\frac{d}{dC}\right)\left\{\tfrac{1}{3} + \frac{A^2a^2 + \ldots}{L\,(3)\,.\,5} + \ldots\right\}(x^2 + y^2 + z^2)^t.$$

Both these expressions are finite where *t* is positive, *i.e.*, the index of the power is a negative odd integer.

47. *This method may be applied to other bodies besides the ellipsoid*, provided only the value of the fundamental integral

$$I = \int dv\,(A\xi + B\eta + C\zeta)^s$$

is known for that body. There are many bodies for which this integral is known, such as triangles, quadrilaterals, tetrahedra, and so on (see a paper by the author in the 'Quarterly Journal,' 1886). As an example, consider the tetrahedron, let (ξ_1, η_1, ζ_1), (ξ_2, η_2, ζ_2), &c., be the coordinates of the four corners, let $\alpha = A\xi_1 + B\eta_1 + C\zeta_1$, $\beta = A\xi_2 + B\eta_2 + C\zeta_2$, &c., so that α, β, γ, δ are proportional to the distances of the four corners from the plane of reference. The fundamental integral I is then equal to the product of the volume of the tetrahedron by the arithmetic mean of the homogeneous products of α, β, γ, δ of the s^{th} degree.

By choosing a proper origin and axes the expressions α, β, γ, δ may be simplified to suit any particular case. Using this expression for the fundamental integral the potential of a tetrahedron for any direct odd law of force can be deduced from the differential coefficients of r^{2t}.

48. When the index κ is not very large, we can often write down the potential

by using the theory of equimomental points. We have already seen that the potential of any body is

$$V = -\frac{\mu}{2t}\int dv\,\{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2\}^t.$$

Expanding the t^{th} power, we see that the highest powers of (ξ, η, ζ) which occur are in the term independent of x, y, z. These only give a constant term to the potential. Neglecting these terms we see that if two bodies are such that $\int \xi^f \eta^g \zeta^h\, dv$ is the same for both, when $f + g + h$ is $=$ or $< 2t - 1$, these bodies have equal potentials at all points. Now $2t - 1$ is the index of the force taken positively. Hence, if two bodies are equimomental for all powers up to the κ^{th}, they are also equipotential, except for a constant.

There are many bodies which may be replaced by equimomental points. For example, we can show that, up to cubes inclusive, a solid ellipsoid is equimomental to six particles placed at the extremities of any set of conjugate diameters, each equal to one-tenth of the mass of the ellipsoid together with a seventh particle placed at the centre of gravity equal to two-fifths of the mass of the ellipsoid. Placing the six particles at the extremities of the axes, we can write down the potential of the ellipsoid when the force varies as the cube of the distance. We clearly have

$$-4V = \tfrac{1}{10}M\,\{x^2 + y^2 + (z - c)^2\}^2 + \ldots + \tfrac{2}{5}M\,(x^2 + y^2 + z^2)^2,$$

therefore, omitting a constant,

$$V = -\tfrac{1}{20}M\,\{5r^4 + 2\,(a^2 + b^2 + c^2)\,r^2 + 4\,(a^2x^2 + b^2y^2 + c^2z^2)\}.$$

In the same way a tetrahedron, up to cubes inclusive, is equimomental to eight points. We collect nine-fortieths of the volume at the centre of gravity of each face and one-fortieth at each corner. The potential of the tetrahedron when the force varies as the direct cube of the distance can therefore be immediately written down. Let the centre of gravity G of the tetrahedron be the origin, and let the axis of z pass through the attracted point P. Then $GP = z$, and (ξ_1, η_1, ζ_1), (ξ_2, η_2, ζ_2), &c., are the coordinates of the four corners. The potential is easily seen to be

$$V = -\frac{M}{4}\left\{z^4 + \frac{z^2}{10}\,\Sigma\,(\xi^2 + \eta^2 + 3\zeta^2) - \frac{z}{15}\,\Sigma\,(\xi^2 + \eta^2 + \zeta^2)\zeta\right\}.$$

49. *To find the potential of a thin homœoid at an internal point P when the law of attraction is the inverse cube.*

Referring to Art. 3, the potential V for an odd inverse κ^{th} power is

$$V = \frac{\mu}{\kappa - 1}\iint \frac{r_1^{3-\kappa} + r_2^{3-\kappa}}{r_1 - r_2}\,D^2\,d\omega.$$

When $\kappa = 3$, this becomes

$$\mathrm{V}_3 = \frac{\mu}{2}\iint \frac{2\mathrm{D}^2}{r_1 - r_2}\,d\omega.$$

Taking the standard quadratic (3) of Art. 1, we have

$$\frac{r_1 - r_2}{2\mathrm{D}^2} = \left\{\mathrm{F}^2 + \frac{\mathrm{E}}{\mathrm{D}^2}\right\}^{\frac{1}{2}} = \{(\alpha lx + \beta my + \gamma nz)^2 + \mathrm{E}\,(\alpha l^2 + \beta m + \gamma n^2)\}^{\frac{1}{2}}.$$

The potential required is, therefore,

$$\mathrm{V} = \tfrac{1}{2}\mu\iint r'\,d\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

where r' is the radius vector of the auxiliary ellipsoid

$$(\alpha x\mathrm{X} + \beta y\mathrm{Y} + \gamma z\mathrm{Z})^2 + \mathrm{E}\,(\alpha\mathrm{X}^2 + \beta\mathrm{Y}^2 + \gamma\mathrm{Z}^2) = 1 \quad . \quad . \quad . \quad . \quad (2),$$

which may also be written in the form

$$(1 - \beta y^2 - \gamma z^2)\,\alpha\mathrm{X}^2 + \ldots + 2\beta\gamma\,yz\,\mathrm{YZ} + \ldots = 1 \quad . \quad . \quad . \quad . \quad (3).$$

To find the integral we require the following lemma :—

Lemma. *Let r' be the radius vector of an ellipsoid whose semi-axes are a' b', c'; r'' the radius vector of the polar reciprocal with regard to a concentric sphere of radius m. Then*

$$\iint r'\,d\omega = \frac{a'b'c'}{m^4}\iint r''^2\,d\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

To prove this, we notice that

$$\iint r'\,d\omega = \iiint dr'\,d\omega = \iiint \frac{dx\,dy\,dz}{x^2 + y^2 + z^2},$$

the integration extending throughout the volume of the ellipsoid. Writing $x = a'\xi$, $y = b'\eta$, $z = c'\zeta$, and integrating throughout the volume of a sphere of radius unity, this becomes

$$\iiint \frac{a'b'c'\,d\xi\,d\eta\,d\zeta}{a'^2\xi^2 + b'^2\eta^2 + c'^2\zeta^2} = a'b'c'\iiint \frac{dr\,d\omega}{c'^2\cos^2\theta + \sin^2\theta\,(b'^2\cos^2\phi + a'^2\sin^2\phi)}$$

by changing to polar coordinates. After integrating with regard to r from 0 to 1, it becomes obvious that the double intregral remaining is the value of $\iint r''^2\,d\omega$ for an ellipsoidal surface whose semi-axes are $1/a'$, $1/b'$, $1/c'$. Introducing the m to preserve the dimensions, the theorem follows at once.

50. It follows from this lemma that the potential of the homœoid is

$$V_3 = \frac{\mu a'b'c'}{2m^4} \iint r''^2 \, d\omega \quad \ldots \ldots \ldots \ldots \quad (5),$$

where a', b', c' are the semi-axes of the auxiliary ellipsoid (3).

This double integral occurs in the ordinary expression for the potential at the centre of a homœoid when the law of force is the inverse square, and its axes are m^2/a', m^2/b', m^2/c'. Taking its value from that investigation, we have

$$\iint r''^2 \, d\omega = \frac{2\pi m^4}{a'b'c'} \int_0^\infty \frac{du}{Q'}$$

where

$$Q'^2 = (\alpha' + u)(\beta' + u)(\gamma' + u) \quad \ldots \ldots \ldots \quad (6),$$

and α', β', γ' are the squared reciprocals of the semi-axes of the auxiliary ellipsoid. We, therefore, have

$$V_3 = \pi\mu \int_0^\infty \frac{du}{Q'} \quad \ldots \ldots \ldots \ldots \quad (7).$$

The axes of the auxiliary ellipsoid are given by its discriminating cubic. Let R be the leading letter of the cubic, so that the three values of R are α', β', γ'. As we want the product represented above by Q'^2, we change the cubic into another having S for its leading letter by writing $R + u = S$. The last term of this second cubic is the product required. In this way we find

$$Q'^2 = \begin{vmatrix} (1-\beta y^2-\gamma z^2)\,\alpha+u, & \alpha\beta xy, & \alpha\gamma xz \\ \alpha\beta xy, & (1-\gamma z^2-\alpha x^2)\,\beta+u, & \beta\gamma yz \\ \alpha\gamma xz, & \beta\gamma yz, & (1-\alpha x^2-\beta y^2)\,\gamma+u \end{vmatrix} \quad . \; . \quad (8).$$

Substituting either this determinant or the product (6) in (7), we have an expression for V_3.

If M be the mass of the homœoid, the potential at an internal point when the law of force is the inverse cube is given by

$$V_3 = \frac{M}{4abc} \int_0^\infty \frac{du}{Q'} \quad \ldots \ldots \ldots \ldots \quad (9).$$

We may put this theorem into a geometrical form by remembering that in the equation (5) the double integral occurs in finding the potential of a homœoid when the law of force is the inverse square. We have, therefore, the following theorem.

Let the potential of a thin homœoid of mass M, and semi-axes a, b, c, be V_3, when the force varies as the inverse cube. Let the potential of a second thin homœoid of mass M'' at an internal point be V_2'', when the force varies as the inverse square. If the bounding surface of the second homœoid be the polar reciprocal with regard to a sphere of radius m of the auxiliary ellipsoid of the first, then

$$\frac{V_3}{M} = \frac{V_2''}{M''}\,\frac{m^2}{2abc}.$$

The potential of the homœoid at an external point P′ is the same as that of a thin confocal homœoid passing through P′ of equal volume and density at the internal corresponding point P.

51. To discover the geometrical relation of the auxiliary ellipsoid to the homœoid whose potential at P is required, we examine the form (2) of Art. 49. Let OP produced cut the homœoid in C, and let OA, OB, OC, be a set of conjugate diameters. Then, if these lines are taken as a new set of axes, the equations of the homœoid and its auxiliary ellipsoid respectively become

$$\frac{x'^2}{OA^2} + \frac{y'^2}{OB^2} + \frac{z'^2}{OC^2} = 1,$$

$$\frac{z'^2}{OP^2} + \left(\frac{x'^2}{OA^2} + \frac{y'^2}{OB^2} + \frac{z'^2}{OC^2}\right)E = 1.$$

The two are thus referred to a common set of conjugate diameters. If we take A′, B′, C′, in OA, OB, OC, respectively, so that

$$\left(\frac{OA}{OA'}\right)^2 = \left(\frac{OB}{OB'}\right)^2 = 1 - \frac{OP^2}{OC^2}, \qquad \left(\frac{OC}{OC'}\right)^2 = 1 + \left(\frac{OC}{OP}\right)^2 - \left(\frac{OP}{OC}\right)^2,$$

the ellipsoid having OA′, OB′, OC′ for conjugate diameters is the auxiliary ellipsoid.

52. *To find the potential of a homogeneous thin homœoid at an internal point* P, *when the law of attraction is any inverse odd power greater than* 3.

Putting $u = 1/r$, the required potential, by Art. 3, becomes

$$V = \frac{\mu}{\kappa - 1}\iint \frac{u_1^{\kappa-3} + u_2^{\kappa-3}}{u_1 - u_2}\,\frac{d\omega}{E}.$$

Referring to the standard quadratic (3) of Art. 1, and putting $p = 2F/E$, $q = 1/ED^2$, we have by the theorem already quoted in Art. 8,

$$u_1^n + u_2^n = p^n + np^{n-2}q + \ldots + n\,\frac{L(n-f-1)}{L(f)\,L(n-2f)}\,p^{n-2f}q^f + \ldots$$

This series stops at the first negative power of p and it is necessary that n should be greater than zero. If $n = 0$, the left hand side must be halved. In our case we put $n = \kappa - 3$, and since κ is odd the series has $\frac{1}{2}n + 1$, *i.e.*, $\frac{1}{2}(\kappa - 1)$ terms.

As in Art. 49, we have

$$u_1 - u_2 = \frac{2}{\mathrm{E}}\left(\mathrm{F}^2 + \frac{\mathrm{E}}{\mathrm{D}^2}\right)^{\frac{1}{2}} = \frac{2}{\mathrm{E}}\frac{1}{r'},$$

where r' is the radius vector of the same auxiliary ellipsoid as that used for the inverse cube. In this way we find

$$\mathrm{V} = \frac{\mu}{2(\kappa - 1)}\iint r'\,d\omega\,\Sigma\,\frac{(\kappa - 3)\,\mathrm{L}(\kappa - f - 4)}{\mathrm{L}(f)\,\mathrm{L}(\kappa - 2f - 3)}\left(\frac{2\mathrm{F}}{\mathrm{E}}\right)^{\kappa - 3 - 2f}\left(\frac{1}{\mathrm{ED}^2}\right)^{f},$$

where Σ implies summation from $f = 0$ to $\frac{1}{2}(\kappa - 3)$.

In these integrations l, m, n are the direction cosines of the moving radius vector r', and x, y, z are the coordinates of the fixed attracted point P. Let $\lambda = \alpha x$, $\mu = \beta y$, $\nu = \gamma z$, for the sake of brevity, then λ, μ, ν are constant during the integration.

Let

$$\mathrm{G} = \iint r'\,d\omega\,(l\lambda + m\mu + n\nu)^{2s}$$

where $2s = \kappa - 3$. As already explained, the integration extends over all positions of the moving radius vector round P.

Remembering that $\mathrm{F} = l\lambda + m\mu + n\nu$, we see that each term of the series for V may be derived from the preceding by a uniform process. Let

$$\Delta = \alpha\,d^2/d\lambda^2 + \beta\,d^2/d\mu^2 + \gamma\,d^2/d\nu^2.$$

After some slight numerical reductions we find

$$\mathrm{V} = \frac{\mu}{\kappa - 1}\frac{1}{2^{\kappa - 2}}\left[\left(\frac{4}{\mathrm{E}}\right)^{\kappa - 3}\mathrm{G} + \left(\frac{4}{\mathrm{E}}\right)^{\kappa - 4}\Delta\mathrm{G}\frac{1}{\kappa - 4} + \left(\frac{4}{\mathrm{E}}\right)^{\kappa - 5}\frac{\Delta^2\mathrm{G}}{\mathrm{L}(2)}\frac{1}{(\kappa - 4)(\kappa - 5)} + \ldots \right.$$
$$\left. + \left(\frac{4}{\mathrm{E}}\right)^{\kappa - 3 - f}\frac{\Delta^f\mathrm{G}}{\mathrm{L}(f)}\cdot\frac{\mathrm{L}(\kappa - f - 4)}{\mathrm{L}(\kappa - 4)} + \ldots\right]$$

where f varies from 0 to $\frac{1}{2}(\kappa - 3)$, so that the series for the potential has $\frac{1}{2}(\kappa - 1)$ terms, κ being the inverse index of the law of force.

When G has been expressed as a function of λ, μ, ν, and the axes of the homœoid whose attraction is required, this formula will give the potential at all internal points.

53. The value of G has been given by the double integral

$$\mathrm{G} = \iint r'\,d\omega\,(l\lambda + m\mu + n\nu)^{2s}$$

where $2s = \kappa - 3$. We shall now show that it can be reduced to a single integral.

Let the moving radius vector r' be called OR. The direction cosines of OR, when referred to the axes of the attracting homœoid, are l, m, n; let its direction cosines referred to the axes of the auxiliary ellipsoid be l', m', n'. Let the fixed straight line whose direction cosines are proportional to λ, μ, ν, when referred to the axes of the homœoid be called OL. Let its direction cosines, when referred to the axes of the auxiliary ellipsoid, be λ', μ', ν'. Then

$$\mathrm{G} = \iint r'\, d\omega\, (\cos \mathrm{ROL})^{2s} (\lambda^2 + \mu^2 + \nu^2)^s.$$

Proceeding as in the lemma (Art. 49) we have

$$\iint r'\, d\omega\, (\cos \mathrm{ROL})^{2s} = \iiint dr'\, d\omega\, (\cos \mathrm{ROL})^{2s},$$

the integration extending throughout the volume of the auxiliary ellipsoid. If x', y', z' be the coordinates of a point on OR referred to the axes of the auxiliary ellipsoid, this is the same as

$$\iiint \frac{dx'\, dy'\, dz'}{r'^2} \left(\frac{\lambda' x' + \mu' y' + \nu' z'}{r'}\right)^{2s}.$$

Putting $x' = a'\xi$, $y' = b'\eta$, $z' = c'\zeta$, where a', b', c' are the semi-axes of the auxiliary ellipsoid, and transforming back to polar coordinates, we have

$$a'b'c' \iiint \frac{r'^2\, dr'\, d\omega\, (\lambda' a'\xi + \mu' b'\eta + \nu' c'\zeta)^{2s}}{(a'^2\xi^2 + b'^2\eta^2 + c'^2\zeta^2)^{s+1}},$$

the integration extending throughout the volume of a sphere of radius unity. Putting $\xi = r'l'$, $\eta = r'm'$, $\zeta = r'n'$, and integrating from $r' = 0$ to 1, we find

$$\iint r'\, d\omega\, (\cos \mathrm{ROL})^{2s} = a'b'c' \iint \frac{(a'\lambda' l' + b'\mu' m' + c'\nu' n')^{2s}\, d\omega}{(a'^2 l'^2 + b'^2 m'^2 + c'^2 n'^2)^{s+1}}.$$

Referring to the lemma proved in Art. 10, and putting $t = 0$, we find that the right-hand side of this equation is

$$\frac{\mathrm{L}\,(2s)}{\mathrm{L}\,(s)\,\mathrm{L}\,(s)} \frac{2\pi}{2^{2s}} \int_0^\infty v^{-\frac{1}{2}}\, dv \left\{\frac{a'^2\lambda'^2}{a'^2 + v} + \frac{b'^2\mu'^2}{b'^2 + v} + \frac{c'^2\nu'^2}{c'^2 + v}\right\}^s \frac{a'b'c'}{(a'^2 + v)^{\frac{1}{2}}\, (b'^2 + v)^{\frac{1}{2}}\, (c'^2 + v)^{\frac{1}{2}}}.$$

Returning to the notation that α', β', γ' (Art. 49) are the squared reciprocals of the axes of the auxiliary ellipsoid, and writing $v = 1/u$, we have

$$\iint r'\, d\omega\, (\cos \mathrm{ROL})^{2s} = \frac{\mathrm{L}\,(2s)}{\mathrm{L}\,(s)\,\mathrm{L}\,(s)} \frac{2\pi}{2^{2s}} \int_0^\infty \mathrm{M}^s \frac{u^s\, du}{\mathrm{Q}'},$$

where

$$Q'^2 = (\alpha' + u)(\beta' + u)(\gamma' + u),$$

$$M = \frac{\lambda'^2}{\alpha' + u} + \frac{\mu'^2}{\beta' + u} + \frac{\nu'^2}{\gamma' + u}.$$

The separate values of the axes $\alpha'\beta'\gamma'$ of the auxiliary ellipsoid may be found by the solution of a cubic, and if this were sufficient, G is here expressed as a single integral. But the product Q′ has been found in Art. 50 as a determinant without requiring the solution of the cubic. We shall now seek for an expression for M of the same kind.

54. Since α', β', γ' are the squared reciprocals of the axes of the auxiliary ellipsoid,

$$S = (\lambda X + \mu Y + \nu Z)^2 + (\alpha X^2 + \beta Y^2 + \gamma Z^2) E = 1 \quad . \quad . \quad . \quad . \quad (\text{I.}),$$

it follows that $\alpha' + u$, $\beta' + u$, $\gamma' + u$ are the squared reciprocals of the axes of

$$S + u(X^2 + Y^2 + Z^2) = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{II.}).$$

Since these quadrics are concyclic, they are also co-axial, and therefore λ', μ', ν' are the direction cosines of OL referred to the axes of the second quadric.

It immediately follows that M is the sum of the squares of the projections of the axes on the straight line OL and is, therefore, equal to the sum of the squares of the projections of any three conjugate diameters on the same straight line.

Let OP_1, OP_2 be any two conjugate semi-diameters of the section $\lambda x + \mu y + \nu z = 0$, and let OP_3 be the third conjugate for the ellipsoid II. Then since OL is perpendicular to the section containing OP_1, OP_2, the sum of the squares of the projections on OL is

$$M = OP_3^2 (\cos P_3OL)^2.$$

Let λ'', μ'', ν'' be the direction cosines of OP_3 referred to the axes of the homœoid. Then by equation (II.)

$$\frac{(\alpha E + u)\lambda''}{\lambda} = \frac{(\beta E + u)\mu''}{\mu} = \frac{(\gamma E + u)\nu''}{\nu} = \rho \text{ (say)},$$

$$\cos P_3OL = \frac{\lambda\lambda'' + \mu\mu'' + \nu\nu''}{(\lambda^2 + \mu^2 + \nu^2)^{\frac{1}{2}}},$$

$$\frac{1}{OP_3^2} = (\lambda\lambda'' + \mu\mu'' + \nu\nu'')(\lambda\lambda'' + \mu\mu'' + \nu\nu'' + \rho).$$

Substituting these values of OP_3 and $\cos P_3OL$, we find, after clearing of fractions,

$$M = \frac{N}{(u + \alpha E)(u + \beta E)(u + \gamma E) + N} \, \frac{1}{\lambda^2 + \mu^2 + \nu^2}$$

where

$$N = \lambda^2 (u + \beta E)(u + \gamma E) + \mu^2 (u + \alpha E)(u + \beta E) + \nu^2 (u + \alpha E)(u + \beta E).$$

If M were written in partial fractions, it must be the same as

$$M = \frac{\lambda'^2}{u + \alpha'} + \frac{\mu'^2}{u + \beta'} + \frac{\nu'^2}{u + \gamma'},$$

as found above. The factors of the denominator must therefore be

$$Q'^2 = (u + \alpha')(u + \beta')(u + \gamma').$$

This is obviously correct since the denominator is the expansion of the determinant

$$Q'^2 = \begin{vmatrix} u + \alpha E + \lambda^2 & \lambda\mu & \lambda\nu \\ \lambda\mu & u + \beta E + \mu^2 & \mu\nu \\ \lambda\nu & \mu\nu & u + \gamma E + \nu^2 \end{vmatrix},$$

which agrees with the determinant already found for Q' in Art. 50, when we substitute $\lambda = \alpha x$, $\mu = \beta y$, $\nu = \gamma z$.

Finally, substituting in the value of G, we have

$$G = \frac{L(2s)}{L(s)\,L(s)} \cdot \frac{2\pi}{2^{2s}} \int_0^\infty \left(\frac{N}{Q'^2}\right)^s \frac{u^s\,du}{Q'}$$

where $2s = \kappa - 3$ and N, Q' have the values given just above. In this way G has been found by a single definite integral in terms of the axes of the attracting homœoid and the coordinates of the attracted point.

55. The potentials of elliptic rings and discs follow from those obtained for an ellipsoidal shell or solid. The transformation may be effected in either of two ways. By making the axis of c equal to zero, the homœoid becomes a disc and the potential at any external or internal point may be deduced from the formulæ of Art. 12. In this case the surface density at any point ξ, η of the disc is proportional to $\rho(1 - \xi^2/a^2 - \eta^2/b^2)^{-\frac{1}{2}}$ where ρ is the density of the homœoid at the point ξ, η, ζ. In this transformation the law of force remains unaltered so that if the formulæ of Art. 12 are used, the index must be an even number.

Another method is to make the axis of c infinite, so that the homœoid becomes a cylinder. Since the potential of an infinite rod, of line density $m\,ds$, at a distance r is $\frac{1}{\kappa - 1}\,\frac{\Gamma\frac{1}{2}(\kappa - 2) . \Gamma(\frac{1}{2})}{\Gamma\frac{1}{2}(\kappa - 1)}\,\frac{1}{r^{\kappa - 2}}$, we see that the potential of the homœoid, when the index of the law of force is κ, becomes proportional to that of an elliptic ring with a law of force whose index is $\kappa - 1$, provided the attracted point lies in the plane of the ring. In this way the potential of a thin elliptic ring, whose line density at the point ξ, η, is proportional to $pf(\xi/a, \eta/b)$, where p is the perpendicular from the centre on the tangent, may be deduced from the formulæ of Art. 12, when the law of force varies as an odd power of the distance.

As these results may be obtained by simple substitution, we shall for the present pass them over and consider a theorem which is derived from neither of these methods.

56. When the force varies as the inverse cube of the distance, it is known that the level surfaces of a homogeneous elliptic disc are confocal quadrics. The potential at any point may then be found in elliptic coordinates by considering the potential at some convenient point of each confocal (see a paper by Mr. WEBB, 'Quarterly Journal,' vol. 14, 1877).

We shall now prove that *if the law of force is the inverse* κ^{th} *power of the distance, and the surface density at* ξ, η *is* $\sigma = (1 - \xi^2/a^2 - \eta^2/b^2)^{\frac{1}{2}(\kappa-3)}$, *the level surfaces are confocal quadrics of the elliptic disc.*

Let P be any point on a confocal, and let the normal at P meet the plane of the disc in G, then PG is the axis of the cone enveloping the disc and having its vertex at P. Draw two consecutive chords through G; let one of these be SGS′ and let $d\theta$ be the angle between them. Then PG makes equal angles with the opposite generators PS, PS′ of the cone.

Take two elements of area at R, R′ on opposite sides of G, such that PG bisects the angle RPR′ as well as the angle SPS′, let GR $= r$, GR′ $= r'$. These elements attract P equally if the surface densities σ, σ' at R, R′ are such that $\dfrac{\sigma r d\theta\, dr}{r^\kappa} = \dfrac{\sigma' r' d\theta\, dr'}{r'^\kappa}$. Supposing this to be true, the resultant attraction of the two elements, and therefore that of the whole disc, will act along the normal PG.

Because PG bisects both the angles SPS′, RPR′, we have by elementary trigonometry

$$\frac{1}{r} - \frac{1}{r'} = \frac{1}{\mathrm{GS}} - \frac{1}{\mathrm{GS'}}$$

$$\frac{\mathrm{SR}\,.\,\mathrm{RS'}}{\mathrm{PR}^2} = \frac{\sin \mathrm{SPR}\,.\,\sin \mathrm{RPS'}}{\sin \mathrm{S}\,.\,\sin \mathrm{S'}} = \frac{\sin \mathrm{S'PR'}\,.\,\sin \mathrm{R'PS}}{\sin \mathrm{S}\,.\,\sin \mathrm{S'}} = \frac{\mathrm{SR'}\,.\,\mathrm{R'S'}}{\mathrm{PR'}^2}.$$

The first of these results shows that $dr/r^2 = dr'/r'^2$. Since PR/GR $=$ PR′/GR′, the second shows that

$$\frac{\sigma}{\sigma'} = \frac{r^{\kappa-3}}{r'^{\kappa-3}} = \left(\frac{\mathrm{SR}\,.\,\mathrm{RS'}}{\mathrm{SR'}\,.\,\mathrm{R'S'}}\right)^{\frac{1}{2}(\kappa-3)}.$$

Produce CR, CR′ to cut the conic in T, T′. If D be the semi-diameter parallel to the chord SGS′, we have by a well-known theorem in conics

$$\frac{\mathrm{SR}\,.\,\mathrm{RS}'}{\mathrm{D}^2} = 1 - \frac{\mathrm{CR}^2}{\mathrm{CT}^2} = 1 - \frac{\xi^2}{a^2} - \frac{\eta^2}{b^2}$$

where ξ, η are the coordinates of R. There is a corresponding equation for the product SR′.R′S′. The necessary relation between σ, σ', is therefore satisfied by the given law of density. This proof does not require κ to be an integer.

57. To find the potential of the disc, we place the attracted point P on the axis of the disc. If h be its distance from the centre C, the potential at P is

$$\mathrm{V}_\kappa = \frac{1}{\kappa - 1}\iint \frac{d\xi\, d\eta\, (1 - \xi^2/a^2 - \eta^2/b^2)^{\frac{1}{2}(\kappa-3)}}{(\xi^2 + \eta^2 + h^2)^{\frac{1}{2}(\kappa-1)}} \quad . \; . \; . \; . \; . \; . \; . \quad (1),$$

writing $\xi = ax$, $\eta = by$ thus becomes

$$\mathrm{V}_\kappa = \frac{ab}{\kappa - 1}\iint \frac{r d\theta\, dr\, (1 - r^2)^{\frac{1}{2}(\kappa-3)}}{\{(a^2\cos^2\theta + b^2\sin^2\theta)\, r^2 + h^2\}^{\frac{1}{2}(\kappa-1)}} \quad . \; . \; . \; . \quad (2),$$

the limits being $r = 0$ to 1, $\theta = 0$ to 2π. This may also be written in the form

$$\mathrm{V}_\kappa = \frac{ab}{\kappa - 1}\iint \frac{r d\theta\, dr\, (1 - r^2)^{\frac{1}{2}(\kappa-3)}}{\{(a'^2\cos^2\theta + b'^2\sin^2\theta)\, r^2 + h^2(1 - r^2)\}^{\frac{1}{2}(\kappa-1)}} \quad . \; . \; . \quad (3),$$

where $a'^2 = a^2 + h^2$, $b'^2 = b^2 + h^2$, so that a', b', h are the axes of the confocal through the attracted point. When $\kappa = 3$ the integral is easily found; we have

$$\mathrm{V}_3 = \pi \log \frac{a'b + ab'}{h\,(a + b)}\,.$$

We see from the third form above that

$$\frac{\mathrm{V}_\kappa}{ab} = \frac{1}{\mathrm{L}\,\frac{1}{2}(\kappa - 1)} \cdot (-1)^{\frac{1}{2}(\kappa-3)} \left(\frac{d}{dh^2}\right)^{\frac{1}{2}(\kappa-3)} \frac{\mathrm{V}_3}{ab}\,,$$

the differentiation being effected on the supposition that a', b' are constant.

When the disc is circular, the first form for V_κ becomes

$$\mathrm{V}_\kappa = \frac{1}{\kappa - 1}\,\frac{1}{a^{\kappa-3}}\iint \frac{r\, d\theta\, dr\, (a^2 - r^2)^{\frac{1}{2}(\kappa-3)}}{(r^2 + h^2)^{\frac{1}{2}(\kappa-1)}}\,.$$

Putting $a^2 - r^2 = (a^2 + h^2)\sin^2\phi$, this immediately reduces to

$$\mathrm{V}_\kappa = \frac{2\pi}{\kappa - 1}\,\frac{1}{a^{\kappa-3}}\int_0^\gamma (\tan\phi)^{\kappa-2}\, d\phi,$$

where γ is the angle subtended by any radius of the disc at the intersection of the confocal with the axis of the disc. The integration can be easily performed by a formula of reduction.

58. Conversely, we may inquire whether this is the only law of density which makes the level surfaces of the disc to be confocals. Taking the law of force to be the inverse κ^{th} power of the distance where $\kappa > 3$, let P be any point on the confocal immediately adjacent to the disc. The potential at P is due to the matter of the disc in its immediate neighbourhood. Let σ be the surface density at this point, z the infinitely small distance of P from the disc. The potential at P of an infinite plane of this surface density is

$$\int \frac{2\pi r\,dr}{(\kappa - 1)\,r^{\kappa - 1}}\,\sigma = \frac{2\pi\sigma}{(\kappa - 1)(\kappa - 3)}\,\frac{1}{z^{\kappa - 3}}.$$

Hence the potential cannot be constant at all points of this confocal unless σ varies as $z^{\kappa - 3}$, *i.e.*, σ varies as $(1 - \xi^2/a^2 - \eta^2/b^2)^{\frac{1}{2}(\kappa - 3)}$.

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THE ROYAL SOCIETY.

30th November, 1895.

THE ROYAL SOCIETY. Nov. 30, 1895.

Her Sacred Majesty QUEEN VICTORIA, Patron.

Date of Election.	
1863. Feb. 12.	HIS ROYAL HIGHNESS THE PRINCE OF WALES, K.G.
1882. Mar. 16.	HIS ROYAL HIGHNESS THE DUKE OF EDINBURGH, K.G.
1893. June 8.	HIS ROYAL HIGHNESS THE DUKE OF YORK, K.G.

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THE ROYAL SOCIETY.

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⁂ *This Council will continue till November* 30, 1896.

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Omissions having occasionally occurred in the Annual List of Deceased Fellows, as announced from the Chair at the Anniversary Meeting of the Royal Society, it is requested that any information on that subject, as also Notice of Changes of Residence, be addressed to the Assistant Secretary.

FELLOWS OF THE SOCIETY.

NOVEMBER 30, 1895.

(C) prefixed to a name indicates the award of the Copley Medal.
(R) .. .. Royal Medal.
(Rm) .. .. Rumford Medal.
(D) .. .. Davy Medal.
(Dw.) .. .. Darwin Medal.
(†) .. .. is liable to an annual payment of £4.
(*) .. .. £3.

Date of Election.	Served on Council.			
June 7, 1860.	'67–69 '77–79	R.	†	Abel, Sir Frederick Augustus, Bart., K.C.B. D.C.L. (Oxon.) D.Sc. (Camb.) V.P.C.S. V.P.S. Arts. Hon. Mem. Inst. C.E., Inst. M.E., Ord. Imp. Bras. Rosae Eq. Hon. Mem. Deutsch. Chem. Gesell., Mem. Soc. d'Encourag. Paris, Sec. and Director of the Imperial Institute. 2 *Whitehall-court,* S.W.; *and Imperial Institute, Imperial Institute-road.* S.W.
June 1, 1876.	'83–85 '91–93	Rm.		Abney, William de Wiveleslie, Capt. R.E. C.B. D.C.L. (Dunelm.) F.I.C. F.C.S. F.R.A.S., Director for Science in the Science and Art Department. *Rathmore Lodge, Bolton-gardens South, Earl's Court,* S.W.; *and Athenæum Club.* S.W.
Jan. 21, 1847.	'58–59			Acland, Sir Henry Wentworth Dyke, Bart., K.C.B. A.M. M.D. LL.D. (Cantab.) F.R.G.S., Coll. Reg. Med. Soc., Hon. Student of Ch. Ch., Radcliffe Librarian and late Reg. Prof. of Medicine in the University of Oxford. *Broad-street, Oxford.*
June 6, 1872.	'82–84			Adams, William Grylls, M.A. D.Sc. F.G.S. F.C.P.S. Vice-President of Physical Soc. Past Pres. Inst. Elec. Eng. Professor of Natural Philosophy and Astronomy in King's College, London. 43 *Campden-hill-square.* W.
June 7, 1883.			*	Aitchison, James Edw. Tierney, M.D. C.I.E., LL.D. (Edin.) F.R.S.E. F.L.S. F.R.C.S. (Edin.) M.R.C.P. (Edin.) Corresp. Fell. Obstet. Soc. Edin. Brigade Surgeon H.M. Bengal Army (retired). 20 *Chester-street, Edinburgh.*
June 6, 1889.			*	Aitken, John, F.R.S.E. *Darroch, Falkirk.* N.B.
June 3, 1880.			*	Allbutt, Thomas Clifford, M.A. M.D. LL.D. F.L.S. Regius Professor of Physic in the University of Cambridge. *St. Radegund's, Cambridge.*
June 1, 1854.	'71–73	R.		Allman, George James, M.D. LL.D. F.R.S.E. F.R.C.S.I. F.L.S. M.R.I.A. Cor. Mem. Z.S. Emeritus Regius Professor of Natural History in the University of Edinburgh, Ex-Pres. of the Linnean Society, Hon. Fell. Roy. Micros. Soc., Hon. Mem. Geol. Soc. Cornwall, Phil. Soc. Yorkshire, Lit. Antiq. Soc. Perth, Trin. Hist. Soc. Dallas, Texas, Soc. Reg. Sci. Dan. Hafn. Soc. Honor. Soc. Zool. Bot. Vindob. Socius Honor. Soc. Hist. Nat. Ind. Batav. Corr. *Ardmore, Parkstone, Poole, Dorsetshire; and Athenæum Club.* S.W.
June 12, 1884.			*	Allman, George Johnston, LL.D. (Dubl.) D.Sc. Emeritus Professor of Mathematics in Queen's College, Galway, Member of Senate of the Royal University of Ireland. *St. Mary's, Galway.*
June 12, 1879.				Anderson, John, M.D. LL.D. (Edin.) F.R.S.E. F.S.A. F.L.S. F.Z.S. late Superintendent, Indian Museum, and Professor of Comparative Anatomy in the Medical College, Calcutta. 71 *Harrington-gardens.* S.W.
June 4, 1891.			*	Anderson, William, D.C.L. Past Pres. Inst. Mech.E., Mem. Inst. C.E., Director General of Royal Ordnance Factories. *Lesney House, Erith; and Royal Arsenal, Woolwich.*

Date of Election.	Served on Council.	
June 7, 1888.		* Andrews, Thomas, F.R.S.E. F.C.S. Mem. Inst. C.E., Telford Medallist and Prizeman. *Ravencrag, Wortley, near Sheffield.*
June 3, 1875.		Archer, William, M.R.I.A. *National Library of Ireland; and* 52 *Lower Mount-street, Dublin.*
June 19, 1851.	'55–56 '83–84	† Argyll, George Douglas Campbell, Duke of, K.G. K.T. D.C.L. (Oxon.) LL.D. (Camb.) Trust. Brit. Mus. Hon. V.P.R.S. Edin. F.G.S. *Argyll Lodge, Kensington,* W.; *and Inverary Castle, Argyleshire.*
June 12, 1873.		† Armstrong, Sir Alexander, K.C.B. M.D. LL.D. (Dubl.) F.R.G.S. late Director-General of the Medical Department of the Navy, Hon. Physician to the Queen and to H.R.H. The Prince of Wales. *The Elms, Sutton Bonnington, Loughborough, Leicestershire.*
June 1, 1876.	'88–90	† Armstrong, Henry Edward, Ph.D. (Lips.) LL.D. (St. Andr.) Pres. Chem. Soc. Professor of Chemistry at the City and Guilds of London Central Institute, South Kensington, Hon. Mem. Pharm. Soc. Lond. 55 *Granville-park, Lewisham.* S.E.
May 7, 1846.	'61–62	Armstrong, William George, Lord, C.B. D.C.L. (Oxon.) LL.D. (Cantab.) M.E. (Dubl.) Ord.SS$^{\text{rum.}}$ Maur. et Lazar. Ital. Gr. Off. Ord. Dannebrog et Ord. Jes. Christ Portog. Com. Ord. Fr. Jos. Austriae, Ord. Car. III. Hisp. et Ord. Imp. Bras. Rosae Eq. *Athenæum Club; Cragside, Rothbury; and Newcastle-upon-Tyne.*
June 3, 1880.		* Attfield, John, M.A. Ph.D. (Tüb.) F.I.C. F.C.S. Professor of Practical Chemistry to the Pharmaceutical Society of Great Britain, Hon. Mem. Amer. Pharm. Assoc., Colls. Pharm. Philad., New York, Mass., Chic., Ontario, and Pharm. Assocs. Liverp., Manch., Maryland, Virg., Georgia, New Hampshire, and Quebec, Hon. Corresp. Mem. Soc. Pharm. Paris, Hon. Mem. Pharm. Soc. Gr. Brit., New South Wales, St. Petersb., Austria, Denmark, East Flanders, Switzerland, Queensland, and Australasia. *Ashlands, Watford; and* 17 *Bloomsbury-sq.* W.C.
June 2, 1881.	'89–91	* Ayrton, William Edward, Mem. Phys. Soc. and Inst. Elect. Eng., Professor of Applied Physics in the Guilds Central Technical College. *Exhibition-road.* S.W.
June 4, 1885.		* Baird, Andrew Wilson, Lieut.-Colonel, R.E. *Care of Messrs. Grindlay, Groom & Co., Bombay, India.*
June 5, 1890.	'92–93	Baker, Sir Benjamin, K.C.M.G. LL.D. M.E. (Dubl.) Mem. Inst. C.E., Hon. Mem. Amer. Soc. Mechan. Engs., Soc of Engs., and Manc. Lit. and Phil. Soc. 2 *Queen-square-place, Queen Anne's Mansions, Westminster; and Athenæum Club.* S.W.
June 6, 1878.	'83–84	† Baker, John Gilbert, F.L.S. Keeper of the Herbarium, Royal Gardens, Kew. *Royal Herbarium, Kew.*
Jan. 12, 1888.		† Balfour, Right Hon. Arthur James, D.C.L. 4 *Carlton-gardens,* S.W.; *and Whittinghame, Prestonkirk,* N.B.
June 12, 1884.	'92–94	* Balfour, Isaac Bayley, D.Sc. M.D. (Edin.) M.A. (Oxon.) F.R.S.E. F.L.S. F.G.S. Keeper of the Royal Botanic Garden, Edinburgh, Queen's Botanist in Scotland, and Professor of Botany in the University of Edinburgh, Corresp. Mem. Deutsch. Bot. Gesell., Soc. Nat. des Sci. Nat. et Math. Cherbourg. *Inverleith House, Edinburgh; and Athenæum Club.* S.W.

Date of Election.	Served on Council.	
June 12, 1873.		Ball, Sir Robert Stawell, Kt., Hon. M.A. (Cantab.) LL.D. M.R.I.A. Soc. Phil. Cam. et Soc. Reg. Edin. Soc. Honor. Lowndean Professor of Astronomy and Geometry in the University of Cambridge. *The Observatory, Cambridge; and Athenæum Club, S.W.*
June 6, 1889.		* Ballard, Edward, M.D. 6 *Ravenscroft-park, High Barnet, Herts.*
June 2, 1864.		† Barkly, Sir Henry, G.C.M.G. K.C.B. F.R.G.S. 1 *Bina-gardens, South Kensington,* S.W.
June 6, 1850.	'80–81	Barlow, William Henry, F.R.S.E. Past Pres. Inst. C.E. *High Combe, Old Charlton, Kent.*
Dec. 12, 1844.		Barrow, John, F.S.A., F.R.G.S. 17 *Hanover-terrace, Regent's Park.* N.W.
June 13, 1895.		* Barry, John Wolfe, C.B. V.P. Inst. C.E. 23 *Delahay-street, Westminster.* S.W.
June 6, 1889.		* Basset, Alfred Barnard, M.A. *Fledborough Hall, Holyport, Berks.*
June 4, 1868.		Bastian, Henry Charlton, M.A. M.D. F.L.S. Coll. Reg. Med. Soc. Professor of the Principles and Practice of Medicine, University College, Physician to University College Hospital, Fellow of Univ. Coll. London, Hon. Fellow Roy. Coll. Phys., Ireland, Corr. Mem. Roy. Acad. Med. Turin and Soc. Psychol. Physiolog. Paris. 8A *Manchester-square.* W.
Feb. 8, 1838.		† Bateman, James, M.A. F.L.S. *Home House, Worthing.*
June 7, 1894.		Bateson, William, M.A., Fellow of St. John's College, Cambridge. *St. John's College, Cambridge.*
June 11, 1857.	'65–67	Beale, Lionel Smith, M.B. Coll. Reg. Med. Socius. Prof. of the Principles and Practice of Medicine, and formerly of Physiology and of General and Morbid Anatomy, and of Pathological Anatomy in King's College, London, and Physician to the Hospital. Government Medical Referee for England. 61 *Grosvenor-street.* W.
June 2, 1892.		* Beddard, Frank Evers, M.A. (Oxon.), F.R.S.E., F.Z.S., Lecturer on Comparative Anatomy, Guy's Hospital, Prosector to the Zoological Society. *Zoological Society's Gardens, Regent's Park.* N.W.
June 12, 1873.		† Beddoe, John, M.D. LL.D. (Edin.) B.A., Officier de l'Instr. Publ. France; Vice-Pres. Anthrop. Inst. Corresp. Mem. Anthrop. Soc. Berlin, Soc. Romana di Antrop.; For. Mem. Soc. Anthrop. Paris; Hon. Mem. Anthrop. Socs. Brussels and Washington, Acad. Anthrop. New York, Amer. Antiq. Soc. and of Imp. Soc. Friends of Sci., Moscow. *The Chantry, Bradford-on-Avon; and Athenæum Club.* S.W.
June 4, 1874.	'81–82	† Bell, Sir Lowthian, Bart., F.C.S. Mem. Inst. C.E. *Rounton Grange, by Northallerton.*
June 12, 1884.		* Bell, James, C.B. D.Sc. (Dubl.) Ph.D. F.I.C. Late Principal of the Inland Revenue Laboratory, Somerset House. *Howell Hill Lodge, Ewell, Surrey.*
June 8, 1871.		Besant, William Henry, D.Sc. F.R.A.S. Fellow of St. John's College, Cambridge. *St. John's College, and Spring Lawn, Harvey-road, Cambridge.*
June 12, 1879.		* Bessemer, Sir Henry, Hon. Mem. Inst. C.E. *Denmark-hill.* S.E.
June 4, 1886.		* Bidwell, Shelford, M.A. LL.B. *Riverstone Lodge, Southfields, Wandsworth* S.W.
April 6, 1843.		Blake, Henry Wollaston, M.A. 8 *Devonshire-place, Portland place.* W.
June 4, 1874.	'91–93	† Blanford, William Thomas, LL.D. (Univ. McGill) A.R.S.M. F.G.S. F.R.G.S. F.Z.S. Ord. SS$^{\text{rum.}}$ Maur. et Lazar. Ital. Eq. Soc. Asiat. Beng. Soc. Honor. 72 *Bedford-gardens, Campden-hill, Kensington.* W.

Date of Election.	Served on Council.	
June 6, 1878.	'80–82 '95	Bonney, Rev. Thomas George, D.Sc. LL.D. (Univ. McGill) Sc.D. (Dubl.) F.S.A. F.G.S. Soc. Phil. Cantab. Soc. et Ebor. Soc. Honor. Soc. Geol. Belg. et Soc. Reg. Canad. Corresp. Corresp. Mem. Soc. Géol. du Nord de France, Professor of Geology in University College, London. 23 *Denning-road, Hampstead.* N.W.
June 5, 1890.		* Bosanquet, Robert Holford Macdowall, M.A. Fellow of St. John's College, Oxford. *Castillo Zamora, Realejo-Alto, Teneriffe; and New University Club, St. James's-street.* S.W.
June 7, 1888.		* Bottomley, James Thomson, M.A. D.Sc. F.R.S.E. F.C.S. Lecturer on Natural Philosophy in the University of Glasgow. 13 *University-gardens, Glasgow.*
June 7, 1894.		* Boulenger, George Albert, F.Z.S. Corresp. Mem. Imp. Soc. Friends of Sci., Moscow, Senckenb. Soc. Frankfort, Linn. Soc. Bordeaux, Sci. Soc. Boston. 8 *Courtfield-road, South Kensington.* S.W.
June 13, 1895.		* Bourne, Alfred Gibbs, D.Sc. Professor of Biology in the Presidency College, Madras. Fellow of University College, London. *Presidency College, Madras.*
June 4, 1891.		Bower, Frederick Orpen, D.Sc. (Camb.) F.L.S. F.R.S.E. Regius Professor of Botany in the University of Glasgow. 45 *Kerrsland-terrace, Hillhead, Glasgow.*
June 3, 1858.		† Boxer, Edward Mounier, Major-General, R.A. *Upton, near Ryde.*
June 7, 1888.		* Boys, Charles Vernon, A.R.S.M. Assistant Professor of Physics in the Royal College of Science, London, Officier de l'Instruction Publique, France. *Royal College of Science, South Kensington; and Glebe House, Glebe Place, Chelsea.* S.W.
June 7, 1894.		* Bradford, John Rose, M.D. D.Sc. Assistant Professor of Clinical Medicine, University College, London. 52 *Upper Berkeley-street.* W.
June 8, 1882.		* Brady, George Stewardson, M.D. LL.D. Professor of Natural History in the Durham College of Science, Newcastle. 2 *Mowbray-villas, Sunderland.*
June 12, 1873.	'77–78	† Bramwell, Sir Frederick Joseph, Bart., D.C.L. (Oxon. et Dunelm) LL.D. (Cantab. et Univ. McGill) M. Inst. C.E. 5 *Great George-street, Westminster.* S.W.
June 3, 1875.		† Brandis, Sir Dietrich, K.C.I.E. Ph.D. F.L.S., late Inspector-General of Forests to the Government of India. 21 *Kaiser Strasse, Bonn, Germany.*
June 12, 1879.	'91–92	Brown, Alexander Crum, D.Sc. LL.D. M.D. Professor of Chemistry in the University of Edinburgh. 8 *Belgrave-crescent, Edinburgh.*
June 6, 1889.		* Brown, Horace T., F.C.S. F.I.C. F.G.S. 52 *Nevern-square, South Kensington.* S.W.
June 7, 1883.		* Browne, Sir James Crichton, Kt., M.D. LL.D. F.R.S.E. 61 *Carlisle-place Mansions, Victoria-street.* S.W.
June 4, 1874.	'82–84	† Brunton, Thomas Lauder, M.D. Sc.D. (Edin.) Hon. LL.D. (Aberdeen) Coll. Reg. Med. Soc. 10 *Stratford-place, Oxford-street,* W.; *and Athenæum Club.*
June 13, 1895.		* Bryan, George Hartley, M.A. Fellow of St. Peter's College, Cambridge. *Thornlea, Chaucer-road, Trumpington-road, Cambridge.*
Dec. 14, 1893.		† Bryce, Right Hon. James, D.C.L. 54 *Portland-place.* W.
June 9, 1887.		* Buchanan, John Young, M.A. F.R.S.E. F.C.S. F.R.G.S. 10 *Moray-place, Edinburgh.*
June 7, 1866.		† Bucknill, Sir John Charles, M.D. (Lond.) Coll. Reg. Med. Socius. Coll. Univ. Lond. Soc. *East Cliff House, Bournemouth.*

Date of Election.	Served on Council.		
June 11, 1857.	'61–63		Buckton, George Bowdler, F.C.S. F.E.S. F.L.S. Corr. Acad. Nat. Sci. Philad., Mem. Soc. Entom. France. *Weycombe, Haslemere, Surrey.*
June 12, 1879.			* Buller, Sir Walter Lawry, K.C.M.G. D.Sc. F.L.S. Z.S. (Corr.) *The Terrace, Wellington, New Zealand.*
June 5, 1890.			* Burbury, Samuel Hawksley, M.A. 17 *Upper Phillimore-gardens, Kensington.* W.
June 1, 1893.			* Burnside, William, M.A. Professor of Mathematics, Royal Naval College, Greenwich. *The Laurels, Hithergreen-lane.* S.E.
June 7, 1894.			* Callendar, Hugh Longbourne, M.A. Late Fellow of Trinity College, Cambridge, Professor of Physics in McGill College, Montreal. *McGill College, Montreal, Canada.*
June 8, 1871.	'77–79		† Carruthers, William, V.P.L.S. F.G.S. Keeper of the Botanical Department, British Museum. *Central House, Central-hill,* S.E.; *and British Museum (Nat. Hist.), Cromwell-road.* S.W.
June 9, 1887.			* Cash, John Theodore, M.D. Regius Professor of Materia Medica in the University of Aberdeen. 25 *Dee-street, Aberdeen.*
Dec. 14, 1882.			† Chamberlain, Right Hon. Joseph, LL.D. (Cantab.). 40 *Prince's-gardens; and Athenæum Club.* S.W.
June 3, 1869.			Chambers, Charles, Director of the Colába Observatory, Bombay. *Colába Observatory, Bombay.*
June 7, 1894.			* Cheyne, William Watson, M.B. C.M. (Edin.) F.R.C.S. (Eng.) Professor of Surgery in King's College, London. 75 *Harley-street.* W.
Jan. 16, 1873.			† Childers, Right Hon. Hugh Culling Eardley, F.R.G.S. 6 *St. George's-place,* S.W.; *Brooks's and Athenæum Clubs.* S.W.
June 2, 1881.	'83–85 '89–91		* Christie, William Henry Mahoney, M.A. F.R.A.S. F.R. Met. Soc. Astronomer Royal, Corr. Mem. Imp. Acad. Sci. St. Petersb. For. Memb. Roy. Acad. Sci. Palermo, Corr. Mem. Soc. Spettros. Ital. Soc. Nationale des Sci. Nat. et Math. Cherbourg. *Royal Observatory, Greenwich.* S.E.
June 7, 1888.			* Church, Arthur Herbert, M.A. (Oxon.) F.C.S. F.I.C. Professor of Chemistry in the Royal Academy of Arts, Lecturer on Organic Chemistry, Royal Indian Engineering College, Cooper's Hill. *Shelsley, Kew.*
June 6, 1889.			* Clark, Latimer, Mem. Inst. C.E. F.R.A.S. 11 *Victoria-street.* S.W.
June 7, 1888.		R.	* Clarke, Alexander Ross, Colonel, R.E. C.B. Hon. F.C.P.S. Hon. F.R.S.E. Corr. Mem. Imp. Acad. Sci. St. Petersb. *Boldrewood, Redhill, Surrey.*
June 8, 1882.	'88–90		* Clarke, Charles Baron, M.A. (Cantab.) F.L.S. F.G.S. 13 *Kew Gardens-road, Kew.*
June 6, 1872.			Cleland, John, M.D. D.Sc. LL.D. Professor of Anatomy in the University of Glasgow. *University, Glasgow.*
June 9, 1848.	'78–80		† Clerk, Henry, Major-General, R.A. 40 *St. Ermin's Mansions, Caxton-street, Westminster.* S.W.
			Clifford-Allbutt (*see* Allbutt).
June 4, 1868.	'71–73 '85–87		Clifton, Robert Bellamy, M.A. (Cantab. et Oxon.), F.R.A.S. Professor of Experimental Philosophy in the University of Oxford, Soc. Lit. Phil. Manc. Soc. Honor. 3 *Bardwell-road, Banbury-road, Oxford; and Athenæum Club.*
June 4, 1886.			* Colenso, Rev. William, F.L.S. Hon. Mem. N.Z. Inst., Mem. Penzance Nat. Hist. and Antiq. Soc., Hon. Mem. Santa Barbara (Cal.) Soc. Nat. Hist. *Napier, New Zealand.*
June 4, 1885.	'93–95		* Common, Andrew Ainslie, Treas. R.A.S. LL.D. (St. And.) D.Sc. 63 *Eaton-rise, Ealing.* W.

Date of Election.	Served on Council.		
June 4, 1891.			* Conroy, Sir John, Bart., M.A. F.C.S. Fellow and Bedford Lecturer of Balliol College, Oxford. *Balliol College, Oxford.*
June 6, 1878.			† Cotterill, James Henry, M.A. Professor of Applied Mechanics, Royal Naval College, Greenwich. 18 *Gloucester-place, Greenwich.* S.E.
June 6, 1878.	'78–79		† Crawford, James Ludovic, Earl of, K.T. LL.D., Trustee of Brit. Mus., F.R.A.S. Leg. Honor. Com. Ord. Imp. Bras. Rosae Com. Acad. Reg. Sci. Berol. Soc. Honor. 2 *Cavendish-square,* W.; *and Haigh Hall, Wigan.*
June 4, 1885.			* Creak, Ettrick William, Captain R.N. M. Inst. Elect. Eng. 36 *Kidbrooke-park-road, Blackheath.* S.E.
June 4, 1868.			† Crofton, Morgan William, D.Sc. Fellow of the Royal University of Ireland 15 *Ambrose-place, Worthing.*
June 4, 1863.	'77–79 '94–95	R. D.	† Crookes, William—VICE-PRESIDENT—Past Pres. Chem. Soc. and Inst. Elect. Eng. 7 *Kensington-park-gardens,* W.; *and Athenæum Club.* S.W.
Apr. 3, 1879.	'80–81		† Cross, Right Hon. Richard Assheton, Viscount, G.C.B. G.C.S.I. D.C.L. LL.D. 12 *Warwick-sq. and Athenæum Club,* S.W.; *and Eccle Riggs, Broughton-in-Furness, Lancashire.*
June 4, 1891.			* Cunningham, Daniel John, M.D. (Edin. and Dubl.), D.Sc. D.C.L. LL.D. Prof. of Anatomy in the University of Dublin. 43 *Fitzwilliam-place, Dublin.*
June 6, 1889.			Cunningham, David Douglas, M.B. C.M. (Edin.) C.I.E. F.L.S. Brigade Surg. Lieut.-Col. Bengal Medical Service. Honorary Surgeon to the Viceroy of India. Professor of Physiology in the Medical College, and Fellow of the University of Calcutta. 9 *Loudon-street, Calcutta.*
June 3, 1880.			* Dallinger, Rev. William Henry, LL.D. Sc.D. (Dubl.) F.L.S. Vice-Pres. R.M.S., Hon. Mem. Amer. Micros. Soc. *Ingleside, Newstead-road, Lee.* S.E.
June 8, 1882.	'94–95		Darwin, Francis, M.A. and M.B. (Cantab.) F.L.S. F.Z.S. F.R.M.C.S. Fellow of Christ's College, and Reader in Botany in the Univ. of Cambridge. Mem. Soc. Nat. Sci. et Math. de Cherbourg. *Wychfield, Huntingdon-road, Cambridge.*
June 12, 1879.	'84–85 '86–87	R.	Darwin, George Howard, M.A. LL.D. (Glasc.) Sc.D. (Dubl.) Ph.D. (Padua) F.R.A.S. F.M.S. Hon. Mem. R.I.A. Fellow of Trinity College and Plumian Professor of Astronomy and Experimental Philosophy in the University of Cambridge. Hon. Fell. Astron. and Phys. Soc. Toronto. *Newnham Grange, Cambridge.*
Jan. 24, 1895.			† Davey, Right Hon. Horace, Lord, M.A. D.C.L. 86 *Brook-street,* W.; *and Verdley-place, Fernhurst, Sussex.*
June 6, 1867.	'89–91		† Dawkins, W. Boyd, M.A. (Oxon.) F.S.A. F.G.S. Assoc. Inst. C.E. Hon. Fellow of Jesus Coll. (Oxford) Professor of Geology and Palæontology in the Victoria University, Owens Coll., Manchester. Soc. Anthrop. Berol. Acad. Sci. Nat. Philad. et Soc. Nat. Hist. Bost. Corresp. Soc. Phil. Amer. et Acad. Sci. Nov.Ebor. et Soc. Geol. Belg. Soc. Honor. *Woodhurst, Fallowfield, Manchester.*
June 4, 1891.			* Dawson, George Mercer, C.M.G. LL.D. F.G.S. A.R.S.M. F.R.S.C. Assistant Director of the Geological Survey of Canada. *Sussex-street, Ottawa, Canada.*
June 5, 1862.			Dawson, Sir J. William, C.M.G. M.A. LL.D. F.G.S. Late Principal and Vice-Chancellor of M'Gill University, Montreal, Hon. Mem. Phil. Soc. Leeds, Glasg., Princeton, Soc. Nat. Hist. Boston, Maryland Academy. Corresp. Fell. Geol. Soc. Edin., Fell. Roy. Soc. Canada, Amer. Acad. Arts and Sciences, Amer. Phil. Soc. Corr. Mem. Acad. Nat. Sci, Philad. 293 *University-street, Montreal.*

Date of Election.	Served on Council.		
June 6, 1861.	'70–72 '81–83		† Debus, Heinrich, Ph.D. F.C.S. Late Prof. of Chemistry at the Royal Naval College, Greenwich, Lecturer on Chemistry at Guy's Hospital. 4 *Schlangenweg, Cassel, Hessen, Germany.*
Mar. 3, 1892.			† Devonshire, Spencer Compton Cavendish, Duke of, K.G. M.A. LL.D. Hon. Mem. Inst. C.E. Chancellor of the University of Cambridge. *Devonshire House, Piccadilly,* W.; *and Chatsworth, Derbyshire.*
June 7, 1877.	'85–86	Rm.	Dewar, James, M.A. F.C.S. F.R.S.E. Hon. LL.D. (Glasc. and St. And.) Jacksonian Prof. of Natural Experimental Philosophy in the University of Cambridge, Fullerian Prof. of Chemistry in the Royal Institution. 1 *Scroope-terrace, Cambridge; and Royal Institution, Albemarle-street.* W.
June 4, 1885.			Divers, Edward, M.D. Professor of Chemistry in the Imperial University, Japan. *Hongo, Tokyo, Japan.*
June 4, 1886.			* Dixon, Harold B., M.A. F.C.S. Prof. of Chemistry and Director of the Chemical Laboratories in Owens College, Manchester. *Birch Hall, Rusholme, Manchester.*
June 7, 1883.			Douglass, Sir James Nicholas, M. Inst. C.E. M. Inst. M.E. M. Inst. E.E. Late Engineer-in-Chief to the Hon. Corporation of Trinity House. *Stella House, Dulwich.* S.E.
Feb. 22, 1855.			Ducie, Henry John Reynolds-Moreton, Earl of, F.G.S. *Tortworth Court, Falfield, Gloucestershire.*
Feb. 9, 1865.			Dufferin and Ava, Frederick Temple Blackwood, Marquis of, K.P. G.C.B. G.C.M.G. G.C.S.I. G.M.I.E. D.C.L. (Oxford) LL.D. (Camb. and Dubl.) F.R.G.S. *Clandeboye, Co. Down, Ireland.*
June 1, 1876.	'79–81		† Dunkin, Edwin, F.R.A.S. Formerly Chief Assistant, Royal Observatory, Greenwich. *Kenwyn, Kidbrooke Park-road, Blackheath.* S.E.
June 1, 1893.			* Dunstan, Wyndham R., M.A. (Oxon.) Sec. Chem. Soc. F.I.C. Professor of Chemistry to, and Director of the Research Laboratory of, the Pharmaceutical Society of Great Britain. Lecturer on Chemistry at St. Thomas's Hospital. 3 *Percy-villas, Campden Hill, Kensington. W.*
June 3, 1875.			† Dupré, August, Ph.D. F.C.S. Lecturer on Chemistry at the Westminster Hospital. *Westminster Hospital Medical School, Caxton-street, Westminster,* S.W.; *and Cliftonville, Sutton, Surrey.*
June 3, 1880.	'86–88		* Dyer, William Turner Thiselton, C.M.G. C.I.E. M.A. (Oxon.) B.Sc. (Lond.) Ph.D. F.L.S. Director, Royal Gardens, Kew; late Fellow Univ. of London; Hon. Fellow, King's Coll., Lond.; Bot. Soc. Edin.; Hon. Mem. Roy. Bot. Soc. Lond., Pharm. Soc. Gt. Britain; Cam. Phil. Soc.; Lit. Phil. Soc. Manchester; Soc. Néerland d'Hort. et de Bot., Amsterdam and New Zealand Institute; Corresp. Acad. Sci. Philad.; Boston Soc. Nat. Hist.; Hort. Soc. Berlin and Massachusetts; Soc. Nat. Sci. et Math. de Cherb.; and Botan. Soc. Copenhagen; Mitg. Kais.-Leop. Deutsch. Acad. der Naturf. in Halle. *Royal Gardens, Kew.*
June 13, 1895.			* Eliot, John, M.A. Meteorological Reporter to the Government of India. *Indian Meteorological Office, Simla.*
June 12, 1873.			† Ellery, Robert Lewis John, C.M.G. F.R.A.S. Late Government Astronomer, and Director of the Observatory. *Melbourne, Victoria.*

Date of Election.	Served on Council.			
June 4, 1891.			*	Elliott, Edwin Bailey, M.A. F.R.A.S. Waynflete Professor of Pure Mathematics in the University of Oxford; Fellow of Magdalen College, Oxford. 4 *Bardwell-road, Oxford.*
June 1, 1893.			*	Ellis, William, F.R.A.S. F.R. Met. Soc. Memb. Inst. Elect. Eng. Late Superintendent of the Magnetical and Meteorological Department, Royal Observatory, Greenwich. 12 *Vanbrugh-hill, Blackheath.* S.E.
June 1, 1876.				Erichsen, Sir John Eric, Bart., F.R.C.S. LL.D. (Edin.) Surgeon Extraordinary to the Queen, Pres. of and Emeritus Prof. of Surgery to University College, and Consulting Surgeon to the Hospital. 6 *Cavendish-place, Cavendish-square.* W.
June 3, 1869.				Esson, William, M.A. F.C.S. F.R.A.S. Deputy Savilian Professor of Geometry in the University of Oxford, Fellow, and Mathematical Tutor of Merton College. *Merton College; and* 13 *Bradmore-road, Oxford.*
June 8, 1871.	'84–85		†	Etheridge, Robert, F.R.S.E. F.G.S. Hon. Memb. Geol. Soc. Belg. N.Z. Inst. Roy. Geol. Soc. Cornwall, Phil. Soc. York, Bristol, Corresp. Imp. Geol. Inst. Vienna. 14 *Carlyle-square, Chelsea.* S.W.
June 2, 1864.	'67–68 '73–75 '78–95			Evans, Sir John—TREASURER and VICE-PRESIDENT—K.C.B. D.C.L. (Oxon.) LL.D. (Dubl.) Sc.D. (Camb.) Trustee, Brit. Mus. F.S.A. F.L.S. F.G.S. F.C.S. F.Z.S. Assoc. I.C.E. Pres. Num. Soc. Hon. M.R.I.A. Hon. F.S.A. (Scot.) Comm. of the Ord. of St. Thiago of Port., Cor. of Inst. de France (Acad. des Inscrip.), Hon. Mem. of the Amer. Phil. Soc., Amer. Acad. Arts and Sciences, Amer. Ethnol. Soc., Num. and Ant. Soc., of Philadelphia, Amer. Num. and Archæol. Soc., Soc. Franç. de Numism., Soc. Ital. d'Anthrop., Soc. Roy. Gr. Duc. de Luxembourg, Soc. Anthrop. de Brux. et de Lyons, Soc. de Borda. Dax., Soc. Polym. du Morbihan, and Soc. Suisse de Numism., For. Mem. of the Soc. Ant. of Sweden, Soc. Anthrop. de Paris, and the Numism. Soc. of the Netherlands, and Corr. Mem. of the Soc. Géol. de Belg., Inst. di Corr. Arch., Acad. Valdarn., Anthrop. Soc. of Berlin, and Soc. d'Emul. d'Abbeville. *Nash Mills, Hemel Hempstead; and Athenæum Club.*
June 12, 1879.			*	Everett, Joseph David, M.A. D.C.L. F.R.S.E. Professor of Natural Philosophy in Queen's College, Belfast. *Derryvolgie Avenue, Belfast.*
June 1, 1893.			*	Ewart, James Cossar, M.D. Professor of Natural History in the University of Edinburgh. *The University, Edinburgh.*
June 9, 1887.		R.	*	Ewing, James Alfred, Hon. M.A. (Camb.) B.Sc. (Edin.) F.R.S.E. M. Inst. C.E. Professor of Mechanism and Applied Mechanics in the University of Cambridge. *Langdale Lodge, Cambridge.*
June 7, 1866.			†	Farrar, Very Rev. Frederic William, M.A. D.D. (Cantab.) Dean of Canterbury.
June 7, 1877.			†	Fayrer, Sir Joseph, K.C.S.I. M.D. LL.D. (Edin. and St. And.) F.R.C.P. (Lond.) F.R.C.S. F.R.S.E. Honorary Physician to the Queen. 16 *Devonshire-street, Portland-place.* W.
June 7, 1877.				Ferrers, Rev. Norman Macleod, D.D. Master of Gonville and Caius College, Cambridge. *The Lodge, Gonville and Caius College, Cambridge.*
June 1, 1876.	'86–88	R.		Ferrier, David, M.A. (Aberd.) M.D. (Edin.) LL.D. F.R.C.P. Professor of Neuro-pathology, King's College, London. 34 *Cavendish-square.* W.
June 4, 1886.			*	Festing, Edward Robert, Major-General, R.E. (retired). Science Museum Director, South Kensington Museum. *South Kensington.* S.W.
June 7, 1883.			*	Fitzgerald, Prof. George Francis, M.A. D.Sc. 40 *Trinity College, Dublin.*
June 2, 1892			*	Fleming, John Ambrose, M.A. (Camb.) D.Sc. (Lond.) Late Fellow of St. John's College, Cambridge, Fellow and Professor of Electrical Engineering in University College, London. *University College, Gower-street.* W.C.

Date of Election.	Served on Council.		
June 6, 1889.			Fletcher, Lazarus, M.A. (Oxon.) F.G.S. F.C.S. Keeper of Minerals in the British Museum. *Natural History Museum, Cromwell-road; and* 36 *Woodville-road, Ealing.* W.
June 2, 1864.	'68–70 '76–78 '84–86	R.	Flower, Sir William Henry, K.C.B. D.C.L. (Oxon. et Dunelm.) LL.D. (Edin. et Dubl.) Sc.D. (Camb.) Ph.D. (Utrecht) F.R.C.S. P.Z.S. F.L.S. F.G.S. V.P. Anthrop. Inst. Hon. Fell. Roy. Med. Chir. Soc. Hon. Memb. Asiat. Soc. Bengal, Camb. Phil. Soc., Manchester Lit. Phil. Soc. N.Z. Inst. Zool. Soc. Amsterdam and France, Acad. Sci. New York, Anthrop. Soc. Washington, Memb. Amer. Phil. Soc. Imp. Soc. Nat. Moscow, Corresp. Memb. Inst. Fr. (Acad. Sci.) Acad. Sci. Bologna, Acad. Sci. Turin, Acad. Nat. Sci. Philad., Soc. Nat. Hist. Boston, Soc. Anthrop. Ethnol. and Prehist. Arch. Berlin, Anthrop. Soc. Rome, For. Assoc. Anthrop. Soc. Paris, Director of the Nat. Hist. Departments, British Museum. *Natural History Museum, Cromwell-road; and* 26 *Stanhope-gardens.* S.W.
June 9, 1887.			* Forbes, George, M.A. F.R.S.E. F.R.A.S. Mem. Inst. C.E. M.I.E.E. Chev. Lég Honor. Memb. Astron. Gesell. Vienna, Amer. Phil. Soc., and Franklin Inst. Formerly Professor of Nat. Phil. in Anderson's College, Glasgow. 34 *Great George-street.* S.W.
June 4, 1886.	'93–95		* Forsyth, Andrew Russell, M.A. Sc.D. F.C.P.S. Sadlerian Professor of Pure Mathematics in the University of Cambridge, Fellow of Trinity College, Cambridge. *Trinity College, Cambridge.*
June 2, 1892.			* Foster, Clement Le Neve, B.A. D.Sc. (Lond.) F.G.S., A.R.S.M., Professor of Mining in the Royal College of Science, and H.M. Inspector of Mines. *Min-y-don, Llandudno.*
June 3, 1869.	'70–72 '77–78 '83–85 '91–93		Foster, George Carey, B.A. F.C.S. Professor of Physics in University College, London. 18 *Daleham-gardens, South Hampstead,* N.W.; *and Athenæum Club.* S.W.
June 6, 1872.	'76–77 '81–95		† Foster, Michael—SECRETARY—M.D. B.A. (Lond.) Hon. M.A. (Cantab.) D.C.L. (Oxon.) LL.D. (Glasg. and St. And.) Sc.D. (Dubl.) F.L.S. F.C.S. For. Mem. R. Accad. dei Lincei, Roma, R. Accad. delle Scienze, Torino, Hon. Mem. Roy. Irish Acad., Lit. and Phil. Soc. Manc., Roy. Soc. N. S. Wales, Med. Chir. Soc., and Pharm. Soc. Lond., Professor of Physiology in the University of Cambridge. *Great Shelford, Cambridge.*
June 2, 1853.	'57–59 '65–67 '75–77 '86–88	C. R.	Frankland, Edward—FOREIGN SECRETARY—D.C.L. (Oxon.) Ph.D. (Marp.) M.D. (Würzburg) LL.D. (Edin. et Univ. McGill) V.P.C.S. Hon. Mem. Inst. C.E., Assoc. Étrang. Inst. Fr. (Acad. Sci.) Par., Imp. Sci. Petrop. et Vindob. Reg. Sci. Berol. Soc. pro fov. Indust. Nat. Corresp., Soc. Reg. Sci. Gött. et Upsal, Soc. Nat. Scrutat Helvet., Acad. Reg. Monac. Socius Honor., Soc. Reg. Edin. Acad. Reg. Hib., Soc. Lit. Phil. Manc., Soc. Chem. Berol. et Acad. Sci. Amer. Nov. Ebor. Lehigh Univ. U.S. Soc. Reg. Sci. Boie. Marob. et Nat. Sanit. Dresd., Chem. Amer. Nov. Ebor. Soc. Reg. Med. Chi. Lond. et Adsoc. Intern. pro Hyg. promov. Brux. Soc. Asiat. Beng. Soc. Honor. *The Yews, Reigate-hill, Reigate; and Athenæum Club.*
June 4, 1891.			* Frankland, Percy Faraday, Ph.D., B.Sc., A.R.S.M. Professor of Chemistry in the Mason College, Birmingham. *Mason College, Birmingham.*

Date of Election.	Served on Council.		
June 4, 1874.			† Franks, Sir Augustus Wollaston, K.C.B. D.C.L. (Oxon.) P.S.A. F.G.S. Trust. Brit. Mus. Keeper of British and Mediæval Antiquities, and of Ethnography at the British Museum. *British Museum,* W.C.; *and* 123 *Victoria-street, Westminster.* S.W.
June 7, 1877.			† Fraser, Thomas Richard, M.D. (Edin.) F.R.C.P. & R.S. (Edin.) LL.D. (Aberdeen) Professor of Materia Medica and Clinical Medicine in the University, Edinburgh. 13 *Drumsheugh-gardens, Edinburgh.*
June 7, 1883.			* Frost, Percival, Sc.D. *Fitzwilliam-street, Cambridge.*
June 7, 1894.			* Froude, Robert Edmund. Superintendent of the Admiralty Experimental Works, Gosport. 1 *Claremont, Alverstoke, Gosport.*
Dec. 13, 1883.			Fry, Right Hon. Sir Edward, B.A. (Lond.) D.C.L. (Oxon.) LL.D. (Edin.) F.S.A. F.L.S. Fellow of the University of London, and of University College, London, and Hon. Fellow, Bailiol Coll. Oxon. *Failand House, Failand, near Bristol.*
June 2, 1892.			* Gadow, Hans Friedrich, Ph.D. (Jena) Hon. M.A. (Camb.) Strickland Curator and Lecturer on the Advanced Morphology of Vertebrata in the University of Cambridge. *Zoological Laboratory, Cambridge.*
June 1, 1893.			* Gairdner, William Tennant, M.D. (Edin.) Hon. M.D. (Dubl.) Hon. LL.D. (Edin.) F.R.C.P. (Edin.) Hon. F.R.C.P. (Irel.) Professor of Medicine in the University of Glasgow. Physician in Ordinary to the Queen in Scotland. *The University, Glasgow.*
June 9, 1859.	'61–63 '67–69 '94–95		Galton, Sir Douglas, K.C.B. D.C.L. LL.D. (Univ. McGill and Durham) F.G.S. F.L.S. F.C.S. F.R.G.S. Hon. Memb. Inst. C.E. 12 *Chester-street, Grosvenor-place.* S.W.
June 7, 1860.	'65–66 '70–72 '76–77 '82–84	R.	Galton, Francis, M.A. (Cantab.) D.C.L. (Oxon.) Sc.D. (Camb.) Officier de l'Instruction Publique, France; Corresp. Memb. of the Geograph. Societies of Berlin and Vienna, and of Anthrop. Soc. of Rome. Hon. Memb. of Geograph. Soc. of Italy, and Inst. Internat. de Statistique. 42 *Rutland-gate.* S.W.
June 6, 1872.	'86–88		† Gamgee, Arthur, M.D. M.R.C.P. (Lond.) & F.R.C.P. (Edin.) 8 *Avenue de la Gare, Lausanne, Switzerland.*
June 5, 1890.			Gardiner, Walter, M.A. F.L.S. Fellow of Clare College, Cambridge, and University Lecturer in Botany. 45 *Hill's-road, Cambridge.*
June 3, 1858.			† Garrod, Sir Alfred Baring, M.D. Coll. Reg. Med. Socius, Physician Extraordinary to the Queen, Consulting Physician to King's College Hospital. 10 *Harley-street.* W.
June 8, 1882.		R.	* Gaskell, Walter Holbrook, M.A. M.D. (Cantab.) LL.D. (Edin.) Lecturer in Physiology at Cambridge. *The Uplands, Great Shelford, near Cambridge.*
June 1, 1865.	'85–87 '89–93		† Geikie, Sir Archibald, Knt. Sc.D. (Cantab. et Dubl.) LL.D. (Edin. et St. And.) F.R.S.E. F.G.S. F.Z.S. Director-General of the Geological Survey of the United Kingdom, and of the Museum of Economic Geology, London.—Inst. Franc. (Acad. Sci.) Acad. Reg. Berol. Acad. Imp. Sci. Vindob. Acad. Reg. Bavar. Monach. Soc. Cor.; Soc. Reg. Sci. Göttingen; Caesar. Leop. Carol. Acad. Sci. Nat.; Soc. Imp. Mineral. Petropol; Soc. Imp. Nat. Sci. Mosquen; Acad. Reg. Valdarnese del Poggio; Soc. Geogr. Ital. et Batav.; Soc. Geol. Edin. Glasc. Liverp. Manchest. Franc. Belg. Stockholm; Soc. Phil. Ebor. et Americ., Soc. Sci. Christiania, Socius. *Geological Survey Office,* 28 *Jermyn-street,* S.W.; 10 *Chester-terrace, Regent's Park.* N.W.

Date of Election.	Served on Council.		
June 3, 1875.			† Geikie, James, D.C.L. (Dunelm.) LL.D. F.R.S.E. F.R.G.S. F.G.S. Murchison, Professor of Geology and Mineralogy in the University of Edinburgh, Hon. Memb. Phil. Soc. York, Geol. Soc. Stockholm, Vidensk.-Selsk. Christiania, Geol. Palæont. Hydrol. Belg., Soc. Geol. Neuchâtel, Memb. Amer. Phil. Soc. Corresp. Memb. Acad. Sci. Philadelphia. 31 *Merchiston-avenue, Edinburgh.*
June 2, 1892.			Giffen, Sir Robert, K.C.B. LL.D. (Glasc.) 44 *Pembroke-road, Kensington.* W.
June 7, 1860.	’86–88	R.	Gilbert, Sir Joseph Henry, M.A. (Oxon.) Sc.D. (Camb.) Ph.D. (Giessen) LL.D. (Edin. et Glasc.) Sc.D. (Cantab.) V.P.C.S. F.L.S. Soc. Reg. Agric. Engl. Soc. Honor. Inst. Fr. (Acad. Sci.) Mem. Cor. Soc. Chem. Agric. Ult. et Acad. Agric. et Sylv. Petrovsk. et Soc. Reg. Agric. Hannov. Soc. Honor. Soc. pro. fov. Ind. Nat. Par. Soc. Agric. Franc. et Inst. Agron. Gorigoretsk. Mem. Corr. Acad. Reg. Agric. Sued. Soc. Late Sibthorpian Professor of Rural Economy in the Univ. of Oxford. *Harpenden, St. Alban's ; and Athenæum Club.*
June 4, 1891.			Gilchrist, Percy Carlyle, A.R.S.M. *Frognal Bank, Finchley-road, Hampstead.* N.W.
June 7, 1883.			* Gill, David, LL.D. (Aberd. et Edin.) Hon. F.R.S.E. F.R.A.S. F.R.G.S. Her Majesty's Astronomer at the Cape of Good Hope, Trustee, S. African Museum, Corresp. Mem. Imp. Acad. Sci. S. Petersb., and Roy. Acad. Sci. Berl., For. Mem. Soc. Holl. des Sci., Haarlem; Corresp. Mem. Soc. degli Spettroscop. Ital. Rome, Soc. Nationale des Sci. Nat. et Math. Cherbourg, Geogr. Soc. Lisbon. *Royal Observatory, Cape of Good Hope.*
June 2, 1853.	’63–64 ’66–68		Gladstone, John Hall, Ph.D. Sc.D. (Dubl.) V.P.C.S. 17 *Pembridge-square.* W.
Jan. 13, 1881.			† Gladstone, Right Hon. William Ewart, D.C.L. *Hawarden, Chester.*
June 7, 1849.			Glaisher, James, F.R.A.S. Ord. Bras. Rosae Eq. *The Shola, Heathfield Road, South Croydon.*
June 3, 1875.	’83–84 ’90–92		Glaisher, James Whitbread Lee, Sc.D. (Camb. and Dubl.) V.P.R.A.S. F.C.P.S. *Trinity College, Cambridge.*
June 8, 1882.	’92–94		* Glazebrook, Richard Tetley, M.A. Treas. C.P.S. Fellow of Trinity College, Cambridge. 7 *Harvey-road, Cambridge.*
June 8, 1882.	’91–93		Godman, Frederick Ducane, F.L.S. F.G.S. F.E.S. 10 *Chandos-street, Cavendish-square,* W.; *and South Lodge, Horsham.*
June 3, 1880.			* Godwin-Austen, Henry Haversham, Lieut.-Col. F.G.S. F.Z.S. F.R.G.S. *Shalford House, Guildford.*
June 1, 1865.			Gore, George LL.D. (Edin.). *Inst. Sci. Research,* 67 *Broad-street, Birmingham.*
Jan. 18, 1872.			Goschen, Right Hon. George Joachim, M.A. 69 *Portland-place.* W.
June 2, 1892.			* Gotch, Francis, B.A. B.Sc. (Lond.) Hon. M.A. (Oxon.) M.R.C.S. Waynflete Professor of Physiology in the University of Oxford. *The Lawn, Banbury-road, Oxford.*
June 9, 1887.			* Gowers, William Richard, M.D. F.R.C.P. Fellow of University College, London, Consulting Physician to University College Hospital. Physician to the National Hospital for the Paralysed and Epileptic. 50 *Queen Anne-street.* W.
Feb. 3, 1881.			† Grant Duff, Right Hon. Sir Mountstuart Elphinstone, G.C.S.I. V.-P.R.G.S. *Athenæum Club; and York House, Twickenham.*

Date of Election.	Served on Council.		
June 4, 1886.	'93–95		* Green, Alexander Henry, M.A. Professor of Geology in the University of Oxford. *University Museum, Oxford.*
June 13, 1895.			* Green, Joseph Reynolds, M.A. B.Sc. (Lond.) D.Sc. (Camb.) F.L.S. Professor of Botany to the Pharmaceutical Society of Great Britain. *Arncliffe, Grange-road, Cambridge.*
June 7, 1888.			* Greenhill, Alfred George, M.A. Professor of Mathematics in the Artillery College, Woolwich. 10 *New Inn.* W.C.
June 6, 1878.			† Greenwell, Rev. William, M.A. D.C.L., Canon of Durham, F.S.A. *Durham.*
June 13, 1895.			* Griffiths, Ernest Howard, M.A. 12 *Parkside, Cambridge.*
Nov. 26, 1840.	'45–50 '56–58 '61–62 '66–67 '78–79	R.	Grove, Right Hon. Sir William Robert, Knt., M.A. D.C.L. (Oxon.) LL.D. (Cantab.) Hon. F.R.S.E. Hon. Mem. Inst. C.E. Ord. Imp. Rosæ Brazil Eq. Acadd. Reg. Sci. Taurin., Lync. Romæ, Soc. Phil. Basil. et Soc. Imp. Carob. Corresp. 115 *Harley-street.* W.
June 7, 1883.			* Groves, Charles Edward, F.C.S. F.I.C. 352 *Kennington-road.* S.E.
June 7, 1883.			* Grubb, Sir Howard, F.R.A.S. 51 *Kenilworth-square, Rathgar, Dublin.*
June 6, 1867.	'74–76	R.	Günther, Albert C. L. G., M.A. M.D. Ph.D. F.L.S. F.Z.S. Late Keeper of the Zoological Department in the British Museum, Reg. Scient. Soc. Upsal Soc. Phys.-Med. ad Rhenum infer. Soc. Zool.-Bot. Vindob. Socius ord. Reg. Acad. Panormit. Scient. Soc. Asiat. Bengal. Instit. Nov. Zel. Soc. Linn. Nov. Gall. Soc. Nat. Scrutat. Basil. Soc. Zool. Gall. Soc. Lit. et Phil. Liverpool Soc. Roman. Zoolog. Socius Honor. Imp. Acad. Scient. Petropol. Reg. Acad. Scient. Taurin. Reg. Acad. Scient. Suec. Soc. Senckenb. Nat. Scrutat. Francci. Acad. Scient. nat. Philad. Acad. Scient. nat. Californ. Soc. Scient. nat. Cherbourg Soc. Human. et Scient. Gall. Merid. Orient. Soc. extran. *Lichfield Road, Kew Gardens, Surrey.*
June 4, 1891.			* Halliburton, William Dobinson, M.D. B.Sc. Professor of Physiology in King's College, London. 9 *Ridgmount-gardens, Gower-street.* W.C.
Jan. 13, 1887.			† Halsbury, Right Hon. Hardinge Stanley Giffard, Lord, M.A. D.C.L. 4 *Ennismore-gardens.* W.
June 4, 1868.	'78–80		Harcourt, Augustus George Vernon, M.A. (Oxon.) D.C.L. (Dunelm.) LL.D. (Univ. McGill) F.C.S., Lee's Reader in Chemistry at Christ Church. *Cowley Grange, Oxford; and Athenæum Club.* S.W.
Dec. 15, 1881.			† Harcourt, Right Hon. Sir William George Granville Venables Vernon, Knt., M.A. *Malwood, Lyndhurst, Hants.*
June 1, 1865.			Harley, George, M.D. F.R.C.P. F.C.S. Acad. Sci. Monach. et Soc. Phys. Med. Herbip. et Acad. Med. Chir. Madrit. Corr. Mem. 25 *Harley-street.* W.
June 4, 1863.			Harley, Rev. Robert, M.A. (Oxon.) F.R.A.S. Lit. et Phil. Soc. Manc. Soc. Reg. Queensl. Soc. Honor. *Rosslyn, Westbourne-road, Forest-hill.* S.E.
June 12, 1884.			* Hartley, Walter Noel, F.R.S.E. F.I.C. Professor of Chemistry in the Royal College of Science for Ireland. *Royal College of Science, Stephen's-green, Dublin; and* 36 *Waterloo-road, Dublin.*
June 3, 1858.			Haughton, Rev. Samuel, M.D. (Dubl. et Bonon.) D.C.L. (Oxon.) LL.D. (Cantab. et Edin.) Fellow of Trinity College, Dublin. *Trinity College, Dublin.*
June 2, 1864.			Hay, Right Hon. Sir John Charles Dalrymple, Bart., Admiral, K.C.B. D.C.L. (Oxon.) F.R.G.S. V.P. Inst. Naval Architects. 108 *St. George's-square,* S.W.; *and Craigenveoch, Wigtownshire,* N.B.

Date of Election.	Served on Council.	
June 1, 1876.		† Hayward, Robert Baldwin, M.A. *Ashcombe, Shanklin, Isle of Wight.*
June 4, 1891.		* Heaviside, Oliver, Hon. Mem. Lit. Phil. Soc. Manchester. *Paignton, Devon.*
June 7, 1866.		Hector, Sir James, K.C.M.G. Ord. Cr. Pruss. M.D. F.G.S. F.L.S. F.R.S.E. C.M.Z.S., Hon. Mem. of the Royal Societies of Victoria, New South Wales, South Australia, and Tasmania; For. Mem. Amer. Acad. Sci., Amer. Inst. Mining Engs., and K. Leop. Carol. Acad.; Director of the Geological Survey, Colonial Laboratory, Meteorological and Weather Departments, and of the New Zealand Institute; Chancellor of the New Zealand University. *Wellington, New Zealand.*
June 6, 1889.		* Hemsley, William Botting, A.L.S. Hon. Memb. Nat. Hist. Soc. Mexico, Principal Assistant in the Herbarium, Royal Gardens, Kew. *Herbarium, Royal Gardens, Kew.*
June 3, 1875.		Hennessey, John Baboneau Nickterlien, C.I.E. M.A. F.R.A.S. F.R.G.S. late Deputy Surveyor-General in charge of the Trigonometrical Surveys, Survey of India. *Merrimu,* 18 *Alleyn-park, West Dulwich,* S.E.; *and Athenæum Club.* S.W.
June 3, 1858.		Hennessy, Henry G., M.R.I.A. Professor of Applied Mathematics and Mechanism in the Roy. Coll. of Science for Ireland. *Clarens, Montreux, Switzerland.*
June 4, 1874.	'82–83	† Henrici, Olaus Magnus Friedrich Erdmann, Ph.D. LL.D. (St. And.) Professor of Mechanics and Mathematics in the City and Guilds of London Institute. *Central Technical College, Exhibition-road,* S.W.; *and* 34 *Clarendon-road, Nottinghill.* W.
June 2, 1892.		* Herdman, William Abbott, D.Sc. F.R.S.E. F.L.S. Professor of Natural History in University College, Liverpool. *University College, Liverpool.*
June 12, 1884.		Herschel, Alexander Stewart, M.A. Hon. D.C.L. (Durham), F.R.A.S. Honorary Professor of Physics and Experimental Philosophy in the Durham College of Science, Newcastle-on-Tyne. *Observatory House, Slough, Bucks.*
June 8, 1871.		† Herschel, John, Col. R.E. F.R.A.S. Late Deputy Superintendent, Great Trigonometrical Survey of India. *Observatory House, Slough, Bucks.*
Jan. 21, 1892.		† Herschell, Right Hon. Farrer, Lord, G.C.B. D.C.L. LL.D. Chancellor of the University of London. 46 *Grosvenor-gardens.* S.W.
June 13, 1895.		Heycock, Charles Thomas, M.A. Lecturer on Natural Science, King's College, Cambridge. 24 *Fitzwilliam-street, Cambridge.*
Feb. 7, 1839.		Heywood, James, M.A. F.G.S. F.S.A. 26 *Kensington Palace-gardens,* W.; *and Athenæum Club.* S.W.
June 4, 1885.		* Hicks, Henry, M.D. M.R.C.S. F.G.S. Corresp. Memb. Acad. Nat. Sci. Philad., Geol. Soc. Liverpool. *Hendon-grove, Hendon.* N.W.
June 5, 1862.		† Hicks, John Braxton, M.D. (Lond.) F.L.S. Coll. Reg. Med. Soc. *The Brackens, Lymington, Hants.*
June 4, 1885.		* Hicks, William Mitchinson, M.A. D.Sc. Late Fellow of St. John's College, Cambridge, Principal and Professor of Physics in Firth College, Sheffield. *Dunheved, Endcliffe-crescent, Sheffield.*
June 13, 1895.		* Hickson, Sidney John, D.Sc. (Lond.) M.A. (Camb.) Hon. M.A. (Oxon.) F.L.S. Fellow of Downing College, Cambridge; Professor of Zoology in Owens College, Manchester. *Ellesmere House, Wilmslow Road, Fallowfield, Manchester.*

Date of Election.	Served on Council.		
June 7, 1894.			* Hill, Micaiah J. M., M.A., D.Sc. Professor of Mathematics, University College, London. 27 *Parliament-hill-road, Hampstead.* N.W.
June 6, 1872.			† Hincks, Rev. Thomas, B.A. (Lond.) *Stokeleigh, Leigh Woods, Clifton, Bristol.*
June 4, 1863.	'76–78	R.	† Hind, John Russell, LL.D. (Glasc.) F.R.A.S. Inst. Fr. (Acad. Sci.) Par. et Acad. Imp. Sci. Petrop. Corr., late Superintendent of the Nautical Almanac. 3 *Cambridge Park-gardens, Twickenham.*
June 7, 1855.			† Hippisley, John, F.R.A.S. *Athenæum Club,* S.W.; *and Stoneaston Park, Bath.*
June 1, 1893.			* Hobson, Ernest William, D.Sc. Fellow of Christ's College, Cambridge. *The Gables, Mount Pleasant, Cambridge.*
June 13, 1895.			* Holden, Henry Capel Lofft, Major, R.A. *The Eaves, Belvedere, Woolwich.*
Apr. 22, 1847.	'53–54 '56–58 '62–64 '70–80 '84–86	C. R. Dw.	Hooker, Sir Joseph Dalton—Past President—K.C.S.I. C.B. M.D. D.C.L. LL.D. F.L.S. F.G.S. F.R.G.S. Hon. Mem. Roy. Bot. Soc. and Roy. Med. Chir. Soc., London; Bot. and Med. Socs., Edin.; Nat. Hist. Soc. Newcastle; Camb. Philos. Soc.; Asiat. Soc. Beng.; and New Zeal. Institute. Corresp. Mem. of Acad. Sci., Paris (Sect. Botan.). Member of Acad. Imp. Sci., St. Petersb.; K. Akad. der Wissensch., K. K. Geogr. Gesell., and Hort. Soc. of Vienna; K. Akad. der Wissensch. Berlin; Accad. delle Sci. dell' Istit. Bologna; Acad. Roy. des Sci. Brussels; Reale Accad. dei Georgofili, Florence; Kong. Dansk. Vidensk. Selsk. Copenh.; K. Gesell. der Wiss. Gött.; K. Danske Vidensk. Selskab., Stockholm; K. Vetensk. Soc., Upsala; K. Phys-oekonom. Gesell. Königsb.; Soc. Vellosiana, Rio de Janeiro; K. Leopold.-Carol. Deut. Akad. der Naturf., Halle; Senck. Naturf. Gesell. Frankf. a M.; K. Baier. Bot. Gesell., Ratisb.; R. Accad. dei Lincei, Rome; Amer. Acad. of Sci., Boston. Corresp. Mem. of Dubl. Nat. Hist. Soc. and Agricult. Soc. of Paris. For. Mem. of Acad. de Méd., Paris and Nat. Acad. of Sci., Washington. *The Camp, Sunningdale, Berkshire.*
June 6, 1878.	'86–87 '91–93	R.	Hopkinson, John, M.A. D.Sc. *Holmwood, Wimbledon.* S.W.
June 4, 1886.		R.	* Horsley, Victor Alexander Haden, B.S. F.R.C.S. M.D. (Halle) Professor of Pathology in University College, London. 25 *Cavendish-square,* W.; *and Athenæum Club.* S.W.
June 1, 1893.			* Howorth, Sir Henry Hoyle, K.C.I.E. D.C.L. 30 *Collingham-place, Cromwell-road.* S.W.
June 12, 1884.			* Hudleston, Wilfrid H., M.A. F.G.S. F.C.S. 8 *Stanhope-gardens, South Kensington.* S.W.
June 6, 1889.			* Hudson, Charles Thomas, M.A. LL.D. (Camb.). 2 *Barton-terrace, Dawlish.*
June 1, 1865.	'66–68 '69–71 '80–82 '88–89	Rm. R.	Huggins, William—Vice-President—D.C.L. (Oxon.) LL.D. (Cantab. Edin. Dubl. et St. And.) Ph.D. Lugd. Bat. Hon. F.R.S.E. F.R.A.S. Ord. Imp. Bras. Rosae Com. Inst. Fr. (Acad. Sci.) Soc. Reg. Sci. Gött. et Soc. Spettros. Ital. Mem. Corr. Acad. Lync. Romæ Soc. Acad. Amer. Art. et. Sci. Boston. Reg. Sci. Hafn. Physiogr. Lund. Reg. Boie. Marob. Acad. Reg. Sci. Acad. Reg. Hib. Soc. Reg. Dubl. Lit. Phil. Manc., Soc. Astr. de France, Soc. Astr. et Phys. Toronto, Soc. Hist. Dallas et Soc. Reg. Nov. Camb. Austr. Soc. Honor. 90 *Upper Tulse-hill; and Athenæum Club.* S.W.
June 3, 1880.		R.	* Hughes, David Edward, Com. of the Leg. d'Honn. Fr. and Com. of Ords. of Charles III. Spain, Iron Crown Austr., SS. Anne Russ., Mich. Bav., Maur. et Lazar. Ital. and Medijie Turkey, Past-Pres. Soc. Teleg. Eng. 40 *Langham-street, Portland-place.* W.

Date of Election.	Served on Council.	
June 6, 1889.		* Hughes, Thomas McKenny, M.A. Trin. Coll. Camb. F.G.S. F.S.A. Professorial Fellow of Clare College, Camb., Chev. Ord. SS$^{\text{rum.}}$ Maur. et Lazar. Ital. Corresp. Memb. Soc. Géol. de Belg. Memb. Soc. Géol. de Fr. Woodwardian Professor of Geology in the University of Cambridge. 18 *Hills-road, Cambridge.*
June 6, 1867.		† Hull, Edward, M.A. F.G.S. LL.D. (Glasc.) late Director of the Geological Survey of Ireland, and Professor of Geology in the Royal College of Science, Master in Engineering (Hon. Caus. Dubl.) Hon. Mem. Acad. Sci. Amer. Philad. Corresp. Soc. Geol. Belg. Soc. Extr. Hon. Mem. Geol. Soc. Edin., Glasg., Manch. 20 *Arundel-gardens, Notting-hill.* W.
June 9, 1859.	'70–71	† Humphry, Sir George Murray, Knt., M.D. LL.D. (Edin.) Sc.D. (Dubl.) Prof. of Surgery in the Univ. of Cambridge, Fellow of King's Coll. Hon. Fellow of Downing Coll. *Grove Lodge, Cambridge.*
June 8, 1882.		* Hutchinson, Jonathan, LL.D. (Glasc. and Camb.) M.D. (Dubl.) F.R.C.S. Corr. Mem. Soc. Chir. Paris, Hon. Mem. Soc. Dermat. Nov. Ebor. Formerly President of and Professor of Pathology and Surgery in the Royal College of Surgeons. 15 *Cavendish-square.* W.
June 2, 1892.		* Hutton, Frederick Wollaston, Captain. F.G.S. C.M.Z.S. Curator of the Canterbury Museum, Christchurch, Cor. du Mus. d'Hist. Nat. Paris, Cor. Mem. Roy. Soc. Tas. Hon. Mem. Roy. Soc. N.S.W. Cor. Acad. Nat. Sci. Philad. Cor. Ornith. Ver. Wien, Cor. K. K. Geol. Reichsanst. Wien. *Canterbury Museum, Christchurch, New Zealand.*
June 6, 1878.		† Jackson, John Hughlings, M.D. Coll. Reg. Med. Soc., Consulting Physician to the London Hospital. 3 *Manchester-square.* W.
Feb. 5, 1891.		Jackson, Right Hon. William Lawies. 27 *Cadogan-square,* S.W.; *and Allerton Hall, Chapel Allerton, Leeds.*
June 4, 1885.		* Japp, Francis Robert, M.A. Ph.D. LL.D. (St. And.) F.I.C. F.C.S. Prof. of Chemistry in the University of Aberdeen. *University, Aberdeen.*
June 2, 1864.		Jenner, Sir William, Bart., G.C.B. M.D. D.C.L. (Oxon.) LL.D. (Cantab. et Edin.) Physician in Ordinary to the Queen, and to H.R.H. the Prince of Wales. *Greenwood, Durley, Bishop's Waltham.*
June 7, 1888.		* Jervois, Sir William Francis Drummond, Lieut.-Gen., R.E., G.C.M.G. C.B. *Merlewood, Virginia Water.*
June 6, 1872.		† Johnson, Sir George, Kt. M.D. (Lond.) Coll. Reg. Med. Soc., Physician Extraordinary to the Queen, Emeritus Professor of Clinical Medicine in King's College, London, and Consulting Physician to King's College Hospital. 11 *Savile-row.* W.
June 2, 1892.		* Joly, John, M.A. B.E. D.Sc. 39 *Waterloo-road, Dublin.*
June 7, 1894.		* Jones, John Viriamu, M.A. (Oxon.) B.Sc. (Lond.) Principal and Professor of Physics in the University College of South Wales and Monmouthshire. Fellow of University College, London. 42 *Park-place, Cardiff.*
June 6, 1872.		Jones, Thomas Rupert, F.G.S. Hon. Mem. Gesell. Isis, Dresden, Soc. Belg. de Microsc., and Soc. Geol. Hydrol. Palæontol. Brux., Geol. Assoc. Lond., Geol. Socs. Edin. and Glasg., Roy. Irish Geol. Soc., and Anthrop. Inst. Lond. Corresp. Mem. of the K.-K. Geolog. Reichsanst. Wien, and Acad. Nat. Sci. Philad. 17 *Parson's Green, Fulham.* S.W.

Date of Election.	Served on Council.			
June 7, 1877.	'87–89			Judd, John Wesley, C.B. F.G.S. Professor of Geology in the Royal College of Science, London, and Dean of the College, Soc. Phil. Ebor. et Sci. Nat. Deva. Soc. Honor, Acad. Sci. Nat. Philad. Soc. Geol. Belg. Brux. Inst. Imp. Geol. Vindob. Corresp. 16 *Cumberland-road, Kew; Royal College of Science, South Kensington; and Athenæum Club.* S.W.
June 5, 1851.	'90–95	C. R.		Kelvin, William Thomson, Lord, Past President—D.C.L. (Oxon) LL.D. (Camb. Dubl. Edin.) F.R.S.E. Hon. Mem. Inst. C.E., and Elect. Eng. Professor of Natural Philosophy in the University of Glasgow, and Fellow of St. Peter's College, Cambridge, Trust. Brit. Mus. Grand Officier of the Legion of Honour of France. Ord. Boruss. "*Pour le Mérite*" Eq. Comm. Ord. of Leopold, Belgium, Comm. Imp. Ord. of the Rose, Brazil. Assoc. Etrang. Inst. Fr. (Acad. Sci.) Par.; Corresp. Mem. R. Inst. Lomb., R. Acad. dei Lincei; For. Mem. Soc. Roy. Sci. Gött., Soc. Ital. delle Scienze, Soc. Reale di Napoli, Acad. Nat. Sci. Philad.; Hon. Mem. Acad. Imp. Sci. Vienna, Acad. Nov. Lync. Rom., United Service Inst. Lond., Lit. and Phil. Soc. Manch., Phil. Soc. Glasg., Roy. Irish Acad. *University, Glasgow; and Athenæum Club.* S.W.
June 2, 1881.				Kempe, Alfred Bray, M.A. 2 *Paper-buildings, Temple,* E.C.; *and* 23 *Gilbert-street, Brook-street.* W.
June 9, 1887.			*	Kennedy, Alexander B. W., LL.D. Mem. Inst. C.E. Pres. Inst. M.E. Emeritus Professor of Engineering and Mechanical Technology in University College, London. 2 *Gloucester-place, Portman-square.* W.
June 5, 1890.			*	Kerr, Rev. John, LL.D. Mathematical Lecturer in the Free Church Training College, Glasgow. 113 *Hill-street, Glasgow.*
June 9, 1887.			*	King, George, M.B. LL.D. C.I.E. F.L.S. Superintendent of the Royal Botanical Gardens, Calcutta, and of the Government Cinchona Plantations, Darjeeling. *Seebpore, Calcutta.*
				Kingsburgh (*see* Macdonald).
June 9, 1887.	'93–95		*	Kirk, Sir John, G.C.M.G. K.C.B. M.D. LL.D. F.L.S. F.R.G.S. *Wavertree, Sevenoaks, Kent; and Athenæum Club.* S.W.
June 3, 1875.	'88–90		†	Klein, Edward Emanuel, M.D. Lecturer on General Anatomy and Physiology in the Medical School, St. Bartholomew's Hospital. 19 *Earl's Court-square.* S.W.
June 12, 1884.	'94–95		*	Lamb, Horace, M.A. (Cantab.) Professor of Mathematics in the Owens College, Manchester. *Burton-road, Didsbury, Manchester.*
June 7, 1883.		R.	*	Langley, John Newport, M.A. Fellow and Lecturer of Trinity College, Lecturer on Histology in the University of Cambridge. *Trinity College, Cambridge.*
June 3, 1875.	'82–83 '88–90 '94–95	R.		Lankester, Edwin Ray—Vice-President—M.A. (Oxon.) LL.D. (St. And.) Linacre Professor of Human and Comparative Anatomy in the University of Oxford, Fellow of Merton College, Honorary Fellow of Exeter College, Oxford; Hon. Mem. Camb. Phil. Soc. and Roy. Phys. Soc. Edin.; Corr. Acad. Nat. Sci., Philadelphia. 2 *Bradmore-road, Oxford; and Athenæum Club.* S.W.
June 7, 1888.		R.	*	Lapworth, Charles, LL.D. (Aberd.), F.G.S. Professor of Geology in the Mason Science College, Birmingham. 13 *Duchess-road, Edgbaston, Birmingham.*
June 2, 1892.				Larmor, Joseph, M.A. D.Sc. (Lond.) Fellow of St. John's College, Cambridge; late Professor of Natural Philosophy in Queen's College, Galway, and Fellow of the Royal University of Ireland. *St. John's College, Cambridge.*

Date of Election.	Served on Council.		
June 1, 1854.		R.	Lawes, Sir John Bennet, Bart., D.C.L. (Oxon.) LL.D. (Edin.) Sc.D. (Cantab.) F.C.S. Inst. Fr. (Acad. Sci.) Corr. *Rothamsted, St. Albans.*
June 5, 1890.			* Lea, Arthur Sheridan, Sc.D. Fellow, Lecturer in Physiology, and Assistant Tutor of Gonville and Caius College, Assistant Lecturer of Trinity College, and University Lecturer, Cambridge. *Gonville and Caius College, Cambridge.*
June 3, 1880.			* Limerick, Charles Graves, Lord Bishop of, D.D., D.C.L. *The Palace, Limerick, Ireland.*
June 7, 1860.	'81–83 '93–95	R.	Lister, Sir Joseph, Bart.—PRESIDENT—F.R.C.S. D.C.L. (Oxon.) Hon. M.D. (Dubl. Würzburg, Bologna) LL.D. (Camb. Edin. Glasg.) Emeritus Professor of Clinical Surgery, King's College, London, Surgeon Extraordinary to the Queen. Knt. Comm. Ord. Danebrog, Knt. Pruss. Ord. "*Pour le Mérite.*" Inst. Fr. (Acad. Sci.) Mem. Assoc. Hon. Mem. Amer. Acad. Arts and Sci. 12 *Park-crescent, Portland-place.* W.
June 12, 1879.	'91–92		Liveing, George Downing, M.A. Sc.D. (Dubl.) Professor of Chemistry, Fellow of St. John's College, Cambridge. *Newnham, Cambridge.*
June 8, 1882.			* Liversidge, Archibald, M.A. (Camb.) F.G.S. F.C.S. F.I.C. F.R.G.S., Assoc. R.S.M. V.P. Roy. Soc. N.S.W. Memb. Phys. Soc. Lond. Min. Soc. Gr. Brit. Min. Soc. Fr. Corr. Mem. Roy. Soc. Tasm. Roy. Soc. Queensland, Soc. d'Acclimat. Maur. Senck. Naturf. Gesell. Frankf. Hon. Mem. Roy. Soc. Vict. New Zeal. Inst. and K. Leop. Carol. Acad. Halle. Corresp. Edin. Geol. Soc. Professor of Chemistry in the University of Sydney. *The University, Sydney, New South Wales.*
June 3, 1869.	'74–76 '85–87 '91–93	Rm.	† Lockyer, Joseph Norman, C.B. F.R.A.S. Phys. Soc. Lond., Ord. Imp. Bras. Rosae. Eq. Inst. Fr. (Acad. Sci.) Soc. pro fov. Indust. Nat. Par. Soc. Reg. Sci. Gött. Frank. Inst. Philad. Soc. Phys. Soc. Reg. Med. Brux. Soc. Spettros. Ital. Reg. Sci. Panorm. et Hist. Nat. Genev. Mem. Corr. Acad. Reg. Linc. Romæ. et Soc. Phil. Amer. Philad. Socius. Soc. Lit. et Phil. Manc. Acad. Gioen. Sci. Nat. Catan. Soc. Phil. Ebor. et Univ. Lehigh Soc. Honor. 16 *Penywern-road,* S.W.; *Observatory House, Westgate-on-Sea; and Royal College of Science, South Kensington.* S.W.
June 9, 1887.	'93–94		* Lodge, Oliver Joseph, D.Sc. LL.D. (St. And.) Professor of Physics in University College, Liverpool. 2 *Grove-park, Liverpool.*
June 7, 1894.			* Love, Augustus Edward Hough, M.A. Fellow of St. John's College, Cambridge. *St. John's College Cambridge.*
June 6, 1867.			† Lowe, Edward Joseph, F.R.A.S. F.L.S. F.G.S. F.M.S. Soc. Lit. et Phil. Manc. et Lyc. Hist. Nat. Nov. Ebor. Mem. Corr. *Shirenewton Hall, near Chepstow, Monmouthshire.*
June 3, 1858.	'61–63 '70–72 '78–79 '93–94		Lubbock, Right Hon. Sir John, Bart., D.C.L. (Oxon.) LL.D. (Cantab., Dubl. et Edin.) M.D. (Würzb.) V.P.L.S. F.G.S. F.Z.S. F.S.A. F.E.S. Trust. Brit. Mus., Assoc. Acad. Roy. des Sci. Brux., Hon. Mem. Amer. Ethnol. Soc. Anthrop. Socc. Wash. (U.S.), Brux., Firenze., Anthrop. Verein, Graz., Soc. Entom. de France, Soc. Géol. de la Suisse, and Soc. Helvét. des Sci. Nat., Mem. Amer. Phil. Soc. Philad. Corresp. Mem. Soc. Nat. des Sci. Nat. de Cherb., Berl. Gesell. für Anthrop., Soc. Romana di Antrop., Soc. d'Emul. d'Abbeville, Soc. d'Ethn. de Paris, Soc. Cient. Argentina, Soc. de Géog. de Lisb., Acad. Nat. Sci. Philad., Numis. and Ant. Soc. Philad., For. Assoc. Mem. Soc. d'Anthrop. de Paris, For. Mem. Amer. Antiq. Soc. *High Elms, Down, Kent.*

Date of Election.	Served on Council.	
June 7, 1894.		* Lydekker, Richard, B.A. (Camb.). *The Lodge, Harpenden, Herts.*
June 2, 1881.	'94–95	* Macalister, Alexander, M.A. M.D. (Dubl. & Camb.) M.D. Sc.D. (Dubl.) LL.D. (Glasg.) Professor of Anatomy in the University of Cambridge. *Torrisdale, Cambridge.*
June 13, 1895.		* McClean, Frank, M.A. LL.D. (Glasg.) F.R.A.S. M. Inst. C.E. *Rusthall House, Tunbridge Wells.*
June 1, 1865.		McClintock, Sir Francis Leopold, Admiral, K.C.B. D.C.L. L.L.D. 8 *Atherstone-terrace, Gloucester-road.* S.W.
June 3, 1880.		* M'Coy, Sir Frederick, K.C.M.G. M.A. D.Sc. (Cantab.) F.G.S. Professor of Natural Sciences in the University, Melbourne, Soc. Phil. Cantab. Soc. Honor. *Melbourne, Australia.*
June 9, 1859.		† Macdonald, John Denis, M.D. Inspector-General of Hospitals and Fleets, R.N. 82 *Messina-avenue, West Hampstead.* N.W.
May 3, 1888.		† Macdonald, Right Hon. John Hay Athole, C.B. LL.D. F.R.S.E. M.I.E.E. Lord Justice-Clerk of Scotland, and Lord President of the Second Division of the Court of Session. 15 *Abercromby-place, Edinburgh.*
June 13, 1895.		* Macewen, William, M.D. (Glasg.) Hon. LL.D. (Glasg.). Professor of Surgery in the University of Glasgow. 3 *Woodside-crescent, Glasgow.*
June 7, 1877.		McIntosh, William Carmichael, M.D. (Edin.) LL.D. (St. And.) F.L.S. F.R.S.E. L.R.C.S.E. C.M.Z.S. Professor of Natural History in the University of St. Andrews; Director of the University Museum, and of the Marine Laboratory, St. Andrews; Mem. Fishery Board for Scotland; Hon. Mem. Psychol. Soc. Paris and Soc. Centrale d'Aquicult. de France. 2 *Abbotsford-crescent, St. Andrews, Scotland.*
June 12, 1884.	'92–93	* McKendrick, John Gray, M.D. LL.D. F.R.S.E. F.R.C.P.E. Professor of Physiology in the University of Glasgow. 2 *Florentine-gardens, Glasgow.*
June 7, 1877.		McLachlan, Robert, F.L.S. F.Z.S. F.E.S. Soc. Imp. Ami. Sci. Nat. Mosq. Inst. Nov. Zel. Soc. pro Faun. et Flo. Fenn. Soc. Entom. Batav. Soc. Entom. Belg. Soc. Entom. Helvet. Soc. Nat. Hist. Glasc. Soc. Honor. Soc. Reg. Sci. Leodii. Corresp. *Westview,* 23 *Clarendon-road, Lewisham.* S.E.
June 2, 1881.	'87–89	* McLeod, Herbert, F.I.C. F.C.S. Professor of Chemistry in the Royal Indian Engineering College, Cooper's Hill. *The College, Cooper's-hill, Staines.*
June 5, 1890.		* MacMahon, Percy Alexander, Major, R.A. Pres. Lond. Math. Soc. *Artillery College, Woolwich; and Regency-mansions,* 40 *Shaftesbury-avenue.* W.
June 8, 1882.		* Malet, John Christian, M.A. Assistant Commissioner of Intermediate Education, Ireland. *Carbery, Silchester-road, Kingstown, Co. Dublin.*
June 7, 1877.		† Mallet, John William, Ph.D. (Gött.) M.D. LL.D. F.C.S. Mem. of the Chem. Socs. of Paris, Berlin, and New York, and of the Amer. Phil. Soc. Philad. Fellow of the Coll. Phys. Philad. and Hon. Fellow of the Med. Chir. Faculty of Maryland. *University of Virginia, Albemarle Co., Virginia, United States.*
June 11, 1857.		Marcet, William, M.D. F.C.S. Past Pres. Royal Meteorol. Soc. *Flower-mead, Wimbledon-park,* S.W.; *and Athenæum Club.* S.W.
June 12, 1873.		† Markham, Clements Robert, C.B. P.R.G.S. F.S.A. Acad. Caes. Nat. Cur. Socius Soc. Geog. Par. Berol. Vindob. Hist. Philad. et Univ. Chil. Soc. Honor. *Athenæum Club; and* 21 *Eccleston-square.* S.W.

Date of Election.	Served on Council.	
June 4, 1891.		* Marr, John Edward, M.A. F.G.S. Fellow and Lecturer of St. John's College, Cambridge, and University Lecturer in Geology. *St. John's College, Cambridge.*
June 4, 1885.		* Martin, Henry Newell, M.A. (Camb.) M.B. and D.Sc. (Lond.) (Hon.) M.D. Univ. of Georgia. *Physiological Laboratory, University, Cambridge.*
June 13, 1895.		* Martin, Sidney, M.D. B.S. B.Sc. F.R.C.P. Assistant Physician in University College Hospital, and in the Hospital for Consumption, Brompton. 10 *Mansfield-street, Cavendish-square.* W.
June 2, 1870.	'73–75	Maskelyne, Nevil Story, M.A. F.G.S. Late Professor of Mineralogy in the University of Oxford, Hon. Fellow Wadham Coll. Oxon. Soc. Reg. Geol. Cornub. Soc. Imp. Min. Petrop. et Soc. Hist. Nat. Bost. Soc., Acad. Reg. Bavar. Monach. Soc. Cor. *Basset Down House, Swindon.*
June 2, 1870.		† Masters, Maxwell Tylden, M.D. M.R.C.S. F.L.S. Ord. Leopold Eques. Inst. Fr. (Acad. Sci.), Acad. Sci. Nat. Philad., Soc. Reg. Liège et Soc. Sci. Nat. Cherbourg Soc. Corr. *Mount Avenue, Ealing.* W.
June 12, 1879.		* Matthey, George, F.C.S. Assoc. Inst. C.E. Leg. Honor. (France), Ord. Franz Josef (Austria), Great Gold Medal for Arts and Science (Germany). *Cheyne House, Chelsea Embankment.* S.W.
June 7, 1877.		† Medlicott, Henry Benedict, M.A. (Dubl.) F.G.S. Late Director (1876–87) of the Geol. Survey of India. 43 *St. John's-road, Clifton, Bristol.*
June 4, 1886.		* Meldola, Raphael, For. Sec. C.S. F.I.C. F.R.A.S. F.E.S. Professor of Chemistry in the Finsbury Technical College, City and Guilds of London Institute. 6 *Brunswick-square.* W.C.
June 1, 1876.		† Meldrum, Charles, C.M.G. M.A. LL.D. F.R.A.S. F.R. Met. Soc. Director of the Royal Alfred Observatory, Mauritius. *Mauritius.*
June 2, 1892.		* Miall, Louis Compton, F.L.S. F.G.S. Professor of Biology in the Yorkshire College, Leeds. *Crag Foot, Ben Rhydding, Leeds.*
June 4, 1874.		Mills, Edmund James, D.Sc. F.C.S. F.I.C. Young Professor of Technical Chemistry in the Glasgow and West of Scotland Technical College, Glasgow. 60 *John-street, Glasgow.*
June 9, 1887.		Milne, John, F.G.S. Assoc. and Hon. Fellow of King's College, London, Late Professor of Mining and Geology in the Imperial College of Engineering, Japan. *Shide Hill House, Shide, Newport, Isle of Wight.*
June 13, 1895.		* Minchin, George M., M.A. (Dubl.). Professor of Mathematics in the Royal Indian Engineering College, Cooper's-hill. *The College, Cooper's-hill, Staines.*
June 3, 1869.		Mivart, St. George, Ph.D. M.D. F.L.S. F.Z.S. Acad. Sci. Philad. Corr. Mem. Professor of the Philosophy of Biology in the University of Louvain. 15 *Duke-street, St. James's.* S.W.
June 8, 1871.		Moncrieff, Sir Alexander, Colonel (late R.A.), K.C.B. 15 *Vicarage-gate, Kensington,* W.; *and Athenæum Club.* S.W.
June 4, 1891.		Mond, Ludwig, Ph.D. F.I.C. F.C.S. *The Poplars,* 20 *Avenue-road, Regent's-park,* N.W.; *and Winnington Hall, Northwich.*
June 5, 1856.		† Moore, John Carrick, M.A. F.G.S. 113 *Eaton-square,* S.W.; *and Corswall, Stranraer, Wigtonshire.*
Dec. 15, 1892.		† Morley, Right Hon. John, M.A. (Oxon.) Hon. LL.D. (Camb. and Glasg.) 95 *Elm Park Gardens; and Athenæum Club.* S.W.
June 3, 1880.		* Moulton, John Fletcher, M.A. Q.C. 57 *Onslow-square.* S.W.

Date of Election.	Served on Council.			
June 6, 1861.		R.		Mueller, Baron Ferdinand von, K.C.M.G. M.D. Ph.D. Hon. LL.D. (Univ. McGill) F.L.S. F.C.S. F.G.S. F.R.G.S. C.M.Z.S. Corr. Mem. Geol. Soc. Edin., Bot. Soc. Lond. and Edin., and Pharm. Soc. Lond. Hon. Mem. Soc. of Arts, Roy. Hort. Soc., Roy. Irish Acad., Roy. Hist. Soc., Inst. Egypt, Inst. Nat. Genevois, R. Scott. Geogr. Soc. and Geogr. Soc. Manch. For. Mem. of the Academies of Stockholm, Copenhagen, Boston, Munich, and Halle. Corr. Mem. K. Gesell. der Wiss. Göttingen. Pres. Roy. Geogr. Soc. Austr. (Vict. Branch), Government Botanist, and Director of the Botanic Museum, Melbourne. *Melbourne, Victoria.*
June 7, 1866.	'83–85 '89–91			Müller, Hugo, Ph.D. LL.D. (St. And.) V.P.C.S. Ord. SS$^{\text{rum}}$ Lazar. et Maurit. Eq. 13 *Park-square,* N.W.; *Crosby-hill, Camberley, Surrey; and Athenæum Club.* S.W.
Mar. 9, 1882.				Mundella, Right Hon. Anthony John. 16 *Elvaston-place; and Athenæum and Reform Clubs.* S.W.
June 3, 1875.			†	Nares, Sir George Strong, Vice-Admiral, K.C.B. 1 *Beaufort-villas, Surbiton.*
June 2, 1870.	'79–81 '89–91			Newton, Alfred, M.A. F.L.S. V.P.Z.S. Professor of Zoology and Comparative Anatomy in the University of Cambridge. *Magdalene College, Cambridge.*
June 1, 1893.			*	Newton, Edwin Tulley, F.G.S. F.Z.S. *Geological Museum, Jermyn-street.* S.W.
June 3, 1880.				Niven, Charles, M.A. F.R.A.S., Professor of Natural Philosophy in the University, Aberdeen. 6 *Chanonry, Old Aberdeen.*
June 8, 1882.	'92–94		*	Niven, William Davidson, M.A., Director of Studies in the Royal Naval College, Greenwich. *Greenwich.* S.E.
June 2, 1870.	'84–85 '89–90	R.		Noble, Sir Andrew, Capt., K.C.B. F.R.A.S. F.C.S. Ord. Coron. Ital. et Ord. Jes. Christ Portog. Com., Ord. Imp. Bras. Rosae Gr. Off. et Ord. Car. III. Hisp. Eq. *Jesmond Dene House, Newcastle-upon-Tyne; and Athenæum Club.* S.W.
June 5, 1890.			*	Norman, Rev. Alfred Merle, M.A. D.C.L. F.L.S. Hon. Canon of Durham. *Burnmoor Rectory, Fence Houses, Co. Durham.*
Jan. 8, 1880.			†	Northbrook, Thomas George Baring, Earl of, LL.D. D.C.L. G.C.S.I. 4 *Hamilton-place,* W.; *and Stratton, Micheldever Station, Hants.*
June 9, 1859.	'64–66 '79–81		†	Odling, William, M.B. Coll. Reg. Med. Socius. V.P.C.S. Hon. Math. Phys. Doct. (Lugd. Bat.) Waynflete Professor of Chemistry in the University of Oxford. *Museum; and* 15 *Norham-gardens, Oxford.*
June 4, 1863.	'75–76 '80–82	R.		Oliver, Daniel, LL.D. (Aberd.) F.L.S. late Keeper of the Herbarium and Library, Royal Gardens, Kew, Emeritus Professor of Botany, University College, London. 10 *Kew Gardens-road, Kew.*
June 4, 1868.				Ommanney, Sir Erasmus, Admiral, Knt., C.B. LL.D. (Univ. McGill) F.R.A.S. F.R.G.S. Cross of Grand Comm. of Royal Ord. of the Saviour, Greece. 29 *Connaught-square, Hyde Park,* W.; *and United Service Club.*
June 7, 1855.			†	Osler, Abraham Follett. *South Bank, Edgbaston, Birmingham.*
June 4, 1885.			*	O'Sullivan, Cornelius, F.I.C. F.C.S. 140 *High-street, Burton-on-Trent.*
June 5, 1851.	'54–56 '60–62 '71–72 '79–81 '88–89		†	Paget, Sir James, Bart., D.C.L. (Oxon.) LL.D. (Cantab. et Edin.) Hon. M.D. (Dubl.) Corr. Mem. Inst. Fr. (Acad. Sci.) Late Vice-Chancellor of the University of London, Serjeant-Surgeon to the Queen, Surgeon in Ordinary to H.R.H. the Prince of Wales. 5 *Park-square West, Regent's Park.* N.W.
June 8, 1882.				Palgrave, Robert Harry Inglis, F.S.S. *Belton, Great Yarmouth.*

Date of Election.	Served on Council.		
June 7, 1888.			* Parker, T. Jeffery, D.Sc. (Lond.), A.R.S.M. A.L.S. C.M.Z.S. F.R.M.S. Mem. Imp. Soc. Nat. Moscow, Corr. Mem. Linn. Soc. N.S.W. Professor of Biology at the University of Otago. *Dunedin, New Zealand.*
June 2, 1870.			† Parsons, R. Mann, Major-General R.E. *Hyde-vale, Blackheath.* S.E.
June 4, 1863.			† Pavy, Frederick William, M.D. (Lond.) LL.D. (Glasc.) Coll. Reg. Med. Socius, Consulting Physician and formerly Lecturer on Physiology and Comparative Anatomy and Zoology, and on Medicine, at Guy's Hospital. 35 *Grosvenor-street.* W.
June 2, 1892.			* Peach, Benjamin Neeve, F.R.S.E. F.G.S. *Geological Survey Office, Sheriff Court-buildings, Edinburgh.*
June 2, 1892.			Pedler, Alexander, F.C.S. F.I.C. Fellow of the University of Calcutta; Professor of Chemistry, Presidency College, Calcutta; and Meteorological Reporter to the Government of Bengal. *Presidency College, Calcutta.*
June 7, 1894.			* Penrose, Francis Cranmer, M.A. F.R.A.S. Honorary Fellow of Magdalene College, Cambridge. *Colebyfield, Copse Hill, Wimbledon.* S.W.
June 7, 1866.	'79–81 '92–94	D. R.	Perkin, William Henry, V.P.C.S. LL.D. (St. And.) Ph.D. *The Chestnuts, Sudbury, Harrow.*
June 5, 1890.			* Perkin, William Henry, junior, Ph.D. F.R.S.E. F.I.C. F.C.S. Professor of Organic Chemistry in Owens College, Manchester. *Fairview, Wilbraham-road, Fallowfield, Manchester.*
June 4, 1885.			* Perry, John, D.Sc. Professor of Mechanical Engineering and Applied Mathematics in the City and Guilds of London Technical College, Finsbury. 31 *Brunswick-square.* W.C.
June 4, 1868.			Pettigrew, James Bell, M.D. and F.R.C.P. (Edin.), LL.D. (Glasg.) Professor of Medicine and Anatomy and Dean of the Medical Faculty in the University of St. Andrews; Laureate Inst. Fr. *St. Andrews.* N.B.
June 9, 1887.			* Pickard-Cambridge, Rev. Octavius, M.A. *Bloxworth, Wareham, Dorset.*
June 5, 1890.			* Pickering, Spencer Percival Umfreville, M.A. F.C.S. F.I.C. Mem. Phys. Soc. Lond. Deutsch. Chem. Gesell. Berlin. *Harpenden, Herts.*
April 4, 1889.			† Pirbright, Right Hon. Baron Henry de Worms, Lord. 42 *Grosvenor-place,* S.W.; *Henley-park, Guildford.*
June 1, 1876.			† Pitt-Rivers, Augustus Henry Lane-Fox, Lieut.-General, D.C.L. (Oxon.) V.P. Anthrop. Inst. F.S.A. F.G.S. 4 *Grosvenor-gardens,* S.W.; *and Rushmore, Salisbury.*
June 9, 1848.	'56–58 '74–76 '86–87		† Playfair (of St. Andrew's), Right Hon. Lyon, Lord, G.C.B. LL.D. Ph.D. V.P.C.S. & R.S.E. Comm. Leg. Honor. and of Ord. Fran. Joseph Austr., Stell. Bor. Suec. Eq. 68 *Onslow-gardens.* S.W.
June 6, 1861.	'63–65 '75–77 '87–89		Pole, William, Mus. Doc. Oxon., Knt. Commander, Imp. Ord. of the Rising Sun, Japan. Mem. and Hon. Sec. Inst. C.E. Soc. Reg. Edin. Soc. *Athenæum Club.* S.W.
June 6, 1889.			* Poulton, Edward Bagnall, M.A. (Oxon.) F.L.S. F.Z.S. F.G.S. Hope Professor of Zoology in the University of Oxford. *Wykeham House, Banbury-road, Oxford; and St. Helen's Cottage, St. Helen's, Isle of Wight.*
June 13, 1895.			* Power, William Henry, Assistant Medical Officer, Local Government Board. *Glenbrook, Greenhithe; and Local Government Board, Whitehall.* S.W.
June 7, 1888.	'94–95		* Poynting, John Henry, D.Sc. Professor of Physics at the Mason College, Birmingham. *Foxhill, Alvechurch, Worcestershire.*

Date of Election.	Served on Council.		
June 2, 1881.	'87–89		* Preece, William Henry, C.B. Fellow of King's College, London, M. Inst. Electr. Eng. Mem. Inst. C.E. Hon. Mem. Inst. E.E. (America) Officier Lég. Honor, France. *Telegraph Department, General Post Office; and Gothic Lodge, Wimbledon.*
June 2, 1853.	'60–62 '68–69 '72–74 '82–84	R.	† Prestwich, Joseph, D.C.L. (Oxon.) F.G.S. F.C.S. Assoc. Inst. C.E. Inst. Fr. (Acad. Sci.) Corresp. Soc. Geol. Fr. Socius. Inst. Imp. Geol. Vindob. Acad. Lync. Romæ. Acad. Reg. Sci. Belg. Soc. Anthrop. Brux. Soc. Phil. Amer. Philad. Pont. Acad. Romæ. Sci. Nat. Helvet. Soc. Vaud. Hist. Nat. et Soc. Lit. Phil. Manc. Soc. Honor. Soc. Reg. Geol. Cornub. Soc. Hist. Nat. Bost. Mem. Corr. *Darent-Hulme, Shoreham, Sevenoaks.*
June 3, 1852.	'56–58 '64–65 '75–77 '85–87 '92–93		Price, Rev. Bartholomew, D.D. F.R.A.S., Master of Pemb. Coll., Sedleian Prof. of Nat. Phil., and Hon. Fellow of Queen's College, Oxford. Canon of Gloucester. *Master's Lodge, Pembroke College, Oxford; and Athenæum Club.* S.W.
June 13, 1895.			* Purdie, Thomas, B.Sc. Ph.D. A.R.S.M. Professor of Chemistry in the University of St. Andrews. *The University, St. Andrews.*
June 4, 1886.	91–92		Pye-Smith, Philip Henry, M.D. B.A. F.R.C.P. Physician to Guy's Hospital, late Lect. on Physiol. at Guy's Hospital. 48 *Brook-street.* W.
June 8, 1871.			Quain, Sir Richard, Bart., M.D. (Lond.) Hon. M.D. (Dubl. and Roy. Univ. Irel.) Hon. LL.D. (Edin.) F.R.C.P. Hon. F.R.C.P. Irel. President of the General Medical Council, Physician Extraordinary to the Queen, Fellow of the University of London. 67 *Harley-street,* W.; *and Athenæum and Union Clubs.* S.W.
June 7, 1888.		D.	* Ramsay, William Ph.D. (Tüb.) F.C.S. F.I.C. Professor of Chemistry in University College, London. Corresp. Inst. Fr. (Acad. Sci.). 12 *Arundel-gardens, Notting-hill.* W.
June 2, 1870.			Ransom, William Henry, M.D. Coll. Reg. Med. Soc. Consulting Physician to the General Hospital, Nottingham. *The Pavement, Nottingham.*
June 12, 1884.			* Ransome, Arthur, M.A. M.D. Professor of Public Health in Owens College. Examiner in Sanitary Science in Cambridge and Victoria Universities. *Sunninghurst, Dean-park, Bournemouth.*
June 12, 1873.	'77–79 '84–95	R.	Rayleigh, John William Strutt, Lord—SECRETARY—M.A. D.C.L. (Oxon.) Sc.D. (Camb. and Dubl.) LL.D. (Edin. Glasg. and Univ. McGill) Ph.D. (Heidel.) Hon. Fellow of Trinity College, Cambridge, Hon. Mem. Inst. C.E. F.R.A.S. Soc. Reg. Edin. Soc. Lit. et Phil. Manc. Acad. Reg. Sci. Monach. Soc. Honor., Inst. Fr. (Acad. Sci.). Par. Corresp., Soc. Reg. Sci. Gött. Corr. Soc., Professor of Natural Philosophy in the Royal Institution. *Terling Place, Witham, Essex.*
June 1, 1876.			† Reed, Sir Edward James, K.C.B. *Broadway-chambers, Westminster.* S.W.
June 7, 1883			* Reinold, Arnold William, M.A. Professor of Physics in the Royal Naval College, Greenwich. 28 *Belmont Hill, Lee.* S.E.
June 3, 1880.			* Reynolds, J. Emerson, M.D. Sc.D. (Dubl.) M.R.I.A. F.C.S. Professor of Chemistry, University of Dublin. 70 *Morehampton-road, Dublin.*
June 3, 1869			† Reynolds, Sir John Russell, Bart., M.D. Pres. R.C.P. Emeritus Professor of Medicine in University College, London, Acad. Caes. Leop. et Soc. Neurol. Nov. Ebor. Socius. Soc. Psychol. Physiol. Par. Soc. Corresp., Physician in Ordinary to Her Majesty's Household. 38 *Grosvenor-street.* W.

Date of Election.	Served on Council.		
June 7, 1877.	'82–84	R.	Reynolds, Osborne, M.A. (Cantab.) LL.D. (Glasgow), Mem. Inst. C.E. Hon. Fellow, Queen's Coll. Camb. Professor of Engineering in Owens College, Victoria University, Manchester. 23 *Lady Barn-road, Fallowfield, Manchester.*
June 7, 1866.	'68–70 '72–74 '76–78 '86–88		† Richards, Sir George Henry, Admiral, K.C.B., F.R.G.S. Inst. Fr. (Acad. Sci.) Par. Corresp. Soc. Geog. Berol. et Soc. Geog. Ital. Flor. Inst. Nov. Zel. Soc. Honor. *The Cottage, Fetcham, Leatherhead; and Athenæum Club.* S.W.
June 6, 1867.			† Richardson, Sir Benjamin Ward, M.D. LL.D. M.A. F.S.A. Coll. Reg. Med. Soc. Honorary Physician to the Royal Literary Fund. Mem. Acad. Caes. Leop. Nat. Cur. Dresd.; Amer. Phil. Soc. Philad.; Acad. Phys. Med. Stat. Milan; Phil. Soc. St. And. Soc. Ital. d'Hygiène, and Soc. Franc. d'Hygiène. 25 *Manchester-square.* W.
Jan. 13, 1842.			Riddell, Charles James Buchanan, Major-Gen. C.B. *Oaklands, Chudleigh, Devonshire.*
June 4, 1885.			* Ringer, Sydney, M.D. (Lond.) Holme Professor of Clinical Medicine, University College, London. 15 *Cavendish-place.* W.
May 24. 1860.			† Ripon, George Frederick Samuel Robinson, Marquis of, K.G. G.C.S.I. C.I.E. D.C.L. (Oxon.) F.L.S. F.R.G.S. 9 *Chelsea Embankment,* S.W.; *and Studley Royal, Ripon, Yorkshire.*
June 5, 1890.			Roberts, Isaac, Sc.D. (Dubl.) F.R.A.S. F.G.S. *Starfield, Crowborough, Sussex.*
June 6, 1878.			† Roberts, Samuel, M.A. (Lond.). 34 *Lady Margaret-road, St. John's College Park.* N.W.
June 7, 1877.	'90–91		Roberts, Sir William, B.A. M.D. (Lond.) Coll. Reg. Med. Soc. 8 *Manchester-square.* W.
June 3, 1875.	'90–92		Roberts-Austen, William Chandler, C.B. F.C.S. Prof. of Metallurgy, Royal College of Science, Chemist of the Royal Mint, Chev. Lég. Hon. France, Mem. Chem. Soc. Berlin. *Royal Mint, Tower-hill,* E.; *Chilworth, Guildford; and Athenæum Club.* S.W.
June 4, 1863.	'72–73 '81–83 '88–90	R.	Roscoe, Sir Henry Enfield, Knt., B.A. D.C.L. (Oxon.) LL.D. (Cantab. et Dubl. et Montr.) Hon. M.D. (Heidelb.) Ph.D. V.P.C.S. Officier Lég. Hon. France, Inst. Fr. (Acad. Sci.) Fellow of Univ. of Lond., Univ. Coll., and Eton College, Emeritus Professor of Chemistry in Victoria University (Owens College), Vice-President of Literary and Phil. Soc. Manchester, For. Mem. Acad. Sci. Paris. Hon. Mem. of the New York Acad. Sci. Berlin Chem. Soc. of the Verein für Naturwiss. Brunswick, and of the Physikal. Verein, Frankfort-on-Main, Acad. Sci. Monach. Acad. Reg. Sci. Gött. et Soc. Sci. Catan. Corresp. Acad. Cæs. Leop. Soc. Reg. Physiogr. Lund. Soc. 10 *Bramham-gardens, South Kensington,* S.W.; *and Athenæum Club.*
June 10, 1886.			Rosebery, Right Hon. Archibald Philip Primrose, Earl of, K.G. D.C.L. 38 *Berkeley-square,* W.; *and Dalmeny Park, Linlithgowshire.*
Dec. 19, 1867.	'71–72 '87–88		Rosse, Laurence Parsons, Earl of, K.P. B.A. D.C.L. (Oxon.) LL.D. (Dubl.) F.R.A.S. Chancellor of the University of Dublin. *Birr Castle, Parsonstown, Ireland.*
June 6, 1872.	'88–90		Routh, Edward John, D.Sc. (Cantab. et Dubl.) LL.D. (Glasc.) M.A. (Lond.) Fellow of the University of London, Hon. Fellow St. Peter's College, Cambridge, F.R.A.S. F.G.S. *Newnham Cottage, Queen's-road, Cambridge.*
June 12, 1884.			* Roy, Charles Smart, M.D. (Edin.) Professor of Pathology in the University of Cambridge. *Trinity College, Cambridge.*

Date of Election.	Served on Council.		
June 12, 1884.	'87–89 '94–95	R.	* Rücker, Arthur William, M.A. (Oxon.) D.Sc. (Vict.) Professor of Physics, Royal College of Science, London, Hon. Fellow of Brasenose Coll. Oxford, Fellow of the University of London, Treasurer, British Association, Corr. Mem. Leeds Lit. and Phil. Soc. Hon. Mem. Royal Cornwall Polytechnique Society. 19 *Gledhow-gardens, South Kensington,* S.W.; *and Athenæum Club.* S.W.
June 4, 1886.			* Russell, Henry Chamberlaine, C.M.G. B.A. (Sydn.) F.R.A.S. F.R. Met. Soc. Government Astronomer of New South Wales. *The Observatory, Sydney, N.S. Wales.*
June 6, 1872.	'85–86		† Russell, William James, Ph.D. V.P.C.S. Lecturer on Chemistry at the Medical School of St. Bartholomew's Hospital. 34 *Upper Hamilton-terrace.* N.W.
June 1, 1876.			† Rutherford, William, M.D. F.R.S.E. Professor of Physiology in the Univ. of Edinburgh. *The University; and* 14 *Douglas-crescent, Edinburgh.*
Jan. 28, 1869.	'69–70 '82–83 '92–94		† Salisbury, The Most Hon. Robert Arthur Talbot Gascoigne-Cecil, Marquis of, K.G. M.A. D.C.L. (Oxon.) Chancellor of the University of Oxford. 20 *Arlington-street,* S.W.; *and Hatfield House, Hatfield, Herts.*
June 4, 1863.		C. R.	Salmon, Rev. George, D.D. (Dubl. et Edin.) D.C.L. (Oxon.) LL.D. (Cantab. Provost of Trin. Coll. Dubl. Inst. Fr. (Acad. Sci.) Paris, Acad. Imp. Sci. Berol. Soc. Reg. Sci. Gött. Corresp. Soc. Reg. Sci. Hafn. Soc. Extr. *Trinity College, Dublin.*
June 4, 1863.			Salter, Samuel James Augustus, M.B. London, Hon. Fellow King's College, London, F.L.S. *Athenæum Club; and Basingfield, near Basingstoke, Hants.*
June 12, 1873.	'82–84 '94–95		Salvin, Osbert, M.A. F.L.S. F.Z.S. *Hawksfold, Fernhurst, Haslemere.*
June 2, 1881.	'87–88		* Samuelson, Right. Hon. Sir Bernhard, Bart. Mem. Inst. C.E. 56 *Princes-gate.* S.W.
June 6, 1867.	'73–75 '84–86 '93–95	R.	Sanderson, J. S. Burdon, M.A. (Oxon.) M.D. LL.D. Sc.D. (Dubl.) LL.D. (Edin.) D.C.L. (Dunelm.) F.R.S.E. F.R.C.P. Regius Professor of Medicine in the University of Oxford, Hon. Fellow of Magdalen College. *Physiological Laboratory; and* 64 *Banbury-road, Oxford.*
June 6, 1878.	'90–92		† Schäfer, Edward Albert, M.R.C.S. Jodrell Professor of Physiology, University College, London. *Croxley Green, Rickmansworth.*
June 6, 1850.			† Schunck, Edward, F.C.S. *Kersal, Manchester.*
June 12, 1879.	'85–87	R.	* Schuster, Arthur, Ph.D. F.R.A.S. Professor of Physics in Owens College, Victoria University, Manchester. 4 *Anson-road, Victoria-park, Manchester.*
June 6, 1861.	'72–73 '86–87		Sclater, Philip Lutley, M.A. Ph.D. (Bonn) Hon. Fellow of Corpus Christi College. F.L.S. F.G.S. F.R.G.S. Secretary of the Zoological Society of London. 3 *Hanover-square,* W.; *and Odiham Priory, Winchfield, Hants.*
June 7, 1894.			* Scott, Dukinfield Henry, M.A. (Oxon.) Ph. D. (Würzb.) F.L.S. F.G.S. Honorary Keeper of the Jodrell Laboratory, Royal Gardens, Kew. *Old Palace, Richmond, Surrey.*
June 2, 1870			† Scott, Robert Henry, M.A. F.R. Met. Soc. Secretary to the Meteorological Council. Ord. Coron. Ferr. Austr. Eq. Acad. Cæs. Leop. Soc., Soc. Met. Fr. Par. Soc. Imp. Reg. Zool. Bot. Soc. Met. Austr. Vindob. Soc. Met. Germ. Berol. et Soc. Nat. Scrutat. Emb. Soc. Honor. Inst. Geol. Imp. Vindob. et Soc. Isis Dresd. Mem. Corr. *Meteorological Office,* 63 *Victoria-street; and* 6 *Elm-park-gardens.* S.W.

Date of Election.	Served on Council.		
June 4, 1886.	'92–94		* Sedgwick, Adam, M.A. Fellow and Lecturer of Trin. Coll., Cambridge, and Reader of Animal Morphology in the University. *Whitefield, Great Shelford, Cambridge.*
June 12, 1879.			* Seeley, Harry Govier, F.L.S. F.G.S. F.Z.S. F.R.G.S. Professor of Geography and Lecturer on Geology in King's College, London, Inst. Imp. Reg. Geol. Vindob. et Acad. Sci. Nat. Philad. Corresp. Soc. Phil. Ebor. Soc. Imp. Sci. Nat. Hist. Mosq. Soc. 25 *Palace-gardens-terrace, Kensington.* W.
June 4, 1874.			Selwyn, Alfred Richard Cecil, C.M.G. F.G.S. Late Director of the Geological Survey of Canada. *Sussex-street, Ottawa, Canada.*
June 5, 1890.			* Sharp, David, M.B. C.M. (Edin.) Hon. M.A. (Camb.) F.L.S. F.Z.S. Hon. Mem. New Zealand Inst. *Museum of Zoology, Cambridge; and Hawthorndene, Hills-road, Cambridge.*
May 7, 1840.			† Sharp, William, M.D. F.G.S. *Horton House, Rugby.*
June 4, 1891.			Shaw, William Napier, M.A. Fellow and Senior Tutor of Emmanuel College, Cambridge. *Emmanuel College, Cambridge.*
June 1, 1893.			* Sherrington, Charles Scott, M.A. M.D. (Camb.) Professor of Physiology in University College, Liverpool. 16 *Grove-road, Liverpool.*
Jan. 9, 1845.	'69–70 '78–80		Simon, Sir John, K.C.B. F.R.C.S. D.C.L. (Oxon.) LL.D. (Cantab. et Edin.) M.D. (Dubl). M.Chir.D. (Munich), Consulting Surgeon to St. Thomas's Hospital. 40 *Kensington-square.* W.
June 5, 1862.			Simpson, Maxwell, B.A. M.B. M.D. & LL.D. (Hon. Dubl.) F.C.S. D.Sc. F.I.C. Hon. F.K.Q.C.P. (Dubl.) Late Professor of Chemistry in Queen's College, Cork. Late Fellow of the Royal University of Ireland. 9 *Barton-street, West Kensington.* W.
June 7, 1894.			* Smith, Rev. Frederick John, M.A. (Oxon.) University Lecturer in Mechanics and Millard Lecturer in Experimental Mechanics, Trinity College, Oxford. 28 *Norham-gardens, Oxford.*
June 9, 1887.			* Snelus, George James, F.C.S. A.R.S.M. Mem. Inst. M.E. Vice-Pres. Iron and Steel Inst. *Ennerdale Hall, Frizington, Cumberland.*
June 6, 1889.			* Sollas, William Johnson, D.Sc. (Camb.) LL.D. (Dubl.) F.R.S.E. F.G.S. Professor of Geology in the University of Dublin. *Lisnabin, Dartry-park-road, Rathgar, Dublin.*
June 11, 1857.	'76–77	R.	Sorby, Henry Clifton, LL.D. (Cantab.) F.L.S. F.G.S. F.Z.S. F.S.A. F.R.M.S. Soc. Min. Petrop. Soc. Holland. Harl. Socius., Acad. Lync. Romæ. Adsoc. Extr., Amer. Acad. Arts et Sci. Soc. Honor. Acad. Sci. Nat. Philad. et Lyc. Hist. Nat. Nov. Ebor. Corr. Mem. *Broomfield, Sheffield.*
June 6, 1878.			† Sprengel, Hermann Johann Philipp, Ph.D. (Heidelb.) F.C.S. Royal Prussian Professor (titular). *Savile Club,* 107 *Piccadilly.* W.
June 1, 1893.			* Stirling, Edward Charles, C.M.G. M.A. M.D. (Camb.) F.R.C.S. C.M.Z.S. Senior Surgeon, Adelaide Hospital; Lecturer on Physiology in the University of Adelaide; Hon. Director of the South Australian Museum. *The University, Adelaide, South Australia.*

Date of Election.	Served on Council.		
June 5, 1851.	'54–92	C. Rm.	Stokes, Sir George Gabriel, Bart.—PAST PRESIDENT—M.A. D.C.L. (Oxon.) LL.D. (Dubl. Edin. et Cant.) D.Sc. Lucasian Professor of Mathematics in the University of Cambridge, F.C.P.S. F.R.S.E., Hon. Memb. Inst. C.E., Soc. Reg. Hib. Lit. et Phil. Manc. et Med. Chi. Lond. Soc. Honor. Ord. Boruss. "*Pour le Mérite*" Eq. Acad. Imp. Sci. Vindob. Inst. Fr. (Acad. Sci.) Par. Acad. Sci. Berol. Soc. Reg. Sci. Gött. Corresp. Soc. Gall. Phys. Reg. Sci. Upsal. Acad. Sci. Bavar. et Acad. Nov. Lync. Rom. Soc. Extr. Soc. Phil. Amer. et Soc. Batav. Roterod. Socius et Acad. Amer. Bost. Soc. Philos. Glasc. et Mach. Civ. Inst. et Soc. Asiat. Beng. Soc. Honor. *Lensfield Cottage, Cambridge; and Athenæum Club.* S.W.
June 4, 1868.	'81–83		† Stone, Edward James, M.A. F.R.A.S. Radcliffe Observer., Ph.D. (Padua) Hon. Fellow Queen's Coll. Camb., Hon. Mem. Lit. Phil. Soc. Manc. Corresp. Mem. Soc. Nat. Sci. Nat. Cherbourg. *Radcliffe Observatory, Oxford; and Athenæum Club.* S.W.
June 2, 1881.			* Stoney, Bindon Blood, LL.D. M. Inst. C.E. M.R.I.A. M.I.N.A. 14 *Elgin-road, Dublin.*
June 6, 1861.			† Stoney, George Johnstone, M.A. D.Sc. Vice-President R.D.S. F.R.A.S. 8 *Upper Hornsey-rise. N.*
June 1, 1854.	'72–74 '80–81 '84–86 '90–91		† Strachey, Richard, Lieut.-General, R.E. C.S.I. LL.D. (Cantab.) V.P.R.G.S. F.G.S. F.L.S. Chairman Meteorological Council. 69 *Lancaster-gate, Hyde-park.* W.
Mar. 22, 1888.			† Sudeley, Charles Douglas Richard Hanbury-Tracy, Lord. *Ormeley Lodge, Ham Common, Surrey.*
June 7, 1894.			* Swan, Joseph Wilson, M.A. (Durh.) F.C.S. F.I.C. M. Inst. Elec. Eng. 58 *Holland-park.* W.
Apr. 25, 1839.	'62–64 '84–85	C. R.	† Sylvester, James Joseph, M.A. D.C.L. (Oxon.) Hon. Sc.D. (Camb.) LL.D. (Dubl. et Edin.) F.R.S.E. Hon. Fellow of St. John's Coll. Camb. and Fellow of New Coll. Oxford, Savilian Professor of Geometry in the University of Oxford, Officer of the Legion of Honour, Inst. Fr. (Acad. Sci.) et Soc. Philom. Par. Acad. Imp. Sci. Petrop. Reg. Sci. Berol. Ist. Lomb. Mediol. Soc. Sci. Nat. Carob. et Soc. Philom. Par. Mem. Corr. Acad. Reg. Sci. Romæ. Acad. Imp. Sci. Vindob. Soc. Reg. Sci. Gött. Reg. Sci. Neapol. Acad. Sci. Amer. Bost. et Acad. Nat. Sci. Wash. et Philad. et Soc. Ital. Sci. Adsoc. Extr. Soc. Reg. Edin. Acad. Reg. Hib. Lit. Phil. Manc. et Univ. Kasan Soc. Honor. *Athenæum Club,* S.W.; *and New College, Oxford.*
June 6, 1878.			† Symons, George James, Sec. Roy. Met. Soc., Mem. of the Scot. Met. and Soc. Met. de Fr., Corresp. Mem. of the Deutsche Met. Gesell. and of the Soc. Reg. de Méd. Pub. de Belg., Registrar of the Sanitary Institute, Chev. de la Légion d'Honneur. 62 *Camden-square.* N.W.
June 7, 1888.			* Teale, Thomas Pridgin, M.A. F.R.C.S. 38 *Cookridge-street, Leeds.*
June 5, 1890.			* Teall, J. J. H., M.A. F.G.S. 2 *Sussex-gardens, West Dulwich,* S.E.; *and Geological Museum, Jermyn-street.* S.W.
June 3, 1869.			Tennant, James Francis, Lieut.-General, R.E. C.I.E. V.P.R.A.S. 11 *Clifton-gardens, Maida-hill.* W.

Date of Election.	Served on Council.		
June 4, 1891.			* Thompson, Silvanus Phillips, B.A. D.Sc. (Lond.), F.R.A.S. M.D. (Regiomont), Reg. Acad. Sci. Suec. Soc. Phys. Verein, Francof. ad Mœnum. Soc. Honor. Principal and Professor of Physics in the City and Guilds of London Technical College, Finsbury. *Morland, Chislett-road, West Hampstead.* N.W.
June 12, 1884.	'89–91	R.	* Thomson, Joseph John, M.A. Sc.D. (Dubl.) Fellow of Trinity College and Cavendish Professor of Experimental Physics, Cambridge. *Trinity College, Cambridge.*
June 5, 1890.			* Thorne, Richard Thorne, C.B. M.B. (Lond.) F.R.C.P. Medical Officer to H.M. Local Government Board, Vice-President of the Epidemiological Society, Lecturer on Public Health at the Medical School of St. Bartholomew's Hospital. 45 *Inverness-terrace.* W.
June 1, 1893.			Thornycroft, John Isaac, M. Inst. C.E. *Eyot Villa, Chiswick Mall, Chiswick.*
June 1, 1876.	'90–91 '93–95	R.	Thorpe, Thomas Edward, B.Sc. (Vict.) Sc.D. (Dubl.) Ph.D. (Heid.) LL.D. (Glasg.) Treas. C.S. Principal of the Government Laboratories, Somerset House, Fellow of the University of London, Soc. Chym. Berol. Socius. Soc. Phil. Glasc. Mem. Corr. Soc. Phil. Leeds, Lit. Phil. Manc. Soc. Honor. *Government Laboratories, Somerset House,* W.C.; *and Athenæum Club.* S.W.
June 3, 1869.			† Thuillier, Sir Henry Edward Landor, General, R.A. C.S.I. F.R.G.S. *Tudor House, Richmond, Surrey; and Oriental Club.* W.
June 3, 1880.	'92–94		* Tilden, William Augustus, D.Sc. (Lond. and Dubl.) F.C.S. F.I.C. Professor of Chemistry in the Royal College of Science, London. Hon. Mem. Pharm. Soc., Soc. Pub. Anal., Soc. Nat. Bristol, Coll. Pharm. Philad. 9 *Ladbroke-gardens, Notting-hill.* W.
June 4, 1891.			* Tizard, Thomas Henry, Staff-Captain R.N., F.R.G.S. *Hydrographic Department, Admiralty, Whitehall.* S.W.
June 6, 1889.			* Todd, Sir Charles, M.A. (Camb.) K.C.M.G. F.R.A.S. Postmaster-General Superintendent of Telegraphs and Government Astronomer, South Australia. *The Observatory, Adelaide, South Australia.*
June 6, 1878.			Tomes, Charles Sissmore, M.A. (Oxon.). 9 *Park-crescent, Portland-place.* W.
June 6, 1867.			† Tomlinson, Charles, F.C.S. 7 *North-road, Highgate.* N.
June 6, 1889.			* Tomlinson, Herbert, B.A. (Oxon.). 88 *Oakley-street, Chelsea.* S.W.
June 1, 1893.			* Trail, James William Helenus, A.M. M.D. C.M. (Aberd.) F.L.S. Regius Professor of Botany in the University of Aberdeen. *The University, Aberdeen.* N.B.
June 2, 1881.			* Traquair, Ramsay H., M.D. LL.D. F.R.S.E. F.G.S. Keeper of the Natural History Collections in the Museum of Science and Art, Edinburgh. 8 *Dean Park-crescent, Edinburgh.*
June 7, 1888.			* Trimen, Henry, M.B. F.L.S. Director of the Royal Botanic Gardens, Ceylon. *Peradeniya, Ceylon; and Union Club.* S.W.
June 7, 1883.			* Trimen, Roland, F.L.S. F.Z.S. F.E.S. Late Curator of the South African Museum. 9, *Osborne Mansions, Northumberland-street, Marylebone-road,* W.; *and Oriental Club.* W.
June 4, 1868.			Tristram, Rev. Henry Baker, M.A. (Oxon.) LL.D. (Edin.) D.D. C.M.Z.S. Canon of Durham. *College, Durham.*

Date of Election.	Served on Council.			
June 7, 1877.	'90–91			Turner, Sir William, M.B. (Lond.) D.C.L. (Dunelm. et Oxon.) LL.D. (Glasg.) Sc.D. (Dubl.) F.R.C.S. (Edin.) F.R.C.S. F.R.S.E. Professor of Anatomy in the University of Edinburgh, Hon. Prof. Anat. Roy. Scot. Acad., Hon. Member Roy. Irish Acad., Hon. Fellow Roy. Med. Chir. Soc. London, Fos. Assoc. Anthrop. Soc. Paris, Cor. Member Soc. Anthrop. Ethnol. and Prehist. Arch. Berlin. 6 *Eton-terrace, Edinburgh; and Athenæum Club.* S.W.
June 8, 1871.			†	Tylor, Edward Burnett, D.C.L. (Oxon.) LL.D. (St. And. Aberd. and McGill) Assoc. Acad. Reg. Belg. Professor of Anthropology in the University of Oxford. *Museum House, Oxford.*
June 4, 1886.	'93–94		*	Unwin, W. Cawthorne, B.Sc. Mem. Inst. C.E. Mem. Inst. M.E. Mem. Amer. Phil. Soc. Professor of Engineering at the Central Technical College of the City and Guilds of London Institute. 7 *Palace Gate-mansions, Kensington.* W.
June 7, 1894.			*	Veley, Victor Herbert, M.A. F.C.S. *University College; and* 22, *Norham-road, Oxford.*
June 7, 1883.			*	Venn, John, Sc.D. 3 *St. Peter's-terrace, Cambridge.*
June 2, 1870.			†	Verdon, The Hon. Sir George Frederic, C.B. K.C.M.G. *The Melbourne Club, Melbourne, Australia; and Athenæum Club.* S.W.
June 4, 1885.	'90–92		*	Vines, Sydney Howard, M.A. (Oxon.) D.Sc. (Camb. and Lond.) F.L.S. Sherardian Professor of Botany in the University of Oxford, Fellow of the University of London, Hon. Mem. Manc. Lit. Phil. Soc. and Roy. Phys. Soc. Edin., Corr. Mem. Soc. Nat. Sci. et Math. de Cherb. and Soc. Nat. Hist. Bost. *Headington Hill, Oxford.*
June 1, 1865.	'85–86 '90–92			Walker, James Thomas, General, C.B. R.E. LL.D. F.R.G.S. 13 *Cromwell-road, South Kensington.* S.W.
June 7, 1883.			*	Walker, John James, M.A. Mem. Lond. Math. Soc. and Mem. Phys. Soc. 12 *Denning-road, Hampstead.* N.W.
June 1, 1893.		R. Dw.	*	Wallace, Alfred Russel, LL.D. D.C.L. F.L.S. F.Z.S. *Corfe View, Parkstone, Dorset.*
June 2, 1892.			*	Waller, Augustus Désiré, M.D. Lecturer on Physiology at St. Mary's Hospital Medical School. 16 *Grove End-road.* N.W.
Nov. 22, 1860.			†	Walpole, Right Hon. Spencer Horatio, M.A. D.C.L. LL.D. Q.C. Trust. Brit. Mus. 109 *Eaton-square,* S.W.; *and Ealing.*
June 9, 1887.			*	Walsingham, Thomas de Grey, Lord, M.A. LL.D., High Steward of the University of Cambridge, Trust. Brit. Mus., F.L.S. F.Z.S. F.E.S., Mem. Soc. Ent. de France, Ent. Ver. zu Berlin, Nederlands Ent. Ver., Soc. Ent. de Russie, Linn. Soc. N.S.W. *Merton Hall, Thetford, Norfolk.*
June 7, 1888.		R.	*	Ward, Harry Marshall, D.Sc. F.L.S. Late Fellow of Christ's College, Cambridge, Professor of Botany in the University of Cambridge. *Botanical Laboratory, New Museums, Cambridge.*
June 4, 1886.			*	Warington, Robert, M.A. (Oxon.) F.C.S. Sibthorpian Professor of Rural Economy in the University of Oxford. *High Bank, Harpenden, St. Albans.*
June 12, 1884.			*	Warren, Sir Charles, Major-General, R.E. K.C.B. G.C.M.G. *Government House, Chatham; and Athenæum Club.* S.W.
June 2, 1881.			*	Watson, Rev. Henry William, D.Sc. *The Rectory, Berkeswell, Coventry.*

Date of Election.	Served on Council.		
June 5, 1890.			* Weldon, Walter Frank Raphael, M.A. Fellow of St. John's College, Jodrell Professor of Comparative Anatomy and Zoology at University College, London. 30A *Wimpole-street.* W.
June 4, 1886.	'88–89		* Wharton, William James Lloyd, Rear-Admiral, C.B. F.R.A.S. F.R.G.S. Hydrographer of the Admiralty. *Florys, Prince's-road, Wimbledon Park; and Athenæum Club.* S.W.
June 9, 1887.			* Whitaker, William, B.A. F.G.S. Assoc. Inst. C.E. Geological Survey of England and Wales. Corr. Acad. Nat. Sci. Philad. 33 *East Park-terrace, Southampton; and Geological Survey Office,* 28 *Jermyn-street.* S.W.
June 7, 1888.	'94–95		* White, Sir William Henry, K.C.B. LL.D. (Glasg.) F.R.S.E. Mem. Inst. C.E. Fellow Royal School of Naval Architecture, V.P. Inst. Naval Architects, Assistant Controller and Director of Naval Construction. *The Admiralty, Whitehall,* S.W.; *and Athenæum Club.* S.W.
June 4, 1886.			Wilde, Henry, Pres. Lit. Phil. Soc. Manch. *The Hurst, Alderley Edge, Cheshire.*
June 2, 1870.			† Wilks, Samuel, M.D. LL.D. Coll. Reg. Med. Soc. Consulting Physician to Guy's Hospital. 72 *Grosvenor-street.* W.
June 5, 1862.			Williams, C. Greville, F.C.S. F.I.C. *Castlemaine, Oakhill-road, Putney.* S.W.
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June 12, 1879.			* Williamson, Benjamin, M.A. D.Sc. D.C.L. (Oxon.) M.R.I.A. Fellow and Senior Tutor of Trinity College, Dublin. *Trinity College, Dublin.*
June 4, 1874.	'89–90		† Wilson, Sir Charles William, Major-General R.E. K.C.B. K.C.M.G. D.C.L. (Oxon.) LL.D. (Edin.) M.E. (Dubl.) F.R.G.S. *Athenæum Club.* S.W.
June 7, 1855.			Wilson, George Fergusson, F.C.S. F.L.S. *Heatherbank, Weybridge Heath, Surrey.*
June 12, 1873.			† Woodward, Henry, LL.D. (St. And.) P.G.S. F.Z.S. F.R.M.S. Lyc. Hist. Nat. Nov. Ebor. et Soc. Phil. Amer. Philad. Soc. Soc. Phil. Ebor. Assoc. Geol. Lond. Socc. Geol. Edin.. Glasc. Liverp. et Nordov. Soc. Honor. Socc. Geol. Belg. Imp. Nat. Hist. Mosq. Hist. Nat. Montreal et Malacol. Belg. Corresp., Keeper of the Department of Geology, British Museum (Natural History), Cromwell-road, S.W. 129 *Beaufort-street, Chelsea.* S.W.
June 1, 1893.			* Worthington, Arthur Mason, M.A. F.R.A.S. Headmaster and Professor of Physics, Royal Naval Engineering College, Devonport. 6 *Osborne Villas, Devonport.*
June 6, 1889.			* Yeo, Gerald Francis, M.D. (Dublin) F.R.C.S. Emeritus Professor of Physiology in King's College, London. *Bowden, Totnes, South Devon.*
June 1, 1893.			* Young, Sydney, D.Sc. (Lond.) F.C.S. F.I.C. Professor of Chemistry in University College, Bristol. 13 *Aberdeen-road, White Ladies'-road, Bristol.*
June 3, 1852			† Younghusband, Charles Wright, Lieut.-General, C.B. *Palace-court-mansions, Bayswater-road,* W.; *and Athenæum Club.* S.W.

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Beetham, Albert William.
Bristowe, John Syer, M.D.
Buchanan, Sir George, M.D.
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Savory, Sir William Scovell, Bart.
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On the Foreign List.

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Dana, James Dwight.
Lovén, Sven Ludwig.
Ludwig, Carl.
Neumann, Franz Ernst.
Pasteur, Louis.
Tchebitchef, Pafnutij.

Change of Name and Title.

Worms, Baron Henry de, *to* Lord Pirbright.

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1895. June 13.	*Bryan, George Hartley, M.A.
1895. Jan. 24.	†Davey, Right Hon. Horace, Lord, D.C.L.
1895. June 13.	*Eliot, John, M.A.
1895. June 13.	*Green, Prof. Joseph Reynolds, D.Sc.
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1895. June 13.	Heycock, Charles Thomas, M.A.
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1895. June 13.	*Holden, Major Henry Capel Lofft, R.A.
1895. June 13.	*McClean, Frank, M.A., LL.D.
1895. June 13.	*Macewen, Prof. William, M.D.
1895. June 13.	*Martin, Sidney, M.D.
1895. June 13	*Minchin, Prof. George M., M.A.
1895. June 13.	*Power, William Henry
1895. June 13.	*Purdie, Prof. Thomas, B.Sc.

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1732. Stephen Gray.
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1736. John Theophilus Desaguliers.
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1738. James Valoue.
1739. Stephen Hales.
1740. Alexander Stuart.
1741. John Theophilus Desaguliers
1742. Captain Christopher Middleton.
1743. Abraham Trembley.
1744. Henry Baker.
1745. Sir William Watson.
1746. Benjamin Robins.
1747. Gowin Knight.
1748. Rev. James Bradley.
1749. John Harrison.
1750. George Edwards.
1751. John Canton.
1752. Sir John Pringle.
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1755. John Huxham.
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1764. John Canton.
1766. William Brownrigg.
Edward Delaval.
Hon. Henry Cavendish.
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